Research on Shape Adjustment of Large Aperture Reflector Based on Differential Equation

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Abstract. The FAST active reflector can transform the reference state's spherical surface into the working condition's parabolic surface by adjusting the system so that the radio telescope has high sensitivity. This paper mainly studies the structure of the ideal paraboloid in the working state and the influence of various factors of the adjustment system on signal reception. The primary cable node distribution model is established based on the paraboloid equation and coordinate rotation. The optimization model, the difference method and the stochastic simulation are used to solve the problem. Firstly, the ideal paraboloid focus selection range is determined as an interval near the feed cabin according to the properties of the focused parallel rays of the paraboloid and the receiving area as the central disk. At the same time, considering the adjustment factors of the reflective panel, the ideal paraboloid endpoint is given. With the constraint interval, an optimization model can be established for the two variables, the distance d from the focal point to the feed cabin and the z-coordinate b of the vertex of the ideal parabola, with the perfect paraboloid and the reference sphere as close as possible as the objective function. It is formed by rotating around the central axis. Therefore, to simplify the operation, the plane because is selected to analyze the ideal parabola, and finally, the optimal solution is obtained by using the two-dimensional grid search algorithm as $b = 300.23$, $d = 1.50$, The resulting ideal paraboloid equation is $z = \frac{x^2 + y^2}{566.12} - 300.23$. The FAST active reflector can transform the reference state's spherical surface into the working condition's parabolic surface by adjusting the system so that the radio telescope has high sensitivity. This paper mainly studies the structure of the ideal paraboloid in the working state and the influence of various factors of the adjustment system on signal reception. The primary cable node distribution model is established based on the paraboloid equation and coordinate rotation. The optimization model, the difference method and the stochastic simulation are used to solve the problem.The average vector of each reflective panel can be obtained according to Cramer's law. Then according to the reflection properties of the wave, the direction vector of the reflected electromagnetic wave can be obtained from the incident electromagnetic wave and the average vector. The end of the panel is projected to the feed along the direction of the reflected electromagnetic wave. The cabin's plane can obtain the area reflected from each panel to the plane of the feed cabin. To simplify the calculation and improve efficiency, the Monte Carlo stochastic simulation algorithm is used to obtain the reflected signal receiving ratio of the working paraboloid. The receiving ratio is that the signal receiving the effect of the working paraboloid is much better than that of the reference reflective spherical surface.

Keywords: Focusing, 2D grid search algorithm, 3D coordinate rotation, differential evolution algorithm, stochastic simulation algorithm.

1. Introduction

Among all the radio telescopes in the world at present, China Sky Eye (FAST) ranks first in single aperture and sensitivity, and my country has its intellectual property rights. The completion and use of FAST have greatly encouraged the development and breakthrough of my country's scientific innovation.

When FAST is working, the reflective panel is controlled by the pull-down cable, actuator and other devices to ensure that the reflective panel reflects the astronomical information in the universe into a circle with a radius of 0.5 meters centred on the feed cabin so that the feed cabin can receive valid information. At this time, the shape of the set of reflecting panels is approximately a paraboloid of revolution, and its diameter is 300 meters.

When FAST is not working, the collective shape of the reflection panel is a spherical surface with a diameter of about 600 meters, called a spherical reference surface, and its diameter is 500 meters in length. The position of the centre of the circular domain of the feed cabin is limited to the focal plane. The focal plane is a sphere with the centre of the reference sphere as the centre. The ratio of the radius of the focal plane to the radius of the reference sphere is 0.534. The actuator is located on the extension of the radius of the reference sphere and can be extended or shortened in this direction.[1-3]

To sum up, it can be seen that how to control the work of the actuator and then indirectly control the pull-down cable to adjust the reflective panel to the appropriate position to construct the working paraboloid is the foundation and focus of ensuring the best quality of star electromagnetic wave reflection and reception.

