Review on Bose-Einstein Condensation

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Abstract. With the prevalence of quantum theory, many physicists have focused on the Bose-Einstein condensation (BEC) because it reveals the quantum behavior macroscopically. This article discusses briefly the discovery of BEC and the difficulties to achieve BEC. After that, the general procedures to achieve BEC are introduced while the mechanism of important techniques to accomplish each procedure is illustrated, such as laser cooling, trapping, and evaporative cooling. Besides, the unique physical properties of BEC are introduced in this article. Finally, the possible application of BEC in the field of an atom laser, simulation, and the atomic clock is evaluated.

Keywords: Atom laser; Atomic clock; Bose-Einstein condensation (BEC); Evaporative cooling; Laser cooling; Trapping.

1. Introduction

In recent years, the observation of Bose-Einstein condensation (BEC) provides researchers with a new perspective on quantum physics. This macroscopic quantum phenomenon has contributed to our understanding of matter-wave and opened new possibilities for investigating strange systems such as Hawking radiation from an analogue event horizon and tests of quantum mechanics with macroscopic superposition states. Moreover, the rapid development of BECs enables them to be applied in manufacturing atom lasers and hypersensitive magnetic sensors [1]. However, this recently achieved goal has a long history.

1.1 Discuss the discovery of BECs.

In 1924, without referring to classic theory, Indian physicist S.N Bose combined the light quantum hypothesis and statistical mechanics to derive Plank's formula, which is the quantum theory's starting point, and sent this paper to Einstein [2]. After reading it, Einstein was inspired by the statistical theory of light quantum and extended the theory to other particle systems such as monoatomic ideal gas. The result of their effort is the advent of Bose-Einstein statistics which governs Bose gas. Bose gas is a new concept of gas with integer spin and indistinguishable properties which are allowed to share a quantum state. Next year, Einstein predicted that at extremely low temperatures, when the de Broglie wavelength of atoms is of the same order of magnitude as the interatomic distance, the ideal gas bosons condense in the lowest energy ground state and BECs occur. At that time, the quantum theory had not fully developed and fermions and bosons had not been distinguished [3]. In 1938, London heard about the experimental work of observation of the superfluidity in liquid helium-4, below the transition at about 2.17 Kelvin. There was a peak in specific heat during the phase transition but nobody could explain it appropriately [4]. London found out that helium-4 was a Boson with zero spin and immediately connected the idea of BECs with this strange phenomenon. He pointed out that both the temperature at which superfluidity occurred and specific heat during phase transition could be well estimated by Einstein's formula. So, London concluded that the superfluidity of helium-4 was a typical BEC phenomenon. However, London had never quantitatively analyzed his thoughts about BEC in the superfluidity of helium-4 so that later physicists such as Landau barely approved of his conclusion. A few years later, Bogoliubov published a paper explaining the occurrence of superfluidity: he showed a strong interaction between liquid He atoms and the long wavelength response of those atoms was completely changed by the interaction [5]. However, because of the strong interactions between liquid He atoms, only a small number of atoms was condensed, so the condensation phenomenon could hardly be analyzed and observed. Therefore, physicists began to work on the search for BECs of weak interaction Bose gases (i.e., dilute cold atoms) with more pronounced condensation phenomena.

1.2 Discuss the Difficulties in Realizing Bose-Einstein Condensation

BEC is a pure quantum phenomenon, which means it is only a result of quantum statistics. Usually, BEC can be described in this way: atoms remain in the gaseous phase at all temperatures starting from absolute zero, and all atoms condense in a condensed cloud where atoms behave like they are in the same one-body state [6].

This phenomenon can be only comprehended quantum mechanically. In quantum physics, wave-particle duality states that every particle or quantum entity can be described as either a particle or wave. In other words, every object has its own de Broglie wavelength. It can be determined whether significant quantum effects will occur in a particle based on its matter wave wavelength. When the de Broglie wavelength of a particle is comparable to the average interparticle separation, matters will exhibit significant quantum characteristics, and the critical conditions for generating quantum effects can be calculated accordingly. So, gas atoms are treated as quantum-mechanical wave packets whose thermal de Broglie wavelength λ_{dB} can be calculated by the formula shown below.

