Using AHP and Time Series Model to Make a Better Forest Harvest Plan

Lujia Long [a], Siyuan Zhang [b], Haoyuan Liang [c]

School of Shanghai, Shanghai University, Shanghai 200444, China

[a] 1092797737@qq.com, [b] 771322978@qq.com, [c] 3253518299@qq.com

Lujia Long, Siyuan Zhang, Haoyuan Liang are co-first authors

Abstract. Nowadays, climate change is the biggest threat to the sustainable development of our society. How to deal with greenhouse gas emissions is an essential prerequisite to solve this problem. Forest carbon sequestration is an important method to offset the release of carbon dioxide. So the balance between the value of forest products and the value of allowing trees to continue growing is closely related to the amount of carbon storage. Therefore, we are expected to find an optimal management strategy that is suitable for all the forests with different factors such as economic benefits, locations and residents' satisfaction, so as to improve forest carbon sequestration as much as possible. Three models are established: Model I: Maximum the Carbon Sequestration; Model II: Score System via Analytic Hierarchy Process (AHP) and Entropy-Weight Method; Model III: Time Series Analysis Model and Comparison. For Model I, we aim at maximizing the carbon sequestration of a forest and collect a series of biological parameters throughout the America. We start with describing the relationship between forest growing stock and carbon sequestration via the conversion ratio. Afterwards, we introduce the S-size Logistic curve to fit the growing stock volume and set different harvest and plant rate to observe the change of growing stock volume so that find an optimal harvest or plant rate to maximize the carbon sequestration amount. The transition point is found as 4% for harvest rate and also 4% for plant rate. We note that although the values of these two parameters are identical, it would arouse a significant difference because they are completely different process for a forest. For Model II, we take citizen attitude and timber value into consideration besides carbon sequestration. With the existing data in Model I and the data of new factors from literatures, we construct a score system of different strategies through analytic hierarchy process and entropy method. As a result, the optimal harvest rate decreases although timber value is included, because the local citizens interviewed in the literature showed a high-level anti-harvest attitude, which is consistent with the output of our model. For Model III, we establish a time series of carbon sequestration (or growing stock, they can be interconverted without difficulty). After recognizing the specific type (ARIMA type) of this time series by solving difference equation, the data in the subsequent years can be predicted. Then, we make the varication of the autocorrelation coefficient and partial autocorrelation coefficient (ACF and PACF) the ensure that the result is reliable and dose make sense. The models constructed in this paper make full use of the data obtained from website and literatures to describe and visualize the results. They are validated by sensitivity analysis and can be widely applied to evaluate the different situations. At the end, we provide a two-page newspaper for the authorities and local residents. According to the output in this paper, we finally draw a conclusion that an appropriate harvest rate is advantageous for both forest and society, and this phenomenon is called tending felling or reproduction felling, which is widely applied in managing forest.

Keywords: Carbon sequestration; Logistic curve; AHP; Time series, ARIMA; Tending felling.

1. Introduction

1.1 Problem Background

Climate change presents a massive threat to life, in order to mitigate the effects of it, we need to take drastic action to reduce the amount of greenhouse gases in the atmosphere. We need to make efforts to enhance our stocks of carbon dioxide sequestered out of the atmosphere by the biosphere or by mechanical means. This process is called carbon sequestration. The biosphere sequesters carbon dioxide in plants (especially large plants like trees), so forests are integral to any climate change mitigation effort.

Forests sequester carbon dioxide in living plants and in the products created from their trees.The carbon sequestered in some forest products combined with the carbon sequestered because of the regrowth of younger forests has the potential to allow for more carbon sequestration over time when compared to the carbon sequestration benefits of not cutting forests at all.

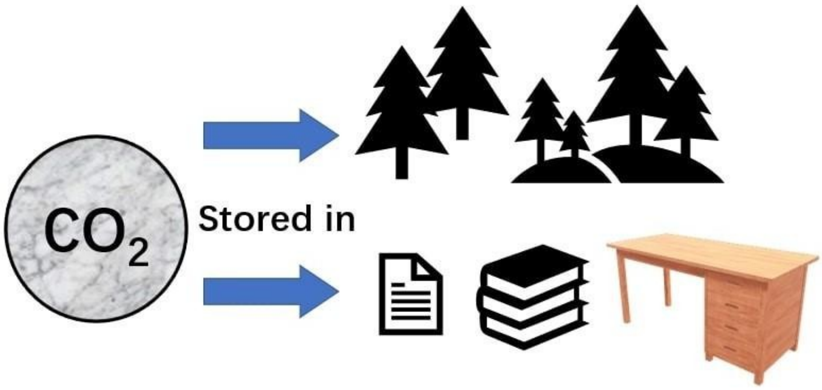

Figure 1. How the carbon is stored via forest and its products

1.2 Our Aim

Develop a carbon sequestration model to determine how much carbon dioxide a forest and its products can be expected to sequester over time. Your model should determine what forest management plan is most effective at sequestering carbon dioxide.

