The reality of Complex numbers

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Abstract. What is a complex number? The question may be well solved today, but about 400 years ago it would have baffled many mathematicians. For example, how was it possible to define the square root of a negative number, which was an "impossible" quantity at the time, but they were all real quantities in mathematics. Complex analysis is now recognized as a fundamental part of mathematics because of its many applications in physics and engineering as well as its connections to other branches of mathematics. By solving some equations of more than one degree, complex numbers were produced. Most mathematicians focused on real analysis and physics applications until the nineteenth century. Cauchy was the first figure to make a considerable effort in a challenging examination. He calculated a number of complex integrals using his integral formula, establishing the foundation for defining the operations and characteristics of complex analysis. Gauss furthered the development of the complex number geometrical theory. The first part of this essay focuses on the history discovery of mathematical and geometrical aspects of complex numbers. The topological features of the complex plane are then carefully understood after that. The beautiful characteristics of complex functions and series are finally compiled based on the topology of the complex field.

Keywords: Complex numbers, topology, complex function.

1. Introduction

Mathematicians have been troubled by these "impossible" quantities for centuries, and in fact, due to the limitations of early mathematics, mathematicians rarely considered the limitations of positive or real numbers [1 - 19]. They do not have any prospects for the development of these undefined quantities. As a result, complex systems have not been extensively exploited for thousands of years. It was only later that mathematicians became convinced that they had discovered something new, leading to developments in the field of complex numbers, such as the Cartesian coordinate system and the roots of functions.

In recent centuries, mathematicians have explored complex numbers more and more deeply. They have gradually solved one problem after another in the field of complex analysis by proving them step by step, and at the same time opened up new horizons for mathematical research.

2. The history of differential equation-Imaginary number

Imaginary numbers appeared in early Italy, hidden in a cubic equation, and algebra was well developed in the 16th century [1-19]. At that time, Nicolo Tartaglia, a mathematician, used his unique method to solve a cubic equation[1-19]. His method was to replace the method of taking values by reasoning. He created many equations to further express his hypothesis, such as the reduced cubic equation shown below

In order to better carry out the next step of reasoning, Tartaglia defined two numbers, namely u and V, defined their difference as D, and defined the legislation that their product is one third of C, so that we can get:the product and the difference between the two numbers and find the two numbers have to be taken, and this need to solve the quadratic equation, but fortunately, the mathematicians before have thought of the solution, and they have worked it out. Tartaglia also tried to find the value of u+v according to previous methods, and then solved the problem according to the value of u-v in the expression. His method of finding u+v was to first find the square of U-V, then 4 times UV, and then find the square root of this quantity [1-21]. Tartaglia's solution is:By Tartaglia's method, the equation is similar. To solve this equation, the values of u and v need to be found at first. According to Tartaglia's scheme, there areBy finding the values of u plus v and u minus v, you can plug them into the equation and find u and v, respectively. And then the cubic equation can be solved. In 1539, however, Tartaglia's scheme was made known to the Italian mathematician Girolamo Cardano, who mentioned it in his work Ars Magna[1-21]. Unfortunately, due to the betrayal of Girolamo Cardano, Tartaglia's method never appeared again. So Cardano, the mathematician, took a lot of heat, and he published this book in an insult to the Italian mathematician Ferro, who had also developed a form of reduced cubed. But Cardano did not praise Tartaglia, although he was inspired by Tartaglia [5], and through his writings the reader can discover a different form from Tartaglia. Cardano solved for x from the perspective of a three-dimensional cube, and the picture below shows his idea in the cube.

Cardano's method is not a very rigorous form, unlike Tartaglia's strict algebraic steps and system of equations. In his method, he still used the solution to Tartaglia's equation 2, except that in order to describe his cube, he defined x=t-u, and after he had solved the geometry, he then devised a general solution. Cardano's "recipe" nevertheless depicts the roots in radical form, concealing many of the roots' true integer values. For example, in equation 3, he represents the solution in its radical form. Cardano's method is far from perfect. When c and d are negative, the approach will include the square root of a negative number in some versions of Equation 1. Cardano did not test these, but he did apply "laws of regular arithmetic" to imaginary quantities and shown that, if such numbers existed, they might satisfy the cube. However, mathematicians soon discovered that this was very impossible because they only employed methods for real numbers. Cardano, on the other hand, did not simply publish a version of Tartaglia's cube solution in the form of equation 1. He also attempted to solve a novel cube form.