$$\lambda_{\mathrm{d}B} = \sqrt{\frac{2\pi\hbar^2}{mk_BT}} \tag{1}$$

Where T is the temperature and m is atom mass. According to this formula, the de Broglie wavelength of gas atoms can be increased by lowering the temperature. The formation process of BECs is depicted in Fig. 1. At very low temperatures, the atoms exhibit classical particle nature, with wave nature becoming more apparent as the temperature decreases. When phase transition conditions are reached, BECs are formed, and then the pure BECs are formed at absolute zero. When temperature reaches to critical temperature T_C, de Broglie wavelengths of all atoms are comparable to the separation between adjacent atoms and then atomic wave packets will overlap, leading to the BEC phase transition [7]. At absolute zero, all atoms are in the ground state of the system, forming a pure BEC, which can be described by a single wave function.

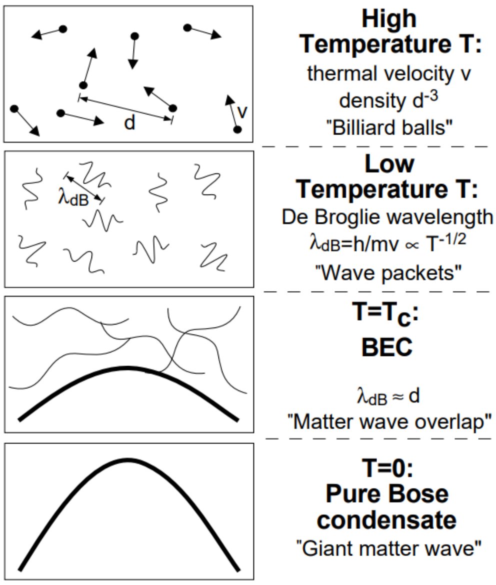

Fig. 1 Graphical explanation of criterion for Bose-Einstein condensation [7].

So generally, BECs can be achieved by lowering the temperature and for ideal Boson gas, the condition for BEC is $n\lambda_{dB} \geq 2.612$ where n is the atomic density [8]. However, the density of Boson gas cannot be as high as usual gas because classic phase transition will involve when temperature decreases and then affect BECs. Normally, the atom density should be controlled within 10^{13}—10^{15} cm^{-3}. As a result, the dilute Bose atomic gas must at least reach a low temperature of the order of μK before it is possible to transfer to BECs. Therefore, finding a way to cool the atomic gas to an extremely low temperature is the only way to realize the Bose-Einstein condensation of the sparse atomic gas.

2. The critical techniques used in BECs.

To achieve BECs, it is necessary to cool atoms to an extremely low temperature of the order of μK and compress them in a region where atoms are thermally isolated from all material walls [7]. Therefore, it is crucial to choose appropriate cooling and trapping techniques to accomplish these tasks.

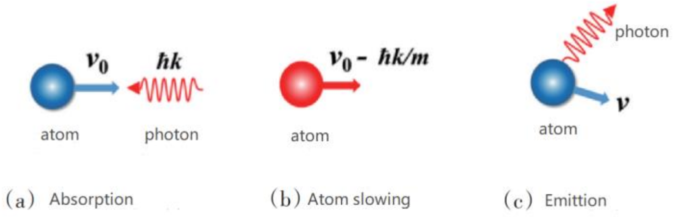

Fig. 2 The fundamental principle of laser cooling [9]

2.1 Laser cooling

Laser cooling relies on the mechanical action of light on atoms and this phenomenon was first discovered by Frisch who used spectral lamps to deflect sodium beams in 1933 [10]. Fig. 2 shows the basic principle of laser cooling in one dimension. When an atom with velocity v_0 collides with a photon which travels oppositely to the atom and has the frequency enabling electrons in the atom to be excited, the atom will absorb the photon and then its velocity decreases to $v_0 - \frac{\hbar k}{m}$. The excited atom will then spontaneously emit the photon in random directions within finite time. After back to the ground state, the atom will absorb photons again and this loop can be processed n times. When n is an extremely large number, the overall emitting momentum is close to zero due to random directions of the emitted photon. Thus, the atom is slowed down. Based on this mechanism, many cooling methods are derived. In most of BEC experiments, cooling methods such as Zeeman slowing, Doppler cooling and optical molasses are mainly used for pre-cooling atoms. Atoms are usually slowed by Zeeman slowing first and then further cooled and trapped by a magneto-optical trap (MOT).