Develop a decision model to inform forest managers of the best use of a forest. Your model should determine a forest management plan that balances the various ways that forests are valued (including carbon sequestration).

Apply our models to various forests. Identify a forest that our decision model would suggest the inclusion of harvesting in its management plan.

2. Assumptions and Model Overview

2.1 Assumptions

To simplify the problem and make it convenient for us to simulate real conditions, we make the following reasonable assumptions.

The area of forest is fixed, since the change of it is little and insufficient to influence the result.

The growth of forest satisfies the SLogistic curve, because the scale of forest should be limited to a number according to external factors while stable internally.

The citizen attitude towards harvest is normally distributed, because it is a social issue that is proved to satisfies normal distribution.

Although the composition ratios of trees species are different, the harvested rate of them is of the same number. That is to say each kind of trees owns a same harvest rate.

The predicted data of the next 100 years is completely decided by the exsiting data without considering other emergencies.

2.2 Model Overview

Firstly, in our basic Model I, we aim at maxmizing the carbon sequestration. We start with describing the relationship between forest growing stock and carbon sequestration via the conversion ratio.

Afterwards, we introduce the S-size Logistic curve to fit the growing stock volume and set different harvest and plant rate to observe the change of growing stock volume so that find an optimal harvest or plant rate to maxmize the carbon sequestration amount.

Secondly, besides carbon sequestration, we take citizen attitude and timber value into consideration. Then we construct a score system of different strategies through analytic hierarchy process and entropy method.

At last, we establish a time series of carbon sequestration (or growing stock, they can be interconverted without difficulty). After recognizing the specific type of this time series, the data in the subsequent years can be predicted.

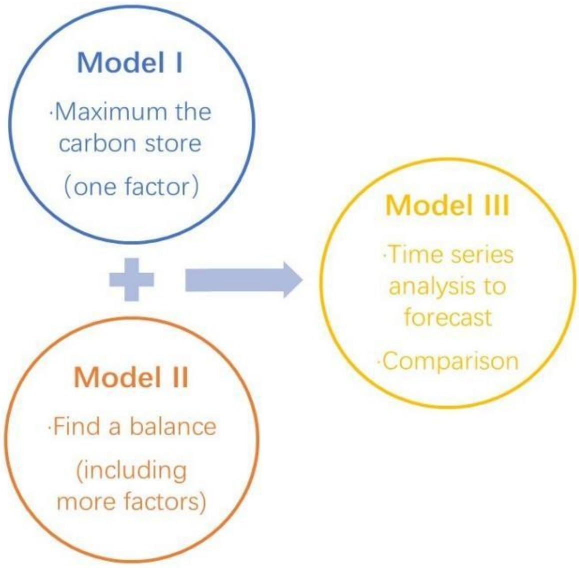

Figure 2. Modeling process

3. Notations

Table 1. We define a list of symbols used in this article.

Symbols	Description	Unit
V	Growing stock volume	$10^6 \cdot m^3$
K	Environmentalcarrying capacity	$10^6 \cdot m^3$
t	Time	year
r	Total growth rate	None
$r1$	Natural growth rate	None
$r2$	Harvest rate	None
$r3$	Mortality rate	None
$r4$	Artificial plant rate	None
C	Carbon sequestration volume	$10^6 \cdot g$
D	Growing stock per hectare	m^3/hm^3
CD	Carbon density	T/hm^3
w	Wastage rate of timber during harvest	%

We list some important parameters there, and we might set new ones but there is nothing to worry about because when a new formula appears we would explain the parameters included in it if they are not in this list.

4. Model I : Maximum the Carbon Sequestration

The Model I is aimed to maximum the growing stock volume so that to enlarge the carbon sequestration amount because these two values are positively related evidently. In our model, the growing stock volume is based on the S-shaped Logistic growth curve. And the calculation of carbon sequestration is given via a series of coefficients describing the relationship between growing stock volume and carbon sequestration amount.