A coordinate system is established by taking the centre of the reference sphere as the origin and the vertically upward direction as the z-axis. When the star to be measured is located on the z-axis, the various requirements of FAST work are integrated, the position of the reflective panel is adjusted, and the ideal working paraboloid is solved. To make the actual reflective surface as close as possible to the perfect working paraboloid, a related model is established to solve the telescopic value of the corresponding actuator. Find the essential parameters of the ideal working paraboloid.

The ratio between the received information and the reflected signal is calculated using the problem-solving process and the required results. At the same time, it is compared with the ratio of the information received in the reference state.

1.1. Problem Analysis

Considering that when the observed star is directly above the reference sphere, observe the cross-section of the reference sphere and the undetermined ideal paraboloid, and combine the modifiable factors of the reflective panel to search for the distance b of the parabola vertex from the centre of the sphere and the proper focus of the paraboloid with the two-dimensional grid search method. The error d that the direction of the feed cabin floats up and down makes it possible to find the ideal paraboloid with the minor deformation from the change of the spherical reference surface under the condition that the absolute value of the difference between each point on the parabola and the reference circle does not exceed 0.6m.

When there is a given angle between the star and the center of the sphere, the determined ideal paraboloid can be considered as the shape and size of the paraboloid at each grade and other parameters do not change, and the vertex coordinates and focus coordinates of the original paraboloid are rotated and transformed to the position of the center of the angle mapping, Determine the central cable nodes and their numbers that need to be moved and changed by approximating the spherical diameter of the paraboloid, determine the quasi-planar equation by the rotation angle, and establish the ideal paraboloid equation according to the distance from the point on the paraboloid to the focus equal to the distance from the issue on the paraboloid to the quasi-plane; In the process of extending and retracting the pull-down cable of the actuator, considering the two inclination changes of the telescopic amount of the actuator and the inclination of the pull-down cable, the transformation relationship is listed, and there are multiple variables; in order to make the adjusted effect more realistic The optimal reflective panel adjustment model is established to minimize the change in the length of each side of the triangular plate after node adjustment and to make the adjusted nodes closest to the ideal paraboloid.[4,5]

It has been determined that the scheme has obtained the acceptance ratio. The electromagnetic wave reflected by three points is used to replace the electromagnetic wave emitted by the entire reflecting panel. The direction vector of a similar incident electromagnetic wave is known, and the average vector of the triangle plate is determined by the coordinates of the three primary cable nodes. Then select the straight line that reflects the electromagnetic wave at this point. The three consecutive lines defined by the three points intersect with the plane where the feeder cabin is located at the other three points, randomly simulate the internal point of the m-th triangle at the intersection point, and record the simulated projection of the triangular reflection panel in the feeder. The number of times n in the source cabin area and then the acceptance ratio of the reflective panel is obtained; the acceptance ratio is calculated for each reflective panel in the paraboloid in turn, and the average is taken as the acceptance ratio of the adjusted source cabin.

1.2. Model Assumptions

1. Assume that the lower end of the actuator is fixed and the direction is fixed;
2. The influence of wind and other reasons on the expansion and contraction of the actuator, the pull-down cable and the plane vibration are not considered;
3. It is assumed that the triangular reflector is fixed on the reference sphere in the boundary area other than the paraboloid;
4. The influence of various frictions of the system on the system is not considered;

2. Definition and notation

Table 1: Description of some symbols

Symbol	Symbol Description
b	The distance from the lower end point of the ideal paraboloid to the center of the sphere
k	Focal Aspect Ratio
p	Focal Length
z_p	Paraboloid equation
z_c	Equation of the base sphere
Vd	Base state actuator lower end point
Vc	Base state actuator upper end point
Vs	main index node
Vc'	Adjusted base state actuator upper end point
Vs'	Adjusted primary index node
e	distance difference between nodes
d	distance difference from point to focal point and quasi-plane
n	point within the feed chamber area
m	Number of times to generate random numbers

We can see Definition and notation in Table 1.