2.1.1 Zeeman slowing.

According to Fig. 2, an atomic beam can be slowed by a laser beam with the same frequency of electronic transition of the atoms. However, when atoms start to slow, the frequency of laser beam will no longer match electronic transition frequency and the deceleration effect of light on atoms cannot be sustained. Therefore, the Doppler shift needs to be compensated continuously during deceleration. The method of Zeeman slowing uses an inhomogeneous magnetic field to adjust electronic transition frequency of atoms by Zeeman effect. As shown in Fig. 3, the magnetic field varies with distance, so the electronic transition frequency of atoms is adjusted and the atoms are decelerated by the continuous interaction between photons (laser beam) and atoms.

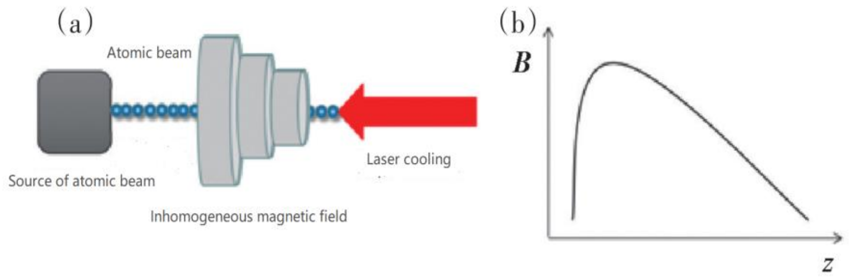

Fig. 3 The Zeeman slowing schematic (a) and magnetic field curve (b) [9].

2.1.2 Doppler cooling and optical molasses

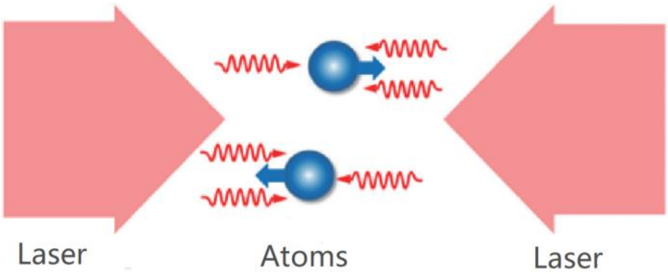

Fig. 4 Diagram of Doppler cooling in one dimension [9]

Doppler cooling was first proposed by Hänsch and Schawlow of Stanford University in 1975 [11]. A decade later, based on the mechanism of Doppler cooling, Steven Chu's group at Bell Labs in the USA reported the implementation of a new method of laser cooling, called "optical molasses" [12]. His group used three pairs of counter-propagating circularly polarized laser beams to shoot on atomic clouds in the directions of the x, y, and z-axis. With this method, they were able to cool sodium atoms to the temperature of 240 μK which is the temperature laser cooling technique can cool and is called the "Doppler limit". In 1987, The National Institute of Standards and Technology of the United States repeated Steven Chu's experiments. The experimental results showed that they cooled sodium atoms to 43 μK, which is far below the Doppler cooling limit [13]. This means laser cooling can break through the Doppler cooling limit and make the atoms cool to a lower temperature. On this basis, Steven Chu [14] and Cohen-Tannoudji of École Normale Supérieure [15] in Paris independently proposed relevant theoretical explanations. In the theoretical analysis, the polarization gradient of the cooling laser, the hyperfine structure of atoms, the optical frequency shift and optical pumping are considered, and the cooling mechanism is called sub-Doppler cooling.

The mechanism of optical molasses is base on Doppler cooling whose one-dimensional principle is illustrated in Fig. 4. Two laser beams irradiating in opposite directions and of the same frequency which is slightly below an electronic transition frequency in an atom. If atoms are assumed to be stationary (which is impossible), there will be no mechanical action of light on atoms due to no absorption and emission of photons. If atoms are moving in one direction, because of the Doppler effect, atoms will receive higher frequency from the laser beam in the opposite direction of its moving and lower frequency from the laser beam in the direction of moving. And only a higher frequency is close to the electronic transition frequency of atoms, so the cooling effect occurs after atoms absorb and emit photons.