4.1 Description of the Growing Stock Volume and Carbon Sequestration Amount

According to the Assumption , we only consider the influence of fixed(or in a range) nature growth rate and morality rate with the changing harvest rate and artificial plant rate based on the S- shaped Logistic growth curve [1]. Therefore, for the target forest, the growing stock volume can be expressed as

$$\frac{dV}{dT} = r \times V \times (1 - \frac{V}{K}), \quad r = r_1 - r_2 - r_3 + r_4 \tag{1}$$

and the amount of carbon sequestration can be expressed as

$$C = V \times D \times CD \tag{2}$$

4.2 Growing Stock Volume Model

The environmental carrying capability K is posed by the figure for growing stock in 2012[2], since the forest is assumed to be stable so it would converge to a fixed value. Then, the carbon sequestration amount from 1953 to 2012 is plotted as shown below in Figure 3.

From Figure 3, it is clear that when we harvest, the sum of forest and harvest is almost twice as much as the figure for not harvesting at all. It is counter-intuitive because as is known to all, harvest dose a big harm to the forest. But it dose make sense in the biology sphere if we control the harvest

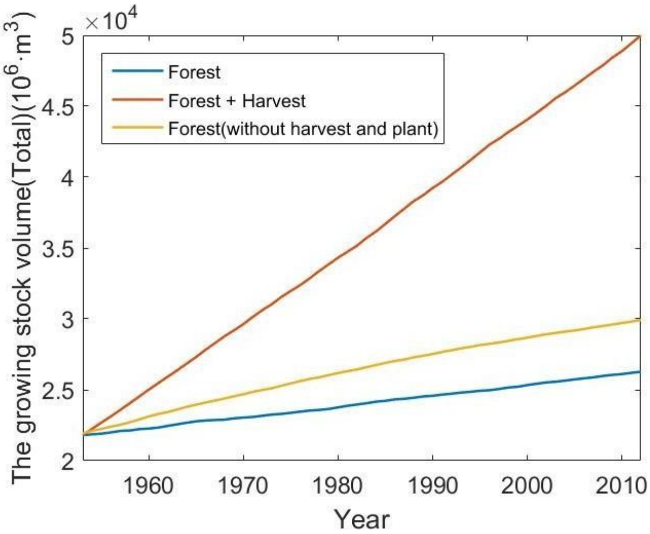

Figure 3. The growing stock volume with or without harvest

rate in a proper range. This phenomenon is called regeneration felling, which is widely applied in the management of forests to maximum the ecological and economical beneits.

4.3 Carbon Sequestration Model with Different r and Cross Validation

The nature growth rate and morality rate is not controlled by human, but we could get a series of carbon sequestration amount depending on different r2 and r4 that are able to be controlled as shown in Figure 4 & 5. However, the data we attain is part of the discrete year and the value of them did not wave significantly, so the method we set ri(i = 1, 2, 3, 4) is to choose a random number in the range of maximum and minimum value from the data. Therefore, it is necessary to do cross validation to weaken the impact of randomness. We made 10-time cross validation, the result processed is shown in Figure. ∗ Before (a) and after (b) acrossvalidation.

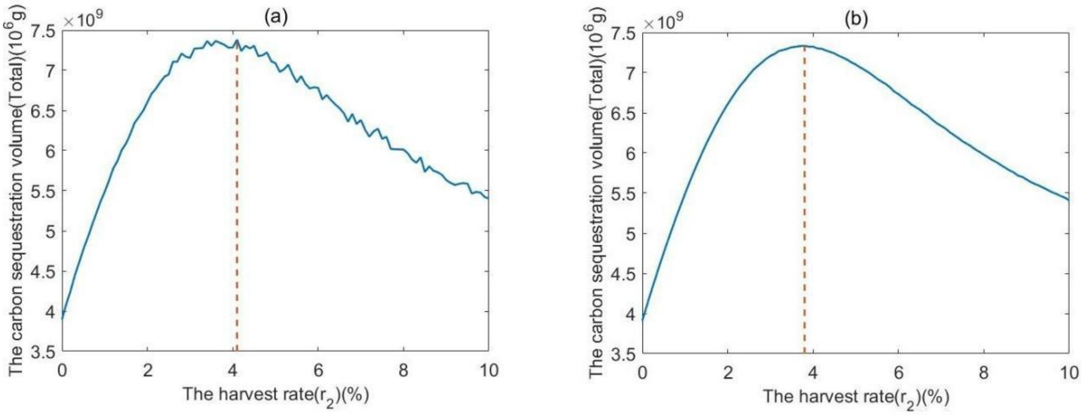

Figure 4. The carbon sequestration volume of different r2

It is clear to see that the final carbon sequestration reaches the maximum value when r≈2 4, which means we could get the highest amount of carbon sequestration when we harvest around 4% forest instead of not cutting down any trees. This strategy should be appled in the management of forest because it is able to store more carbon while producing other values like being used to buildwooden furnitures.