3. Model establishment and solution

$$z = \frac{1}{2p}x^2 - b, x \in [-\frac{c}{2}, \frac{c}{2}] \tag{1}$$

Where c is the length of the lighting area, the value is 300m. Then the equation of the ideal paraboloid obtained by rotating around the z-axis is:

$$z = \frac{x^2 + y^2}{2p} - b \tag{2}$$

The functional relationship of p and b can be obtained as:

$$p(b) = 2[b - (1-k)R + d] \tag{3}$$

Then the ideal parabola equation can be transformed into:

$$z=\frac{1}{4[b-(1-k)R+d]}x^2-b,x\in[-\frac{c}{2},\frac{c}{2}] \tag{4}$$

The equation of the base circle is:

$$x^2+z^2=R^2,x\in[-\frac{e}{2},\frac{e}{2}] \tag{5}$$

Among them, e is the diameter of the spherical surface, and the value is 500m. can be transformed into:

$$z=-\sqrt{R^2-x^2},x\in[-\frac{e}{2},\frac{e}{2}] \tag{6}$$

Denote H as (x_h,z_h), take $x_h=-\frac{c}{2}$, and substitute it into the ideal parabola equation to obtain z_h, connect and extend the straight line CH and the reference circle to intersect at point E, then the equation of the straight line CE is:

$$z=\frac{x_h}{z_h}x \tag{7}$$

By combining the equation of the straight line CE with the equation of the base circle, the coordinates of the point E can be solved as (x_E,z_E).

Since the radial expansion and contraction range of the actuator is ± 0.6 m and the maximum deviation between the ideal parabola and the reference circle is at the endpoint and the symmetry point, that is $|EF|\le 0.6,|AB|\le 0.6$

Due to the repeated operation of the adjustment process of the reflective panel, the cable net structure will be fatigued, which will lead to the damage of the cable net structure. Therefore, in order to enhance the fatigue resistance of the cable net structure, the variation range of the ideal parabola and the reference circle should be within the relevant constraints. conditions to be as small as possible.

where z_p is the z-coordinate of the ideal parabola, and z_c is the z-coordinate of the reference circle.

In summary, the established optimization model is

$$\min\int_{-\frac{c}{2}}^{\frac{c}{2}}(z_p-z_c)^2\mathrm{dx}$$

$$\begin{cases} z_p=\frac{1}{2[b-(1-r)R]}x^2-b \\ z_c=-\sqrt{R^2-x^2} \\ \sqrt{(x_E-x_H)^2+(z_E-z_H)^2}\le 0.6 \\ b-R\le 0.6 \end{cases} \tag{11}$$

The integral expansion is a binary function of $\overline{b}$ and $\overline{d}$. For this function, we use a two-dimensional grid search algorithm to solve it. The final obtained is the z-coordinate of the vertex of the ideal parabola under the optimal target and the distance between the focus and the feed cabin.

According to the above steps, the optimal solution $b=300.23$, $d=1.50$, and the objective function are obtained by MATLAB calculation D=17.7725.

The final ideal parabola is shown in Figure 1, and the equation is:

$$z = \frac{1}{566.12}(x^2 + y^2) - 300.23 \tag{12}$$

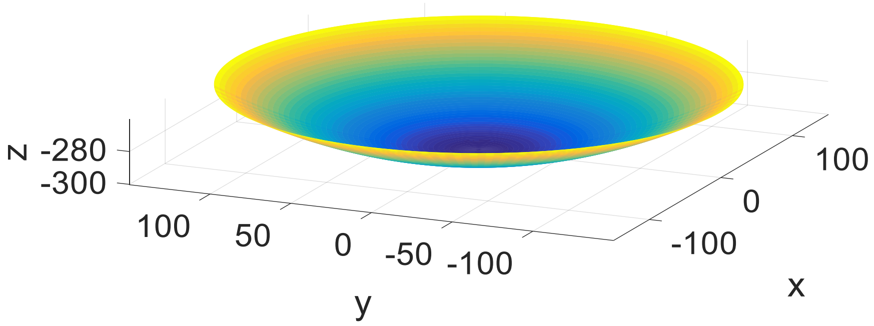

Figure1: Problem one ideal paraboloid diagram

As shown in the Figure 2, if the coordinates of the three vertices of a triangular reflecting panel are $L(x_1, y_1, z_1)$, $M(x_2, y_2, z_2)$, $N(x_3, y_3, z_3)$, Let the direction vector of the incident electromagnetic wave be $\vec{a} = (x_r, y_r, z_r)$, which is determined by the azimuth of the celestial object to be observed. At the same time, set the direction vector of the reflected electromagnetic wave as $\vec{b} = (x_f, y_f, z_f)$.

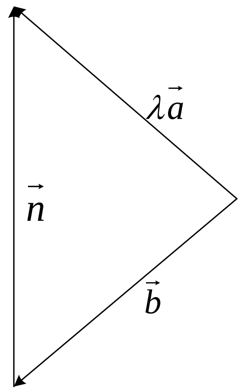

Figure 2 Schematic diagram of reflected electromagnetic wave vector analysis

We can know that:

$$\lambda = \frac{x_t^2 + y_t^2 + z_t^2}{2(x_r x_t + y_r y_t + z_r z_t)} \tag{13}$$

Assuming that the intersection of the reflection line at a certain end point of the i-th reflection panel and the plane where the feed cabin is located is $A_i'(x_i', y_i', z_i')$, then according to the above theory, the existence relation is:

$$\begin{cases} x_i' = x_i + \mu(x_{ti} - \lambda x_r) \\ y_i' = y_i + \mu(y_{ti} - \lambda y_r) \\ z_i' = z_i + \mu(z_{ti} - \lambda z_r) \end{cases} \tag{14}$$

According to the distance between the feed cabin and the origin $(1-k)R$, the plane where the feed cabin is located is:

$$\begin{aligned} K(x,y,z) &= \cos\beta\cos\alpha(x+(1-k)R\cos\beta\cos\alpha) \\ &+\cos\beta\sin\alpha(y+(1-k)R\cos\beta\sin\alpha)+\sin\beta(z+(1-k)R\sin\beta)=0 \end{aligned} \tag{15}$$

Combining equations (14) and (15), the three endpoints corresponding to the reflective triangular panel on the plane where the feed cabin is located are $A'(x_1', y_1', z_1')$, $B'(x_2', y_2', z_2')$, $C'(x_3', y_3', z_3')$.

4. Conclusion

1. The optimization model often uses simplified algorithms such as the differential evolution algorithm and stochastic simulation algorithm. The differential evolution algorithm optimizes 2000 multi-dimensional vectors according to the algorithm, which saves a lot of computing time; the idea of the stochastic simulation algorithm also applies the concept of the central theorem of large numbers and uses the results obtained from a large number of repeated experiments to replace the probability, making the algorithm more concise and efficient.

2. Use transformational thinking. The smallest distance between the actual central cable node and the ideal primary cable node is converted into the closest distance between the existing central cable node, the focal point, and the quasi-plane, simplifying many operations.

3. Considering the actual working situation of the actuator, add the inclination angle allowed by the error within a specific range to the pull-down cable so that the optimization model is more practical and feasible.

4. The optimization model has more feasibility. According to the model, the angle of the observed celestial body given in the title can be calculated, and any tip that meets the size of the divine eye can be applied.

5. Considering the actual situation, the scheme required by the model establishment should minimize the wear and tear of the reflective panel of the sky eye, reduce energy consumption, and prolong the life of each structure.

6. The search interval for the number of iterations in the algorithm is not enough and the range is not wide enough, and the result obtained is only the local optimal solution, not the global optimal solution.

7. The density of the reflected wave surface when the electromagnetic wave is reflected is not considered, which may cause a certain error in the final result, which is not accurate enough.

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