However, Doppler cooling and optical molasses only produce a decelerating effect and do not provide restoring force toward the center. Atoms will be pulled by gravity and gradually escape when they are being cooled. So, MOT is designed for both cooling and trapping the atoms in most BEC experiments. The setup of MOT is shown in Fig. 5. MOT consists of three pairs of counterpropagating red detuned laser beams in three directions, which is the optical molasses, and two coils with the opposite current direction. The aim of optical molasses is to cool the atoms down to the doppler limit and two coils creating a spatially varying magnetic quadrupole field aim to trap the atoms in the center. The magnetic field causes the change in the electronic transition frequency of atoms due to the Zeeman effect. This change in transition frequency will increase with the radial distance from the center of the trap. Therefore, those atoms away from the center will receive frequency close to their electronic transition frequency from the laser and atoms are more likely to be kicked toward the center of trap.

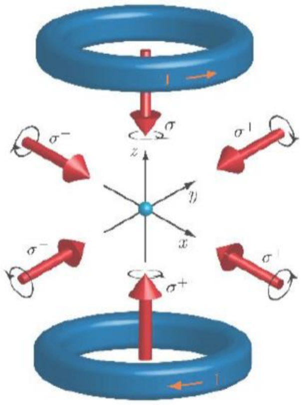

Fig. 5 Diagram of setup of magnetic-optical trap (MOT) [16]

2.2 Trapping and evaporative cooling.

After laser cooling, atoms still do not reach the temperature at which BECs can be observed clearly. Therefore, further cooling on atoms is needed and evaporative cooling is the most effective cooling method to cool atoms to such low temperature. Besides, atoms should be compressed in a region where their wave packets overlap to make BECs happen. So far, only magnetic trap can meet both cooling and holding condensates requirement [7].

2.2.1 Magnetic trapping.

Many neutral atoms have magnetic dipole moments, so they will behave like a tiny bar magnet which experience force from magnetic field [17]. According to Maxwell's electromagnetic theory, an inhomogeneous magnetic field B has a force F on the magnetic dipole moment μ with the equation $F = -\nabla(\mu \cdot B)$. Thus, atoms can be trapped at the minima of the magnetic field. The simplest magnetic trap is a quadrupole trap formed by an inverse Helmholtz coil with a central field of zero strength. The advantages of a reverse Helmholtz coil are the high trapping force and the long trapping time. However, because the magnetic field at the center of the trap is zero and the direction of magnetic field in its vicinity varies strongly, the spin of atoms passing through the central zero-magnetic-field point could be flipped. If spin flips happen, the zero potential point becomes the highest potential energy point instead, and the atoms will escape from the trap. These spin flips are referred to as “Majorana flops” which is the biggest drawback of magnetic trap [18]. To solve this problem, many methods are designed and only time-orbiting-potential trap (TOP) and optically plugged trap are introduced below.

2.2.2 Time-orbiting-potential trap (TOP)

TOP is completed with two parts, as shown in Fig. 6, first part is quadrupole trap configuration which is same as magnetic trap mentioned above. The second part is a rotating magnetic field **B** with a certain frequency in the x-y plane. The frequency of the rotating field is much higher than the orbiting frequency of the atoms, but much lower than the Larmor frequency [7]. So, after adding rotating magnetic field in this way, the original quadrupole potential rotates in space with time. After taking the time average, it is resulting time-orbiting-potential trap is harmonic and its zero potential point is above the origin along a circular orbit [20]. By this method, the minimum potential point in TOP is non-zero and the potential change across the minimum point is smooth, which greatly reduces the possibility of Majorana flops and reduces atom loss.

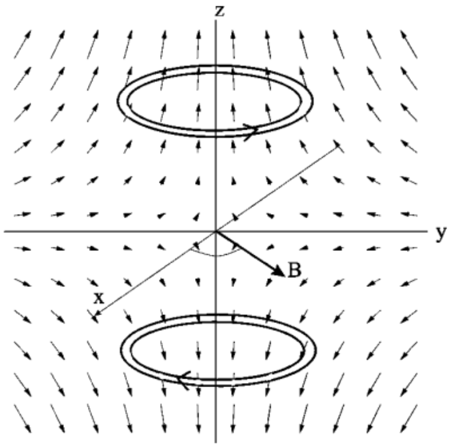

Fig. 6 Scheme of time-orbiting-potential trap [19].