On the other hand, the carbon sequestration shows a monotone increasing trend with thegrowth of plant rate, which is obvious and easy to understand. However, we could not act like that in reality due to the limits of cost and land area. Therefore, it is reasonable to set the plant rate from 2% to 4%. After this interval the growth rate of carbon sequestration begins to slow down and we should

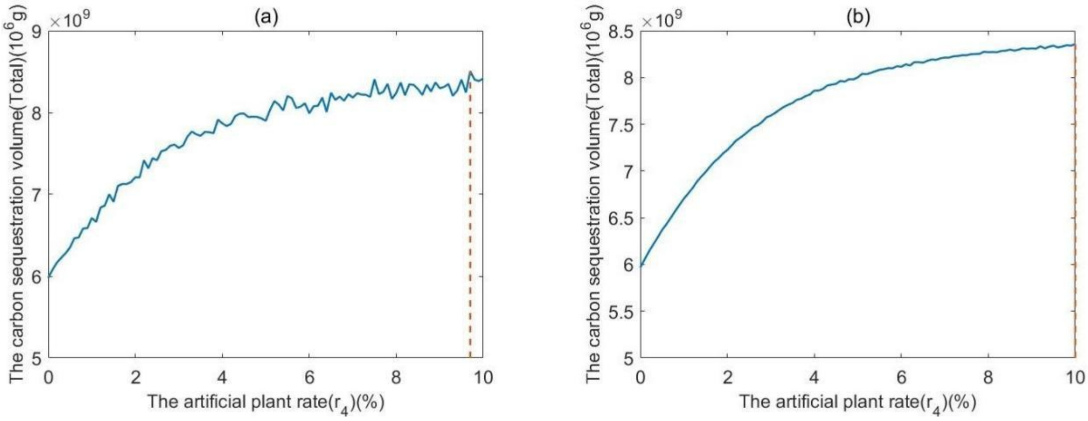

Figure 5. The carbon sequestration volume of different r4

stop planting to save resources, because it will not make a significant contribution to carbon sink subsequently and the cost performance becomes lower.

5. Model II: Score System via Analytic Hierarchy Process(AHP) and Entropy-Weight Method

In model I, we find the optimal r2 and r4 to maximum to carbon sequestration. However, it is impossible to consider a single factor to manage a forest. In other words, the strategy to maximum the carbon sequestration is not necessarily to be the best management for society.

In order to evaluate the impact of these aspects, we take citizen attitude and timber value into account in Model II. After data processing and through the analytic hierarchy process (AHP) and Entropy-Weight Method, we would finally get another optimal r2 and r4 including the two new indexes.

5.1 Data Collecting

5.1.1 Factor 1: Carbon Sequestration

In Model II, we set the target interval of r2 from 2% to 6%. Because this range is the neighbour of optimal r2 in Model I so that the carbon sequestration will not fall rapidly when r2 is changing. Except this assumption, there is no need to do extra calculation as the data about carbon sequestration is already calculated before.

5.1.2 Factor 2: Citizen Attitude

The data about citizen attitude we attain is a score system from 1 to 5 points with given standard deviation [3]. As to this problem, we regard the score system satisfies the normal distribution, which can be seen as known because the parameters of distribution are given. Then, we approximate the probablity of a point to a ratio with a small step size.

$$ratio = \frac{\int_{[r_2-0.1,\, r_2+0.1]} \frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{(x-\mu)^2}{2\sigma^2}}}{\int_{[1,5]} \frac{1}{\sqrt{2\pi}\sigma} \exp^{-\frac{(x-\mu)^2}{2\sigma^2}}} \tag{3}$$

It is worth mentioning that the score is negatively correlated with the harvest willingness and positively with the plant rate, so score one should be corresponding to r2 = 6% and r4 = 2%.