He not only struck an impasse because of his single solution path, but he also "missed" the genuine answer that satisfies the cube. As a result, his comments on a subject he didn't completely comprehend sounded unproductive. Rafael Bombelli later provided the missing steps in his approach. Cardano's failure to convey the entire mathematical theory and rationalize it as acceptable did not harm his reputation, but rather demonstrates the structure of mathematical proofs in the mid-16th century. It is essentially geometric, and the words are not stated explicitly, therefore the demonstration does not provide a compelling foundation for the broader rules that follow. Cardano described nearly all of his mathematical explanations as "demonstrations." He did it with a specific goal in mind: to produce mathematical demonstrations so that "reasoning may encourage conviction." Bombelli's Algebra and the Beginnings of Complex Number Theory. Rafael Bombelli was certainly a character - he has been called “last great six- teenth century Bolognese mathematician”.Bombelli embodies the "ordinary man" fairly well, owing to his lack of academic specialization in mathematics. Unlike Cardano, Bomberi was unable to justify a solution that did not totally reflect the cube. L'Algebra, his work, was a watershed moment in the evolution of complex numbers, since it became a really rigorous representation of imaginary numbers. Language for the Unknown by Bombelli Bombelli's language of unidentified numbers Bombelli regarded the amounts appearing in Cardano's technique as both "actual" and evidence of a newly discovered mathematical potential.not merely a partially invalid method Because he was effectively building "from the ground up," he chose to give these quantities names. He deciphered -1 as pdm, which is an acronym of the Italian piu di meno, which translates as "plus of minus." Similarly, he called its inverse (--1) meno di meno, which translates as "minus of minus" and is abbreviated as dm. At the time, mathematics was mostly represented by words rather than symbols. As a result, expressing the mathematical expressions of the unknowns in Bomberi would be relatively simple.

Bomberi would write this as "seven plus three times plus of minus." Bomberi not only coined the word imaginary quantity, but there is evidence that he recognized some of its features and associated operations. Bomberi "attacked the irreducible case of the cubic, which...leads to the complex number's cube root." He invented his own approach for calculating the cube equation, one that boldly admits the existence of the square root of a negative number. Consider Bomberi's work in the irreducible situation of cubes, or cubes with three real roots. He first demonstrated how Cardano's formula does not allow one to find these roots, which is why Tartaglia and Cardano labeled it irreducible. However, he demonstrated how combining imaginary roots can result in a real number, which provides the "missing step." Take a look at the cubic equation provided by

The solution is obtained by using Cardan's approach to this problem.The equation stated, however, contains three real roots, namely 4, -2 + Bomberi agreed that only one root was produced where three roots should have been obtained, and not these three roots.