2.2.3 Optically plugged trap.

Optically plugged trap is another design to improve the drawback of magnetic traps. It was firstly designed by Ketterle's group to demonstrate the first BEC in sodium in 1995 [21]. His group used a tightly focused blue-detuned laser beam to plug the zero-potential hole, so the optical force will repel atoms from center of the trap. The scheme of optically plugged trap is shown in Fig. 7. Generally, as shown in b), there is a beam of blue-detuned laser passing through center of trap. So, the resulting minimum potential becomes a ring where the spin of atom hardly flips. Another type of optically plugged trap uses elliptical blue-detuned laser beam as shown in c). This shape of laser beam breaks the symmetry of minimum potential ring and creates two distinct minima. For each minimum, 3 distinct frequencies create 3-dimensional, harmonic confinement which make sure atoms will be trapped having less Majorana flops. In addition, the frequency of the laser is selected away from the resonant frequency of atoms, avoiding the heating effect caused by resonant photon scattering. However, the position and shape of trapped atoms in the optical plugged trap are easily affected by the position of the plugged laser beam.

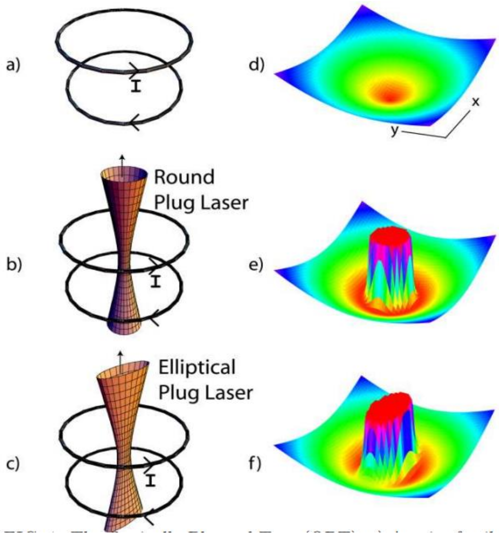

Fig. 7 The optically plugged trap. a) A quadrupole trap. b) An optically plugged trap realized by focusing blue-detuned laser beam to its center. c) An optically plugged trap realized by focusing elliptical blue-detuned laser beam to its center. (d-f) potential energy verses position in x-y plane corresponding to trapping geometrics of (a-c) [22].

2.2.4 Evaporative cooling

This is the key technology to cool the atoms in the trap to a temperature below the order of μK. In the magnetic trap, atoms with higher energy can reach the place where the magnetic field is stronger. In the experiment, a radio frequency field with an appropriate frequency is selected to make the atoms with higher energy transfer to an untrapped state and make those atoms escape from the magnetic trap. The remaining atoms reach a lower temperature thermal equilibrium through elastic collision, and the frequency of the scanning radio frequency field causes the atoms to leave a part in order of energy from large to small, and the last remaining is ultracold. The atomic gas evaporative cooling technology is named because its basic idea is like that of cooling water by evaporation in life. The key to this technology is a very high background vacuum degree and a large elastic collision cross section, a high atomic density.

3. Physical Properties of Bose-Einstein Condensates

The atomic systems in Bose-Einstein condensation state are called Bose-Einstein condensates (BECs) whose physical properties are dramatically different from that of normal atoms. Apart from superfluidity and superconductivity, there are some other physical properties in BECs.

When temperature is lower than critical temperature T_C, The heat capacity of an ideal Bose gas is expressed in the equation below [23].

$$C_v = 1.926Nk\left(\frac{T}{T_C}\right)^{\frac{3}{2}} \tag{2}$$

As shown in Fig. 8, when T=0, C_v=0; when T=T_C, C_v has maximum value $1.926Nk$, which is bigger than classic heat capacity of atoms $\frac{3}{2}Nk$. When $T > T_C$, the properties of an ideal Bose gas approximately obey the Boltzmann distribution so C_v approaches to $\frac{3}{2}Nk$ when temperature increases. At the temperature of T_C, ideal Bose gas undergoes a phase transition.