5.1.3 Factor 3: Timber Value

In Model I, we calculated the harvest amount in terms of carbon sequestration. However, timber has other values such as building furnitures and papermaking, which makes a significant economic benefits. Therefore, it is reasonable to take this factor into consideration in Model II. We should note that there is a rate of wastage during harvesting so the calculating can be done by equation (4) from

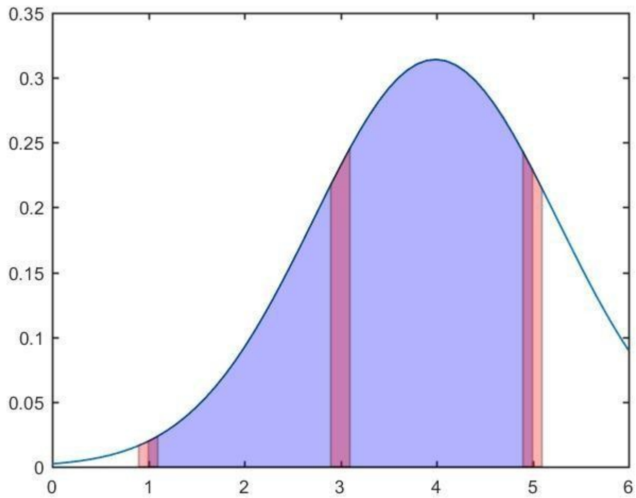

Figure 6. The normal distribution (μ = 3.99, σ = 1.27)

The data [4] [5] in Figure 7

$$TV = \sum_{i=1}^{n} R_i P_i \times (1 - w) \tag{4}$$

	Volume(10^6*inch^3)	Ratio	Price($/BF)
Red Maple	7147	0.241061792	11.125
Black Cherry	4145	0.13980707	8.5
Red Oak	3825	0.129013761	6.75
Sugar Maple	2854	0.096262817	15
Chestnut Oak	2657	0.089618187	16.5
East Hemlock	1961	0.066142742	10
Poplar	1937	0.065333243	5.65
Birch	1728	0.058283864	9
White Ash	1709	0.057643011	6.375
White Oak	1685	0.056833513	12.5

Figure 7. The tree species composition in Pennsylvania and the regular price of timber

5.2 Analytic Hierarchy Process(AHP) Model

The AHP usually divides a problem into 3 steps: Objective, Criterion and Scheme. The relationship between criterion and scheme can be calculated via a consistent matrix. And the relationship between objective and criterion will be given by entropy-weight method.

5.2.1 Consistent Matrix

Now we establish a matrix M

$$M = \begin{matrix} M_{11} & \cdots & M_{1n} \\ \vdots & \ddots & \vdots \\ M_{m1} & \cdots & M_{mn} \end{matrix}$$

Mij represents the importance or willingness of row index scheme compared to the column one. Usually the consistence test is required but due to our target method of filling the matrix, it can be ignored with no error. The consistent matrixes of carbon sequestratoin (CS), Citizen Attitude (CA) and Timber value (TV) are displayed in Figure 9 (next page).

CS	$r_2=2$	$r_2=4$	$r_2=6$
$r_2=2$	1	0.9213	1.002
$r_2=4$	1.0841	1	1.0816
$r_2=6$	0.9973	0.9259	1

CA	$r_2=2$	$r_2=4$	$r_2=6$
$r_2=2$	1	0.729627	0.987714
$r_2=4$	1.370564	1	1.353726
$r_2=6$	1.012438	0.738702	1

TV	$r_2=2$	$r_2=4$	$r_2=6$
$r_2=2$	1	0.6551	0.6093
$r_2=4$	1.5155	1	0.9256
$r_2=6$	1.6402	1.0797	1

CS	$r_4=2$	$r_4=4$	$r_4=6$
$r_4=2$	1	0.9567	0.9310
$r_4=4$	1.0488	1	0.9702
$r_4=6$	1.0712	1.0265	1

CA	$r_4=2$	$r_4=4$	$r_4=6$
$r_4=2$	1	0.3981	0.2941
$r_4=4$	2.5117	1	0.7387
$r_4=6$	3.4001	1.3537	1

TV	$r_4=2$	$r_4=4$	$r_4=6$
$r_4=2$	1	0.75654	0.66134
$r_4=4$	1.32341	1	0.86855
$r_4=6$	1.51445	1.15375	1

Figure 8. The consistent matrix of CS, CA and TV with different r2 and r4

5.2.2 Entropy-Weight Method to Get Weight of Factors

The basic idea of entropy weight method is to determine the weight according to the index variability. Generally speaking, the smaller the information entropy of an index is, the greater the variation degree of the index is, the more information it provides, so that the greater its weight is and vice verse.

We assume a matrix X,

$$X = \begin{matrix} X_{11} & \cdots & X_{1n} \\ \vdots & \ddots & \vdots \\ X_{m1} & \cdots & X_{mn} \end{matrix}$$

Xij means the figure for j-th index in the i-th scheme.

The information entropy Ej is given by

$$E_j = -\frac{1}{\ln n}\sum_{i=1}^{n} p_{ij} \ln p_{ij} \tag{5}$$

Then, the weight of each index can by calculated by

$$\omega_j = \frac{1 - E_j}{\sum_{j=1}^{m} (1 - E_j)} \tag{6}$$

According to the data in Figure 9, we are able to output a weight coefficient of each factor by entropy-weight method.

5.3 Result of Model II

Finally, with the consistent matrix describing the relationship of criterion and scheme and the weight of each factor, we get the final score by

$$\omega_j = \frac{1 - E_j}{\sum_{j=1}^{m} (1 - E_j)} \tag{7}$$

$Score \quad System(r_2)$

	weight	$r_2=2$	$r_2=4$	$r_2=6$
CS	0.312	0.318508	0.352695	0.328797
CA	0.494	0.476431	0.482357	0.041211
TV	0.194	0.241064	0.366032	0.392904
Score		0.293959	0.380652	0.325389

$Score \quad System(r_4)$

	weight	$r_4=2$	$r_4=3$	$r_4=4$
CS	0.312	0.324528	0.295595	0.240628
CA	0.494	0.351821	0.405132	0.364684
TV	0.194	0.323652	0.299272	0.394689
Score		0.222238	0.350482	0. 427135

Figure 9. The score system via AHP

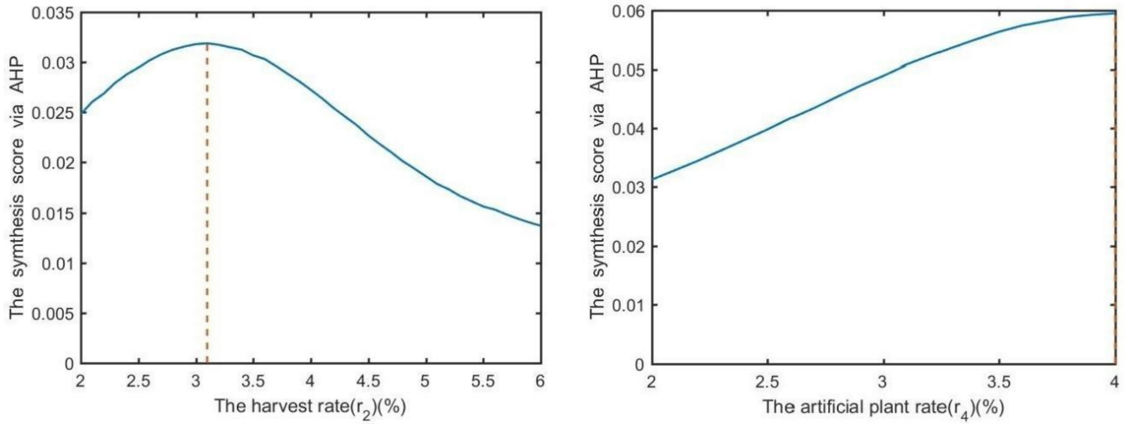

Figure 10. The symthesis score via AHP

The result we present is tested with the data of. Firstly, we use three fixed value to visualize our score system in Figure 10. But in fact, we could extent this process to the continuous case with no difficulty, which is shown in Figure 11. Compared to the result in Model I, the transition point of harvest rate r2 decreased to some extent although the timber value is considered. This can be explain by the citizen's high anti-harvest attitude. So we are supposed to higher the plant rate r4 as a respond to the public and this is consistent with our score system.

There are several parameters in this model such as the scale of forest, the tree species composition and the local residents' attitude to harvest forests, etc. Thus, as to the situation of other regions or forests, the transition point and score system can be attained easily by changing these parameters.

6. Model III: Time Series Analysis Model and Comparison

6.1 The Time Series Model

In this section, we establish a time series analysis model to predict the carbon sequestration amount over time. In model III, the data comes from [6] from 1962 to 1999. Our job in this part

includes analyzing the time series to find suitable regression parameters for this series and forecasting the next 100 years' data. And we note that the data in Model III is of Black Hills NationalForest in America.