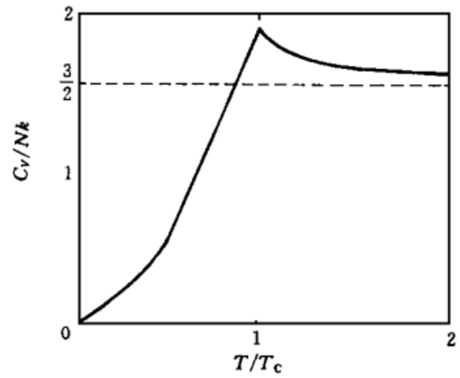

Fig. 8 Phase transition curve of ideal Bose gas [23]

Phase coherence is another important physical property of BEC. By dividing the sodium condenses into two parts, the Ketterle's team observed the interference pattern between them, proving the existence of the phase coherence phenomenon [7]. They divided the sodium condenses into two parts by the repulsive force of the laser beam on the atoms. After the condensed state was divided into two parts, it was expelled from the trap and fell freely in the gravitational field. After 40 milliseconds, the phase-related atomic cloud of the two parts was in the process of falling and expand to each other. Because the phases between them are the same, there is an interference phenomenon in the area where the atomic clouds are superposed. Fig. 9 shows laser absorption image of the interference pattern of BECs. The width of absorption image is 1.1 mm, and the fringe spacing of the interference pattern is 15 μm, which corresponds to a very large wavelength of matter. The atomic de Broglie wavelength at room temperature is only 0.05 nm, which is smaller than the atomic scale. So, this is an important condensed-state phase coherent phenomenon [7].

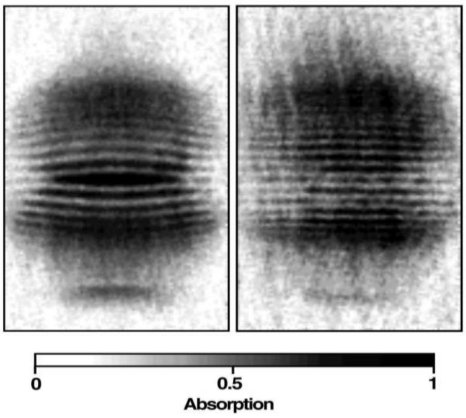

Fig. 9 Interference pattern of two expanding sodium condensates [7].

4. Applications of BEC.

4.1 Atomic clock

Time frequency is the most accurate physical quantity to be measured. Currently, the cesium fountain reference clock used to reproduce the second definition has an uncertainty of $(3\sim6) \times 10^{-16}$ [24]. Bose-Einstein condensation plays a vital role in the new generation of high-end atomic clocks.

The cesium atomic clock is mainly composed of a resonant cavity, a microwave feeding mechanism and a frequency-adjustable crystal oscillator. The cesium atomic vapor in the hyperfine energy state ($F_1 = 3$) is sent into resonant cavity. If the fed microwave frequency is close to ν cesium (the transition frequency between $F_1 = 3$ and $F_2 = 4$), the cesium atoms in resonant cavity will transfer from $F_1 = 3$ to $F_2 = 4$. By detecting the proportion of transition atoms, it is possible to identify whether resonance is achieved. Fine-tune the crystal oscillator frequency, when the maximum transition occurs, the microwave frequency at this time is equal to ν cesium. The oscillating signal of this frequency is sent to the electronic counter, and every time the counter reads out 9192631770 cycles, it means that the time has passed 1 s. At present, the cesium atomic vapor used by the metrology departments of various countries has been cooled by laser. If the cesium atomic vapor is directly taken from the BEC, the signal of the resonance transition will be sharper, and the time measurement accuracy is expected to be improved by another 100 times [25].

4.2 Simulation of black-hole radiation

Since the discovery of black hole, it is hard to observe black-hole radiation. The features of this radiation can be analyzed now by using BECs simulating black hole.

Quantum physics states that, instead of empty, the vacuum of space is filled with particles in pairs which will destroy themselves immediately and overall appear to be empty. Based on General Theory of Relativity, Hawking proposed that black holes are not purely black, but emit thermal radiation at a temperature that depends on their mass. He also investigated what happens when paired particles are close to event horizon of black holes, which is the boundary beyond which nothing can escape, even it is light. He found paired particles could prevent destroy themselves when they are pulled apart by tidal forces of gravity, which means as illustrated by Fig. 10a, one particle is absorbed by black hole and another is emitted into space in the form of thermal radiation which known as Hawking radiation [26].