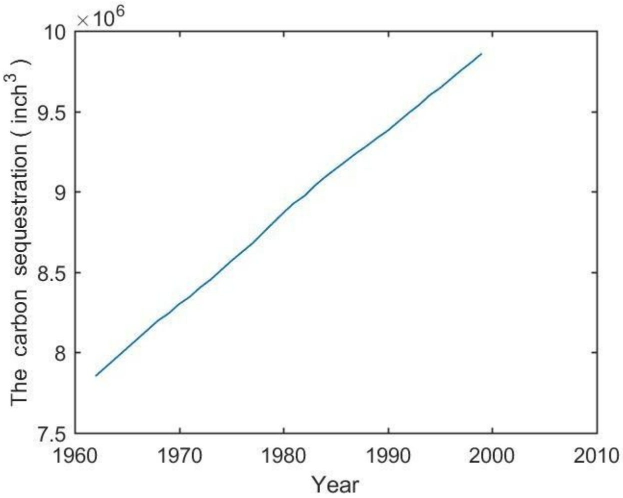

Figure 11. The carbon sequestration in Black Hills

From the Figure 12, the carbon sequestration is influenced by autocorrelation phenomenon in the time series and represents a nearly linear trend. As a result, we choose 1-th-order autoregression formula to fit the curve, namely ARIMA (0,1,0).

ARIMA (0,1,0) formula:

$$(1-L)^d y_t = \alpha_0 + \varepsilon_t \tag{8}$$

i.e.

$$y_t = \alpha_0 + y_{t-1} + \varepsilon_t \tag{9}$$

In the given formula, L is the lag factor, to be more specific, Ld = (1 − L) dyt,

L yt = yt−1; yt is the value of carbon sequestration at time t; α0 is the initial value of y; εt is the white noise sequence.

Firstly, we collect the data of time series of carbon sequestration. By solving the autoregression formula, we find the regression equation of yt:

$$y_t = 46534.061 + y_{t-1} + \varepsilon_t = 46534.061 + y_{t-1} \tag{10}$$

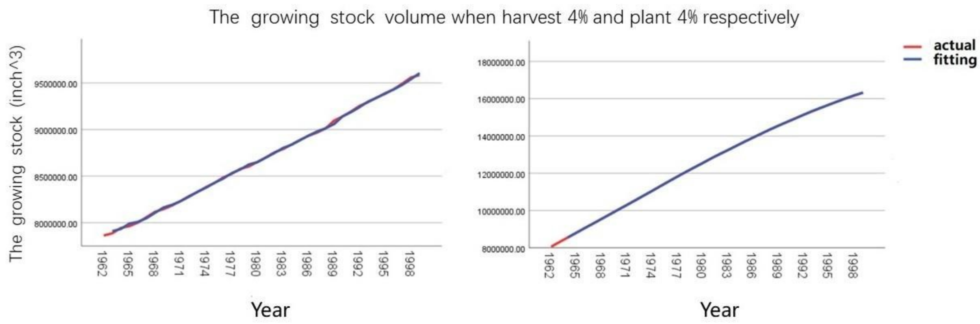

Figure 12. The growing stock volume when harvest 4% and plant 4% respectively

Through the regression equation and time series sequence, we are able to fit the data of carbon sequestration amount from 1962 to 1999 in Figure 13. From the graph, the harvest rate and plant rate are highly related with the growing stock volume and the fitting curve matches the actual situation is of great level.

6.2 The Autocorrelation Coefficient and Partial Autocorrelation Coefficient (ACF and PACF)

Like we calculated before, this time series is close to the ARIMA (0,1,0) model. So we need to make first-order difference to convert the time series to a stable one. Then it is significative to analyze the ACF and PACF because they only make sense when the time series is stable.

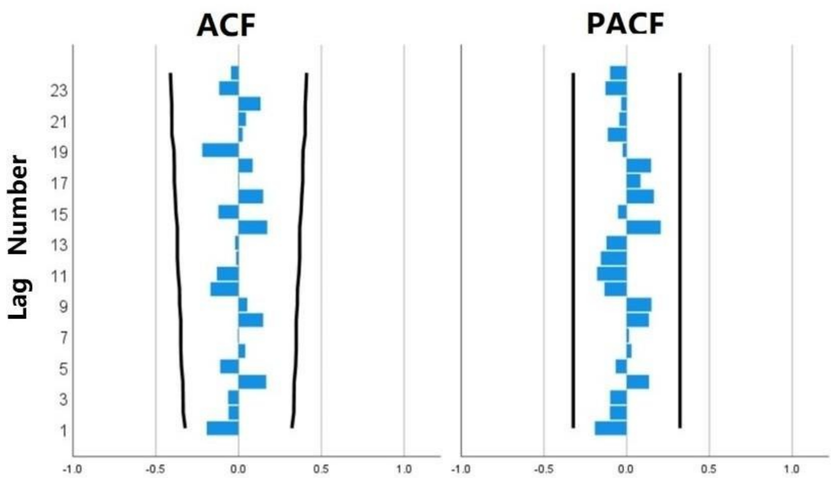

Figure 13. The ACF and PACF of the time series model

The autocorrelation coefficient and partial autocorrelation coefficient of the model is in Figure, and both of the ACF and PACF of all lag orders have no significant difference from 0. This means εt = 0 and we can not refuse the forecast result, so the ARIMA (0,1,0) is able to recognize the pattern of carbon sequestration amount over time.