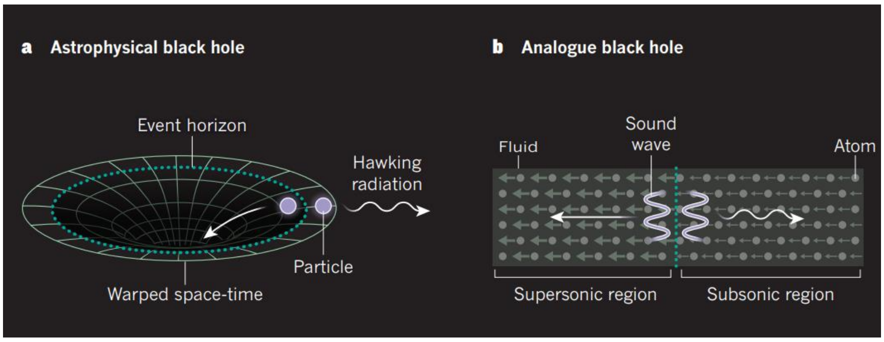

Fig. 10 Simulating black hole using BECs. a, A brief illustration of Hawking radiation. b, An analogue of black hole based on a flowing fluid in BEC phase [26].

Silke in University of Nottingham came up an idea using flowing fluid in BEC phase to simulating black hole and using sound wave to analogue particles [26]. Fig. 10b shows the details of this simulation. There are two regions where fluid flow in different speed. All the fluid travels to left and those in the left region travels supersonically, those in the right region travels subsonic. So, the difference in fluid speed analogues to difference in gravity force produced by black hole and the boundary between supersonic region and subsonic region acts like event horizon of black hole. Silke found that when there is a pair of sound produced close to the boundary, one of the sound waves is absorbed by the supersonic region and another sound wave is emitted to opposite direction, which proves the existence of Hawking radiation.

4.3 Atom laser.

The reason of why laser is so special is that it uses coherent light waves—all the light within the laser vibrates in perfect sync. And quantum mechanics shows that particles like atoms should also be considered waves. Ketterle’s team in MIT showed the possibility to build an atom laser by demonstrating the interference pattern of BECs [7]. This means atomic wave packets in BEC phase are coherent and BEC is the core technology to realize atom laser. However, BEC is very fragile. They will be instantly destroyed when light hits them. But the presence of light is crucial in preparation processes: to cool matter down to a millionth of a degree, a laser is needed to cool its atoms. Therefore, in most cases, BEC can only burst briefly and cannot be maintained continuously.

Florian’s team in University of Amsterdam have solved this problem by changing the cooling routines. Whereas in previous experiments the cooling processes of the atoms was all done in same place, in their experiment they let atoms pass through a path where atoms are cooled down when they move. Finally, the ultracold atoms reach the heart of the experiment, where they can be used to form coherent matter waves in the BEC phase. While these atoms are being used to atom laser, new ones are already on their way to complement the BEC. In this way, BECs can be produced continuously [27]. Having solved the key problem of creating a continuous Bose-Einstein condensate, the researchers' next goal is to use a laser to generate a stable output beam of matter. Once their lasers can run permanently and produce stable beams, nothing will stand in the way of technological applications anymore, and atom lasers may begin to play an equally important role in technology as ordinary lasers.

Atom lasers will further improve the accuracy of atomic clocks, and have seen very bright application prospects in the fields of chip technology, precision measurement and nanotechnology. Condensates have good coherence and can be used to develop high-precision atomic interferometers, measure various potential fields, and measure gravitational field acceleration and acceleration changes. Atom lasers may also be used in the manufacture of integrated circuits. Condensates have also been proposed to process quantum information.

5. Conclusions

Since the concept of Bose-Einstein condensation was proposed by Einstein in 1925, it has drawn many attentions from the researchers and physicists. We hardly image the behavior of BECs because they are microscopic objects but behave coherently as a unity. Therefore, it is vital to investigate BEC that it could provide a valuable comprehension of the behavior of atoms in quantum field. This article reviews the development and basic theories of achieving BEC. The article also introduces the mechanism of laser cooling, such as Zeeman slowing, Doppler cooling and magnetic-optical trap (MOT); trapping techniques: magnetic trapping, time-orbiting-potential trap (TOP) and optically plugged trap; and also evaporative cooling, which are the important techniques to achieve BEC. As for application, BEC has unique properties such as superfluidity and phase coherence, which give it an edge in application of simulation of black hole and atom laser. As investigation on Bose-Einstein condensation continues, more and more quantum behavior will be revealed. These new theories will deepen the understanding of many-body physics and drive the development of applications such as precision measurement, quantum information, and quantum phase transitions.

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