6.3 Contrast of Our Model and ARIMA Model

Now the carbon sequestration amount predicted by Model I $ II and Model III in the next 100 years is shown in Figure 15, we are going to compare them to assess the plausibility and validity of our models.

The results of two models does not show much difference. Thus, it is convinced that the output of Model I is highly believable to predict the further data.

6.4 Conclusion

According to the examination of time series model, we could introduce a series of forest managing strategies below to deal with the assignments we received.

Firstly, the value of growing stock after 100 years is approximately 96 × 105 and 160 × 105 inch3, with the initial value of 78.1 × 105 in the following 100 years from 1999.

Sencondly, the optimal management can be given via the score system established by Model II. To get a final score, we need to know the biological parameters including the initial growing stock volume, natural growth rate, mortality rate, harvest rate and etc. Besides, the figure for social factors such as citizen attitude and timber value should be known as well. It is a pity that these data of Black Hills NationalForest is missing or partly missing. Fortunately, we obtain a piece of detailed data of the whole America [1]. According to it, we recommend to Authorities in the USA setting the forest harvest rate as 4% and plant rate as 4% as well.

At last, the strategy of changing time schedule can be easily achieved by adjusting the time step

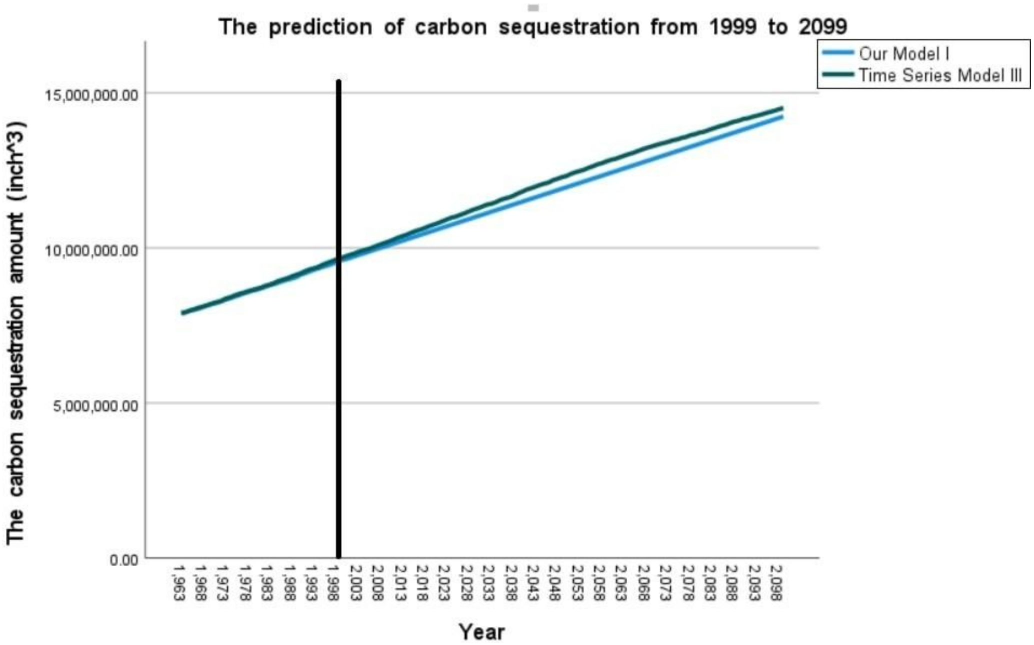

Figure 14. The prediction of carbon sequestration of Model I & III

size in our model. As to this problem, we recommend not to change the harvest interval to 10 years as more frequent harvest plan is actually more efficient for the growth of trees and the total carbon sequestration.

7. Conclusion

We analyze the problem based on a biological formula. Besides, we use entropy-weight method to evaluate the importance of each factor, which is more objective and reasonable. In the end, we predict the development trend of forests in different conditions.

By these jobs, we came to an unexpected and counter-intuitive conclusion: Felling is not always harmful to forests. However, it can be economical and beneficial for both forest and society with a modest felling strategy.

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