3. The reality of complex numbers

More empirically, Bombelli demonstrated that any real number may be stated in complex form. Bombelli's theory on this subject was a mathematical breakthrough that added significant depth to the development of set theory, involving the organizational definition of real-number sets and the intimate relationship between real-number sets and imaginary sets. Creating a New Mathematical Language for Imaginary Numbers The Use of "Imaginary" by Leibniz. Complex number were seen in a more contemporary manner in the 18th century. In addition to producing more contemporary representations of imaginary numbers and expressions, Leibniz's study of complex numbers advanced our grasp of number-theoretic features such touch conjugate as well as algebraic qualities like quadratic and binomial. Leibniz discovered a few intriguing characteristics. Positive rational numbers are produced by complex numbers, particularly by linear combinations of complex conjugates. For example: Leibniz also studied cubics, and the conclusion of the study is that “they will either have three real roots or two imaginary roots and one real root. Leibniz out to demonstrate that, contrary to Bombelli's assertion, Cardano's formula was in fact universally valid and did not require redefinition. His use of the word "imaginary" contributed the most to the complex's growth. In fact, this was one of the first times the phrases used to describe these numbers were utilized. Leibniz actually had a good reason for calling the amounts fictitious. "The Divine Spirit finds a sublime expression in the miracle of analysis..., that gap between being and not-being that we name the imaginative root of negative oneness," he explained. These numbers were utilized by Leibniz, but he was still unaware of their nature; this is related to the fact that the first mathematics was based on geometry and what can be measured. He concluded that complex numbers weren't actually numbers because there wasn't much evidence to support their geometrical validity. This viewpoint is where the word "imaginary" first appeared and gained popularity. Euler invented the sign I to symbolize the square root of the negative identity, $i = \sqrt{-1}$. This is very significant for the future research on the imaginary number, and the concise representation is widely used. Numerous mathematicians began to think that there could be various "orders" or "types" of complex numbers during the 18th century as the study of complex numbers progressed. Since there are many detailed classifications of real numbers, many mathematicians are curious about the classification of imaginary numbers. But d'Alembert proved that every imaginary number can be expressed in the form of a + bi, d’Alembert. The paradox of the set of complex numbers is that while each of its components can only be stated in this one form, the set is infinite, just like the real numbers.

4. Complex plane and its topology

If the imaginary part of z is 0, the number z is real, and if the real part of z is 0, we say that z is imaginary. C admits operations of addition, multiplication and complex conjugation. The natural expansions of the covvespaxliney operations on R with the rule that i=-1 are addition and multiplication. Furthermore, addition and multiplication are associative and commutative, admit identities (0 and 1, respectively), admit increments -z with (for z=/0), and obey the distributive law

C should be represented as a coordinate plane R2 with the coordinate pair z=x+iy displayed (x,y). In this situation, the x-axis is referred to as the real axis, the y-axis is referred to as the imaginary axis, and the coordinate plane is referred to as the complex plane. Recalling the distance formula for R2, we see that |z| is the distance from z to 0 in the complex plane. More generally, |z-2| is the distance from z to w. It is worth noting that addition corresponds to vector space addition on R2 and may thus be viewed in this manner. To illustrate multiplication, remember that every point in R2 may be defined by polar coordinates and convert this from C. any z∈C. Define same notation and terminology for subsets Ω∈C. For Z0∈C and c > 0. The open disc of radius r centered at Z0.Dr (Z0): = {Z∈C: ∣Z-Z0∣ < r}. The closed disc of radius r centered at Z0. Dr (Z0): = {Z∈C: ∣Z-Z0∣≤ r}. The circle disc of radius r centered at Z0: Cr(Z0): = {Z∈C : ∣Z-Z0∣=r. The set of points in mathematics whose distance from a given point in the plane, P, is less than one is known as the open unit disk (or disc) :D1(P) = {Q:∣P-Q∣>1}. The closed unit disk around P is the set of points whose distance from P is less than or equal to one: $\overline{D}_1$(P)={Q:∣P-Q∣≤1}

The subject of topology focuses on certain characteristics of geometric spaces or figures that hold true despite repeated form changes. It ignores the sizes and forms of the items and only takes into account their spatial connections. In topology, the important topological properties include connectivity and compactness. It simply a translation from R^2 to C.

In topology and related mathematical fields, topological properties or topological invariants are properties of topological spaces that are invariant under homogeneous embryos. Or a topological property is a class of topological spaces that are closed under a homogeneous embryo. That is, if a space X has this property, then all spaces that are in the same embryo as X have this property, then the property of this space is a topological property. In layman's terms, a topological property is a spatial property that can be expressed in terms of open sets.

So that, it can be define Ω∈C is open if for all Z∈Ω there exists r > 0 such that we say Ω is closed if its complement $\Omega^c = C \setminus \Omega$ is open.

A common problem in topology is to determine whether two topological spaces are homogeneous embryos. To prove that two spaces are not flush, it is sufficient to find the topological property that they do not share.

A set Ω∈C is closed if for any convergent sequence $(Z_n)u\in(n)$ ∈Ω are has $\lim_{n\to\infty} Zn$∈Ω

Proof: Assume Ω is closed and let (Zn)∈Ω be a convergent sequence. If w∉Ω, then since Ω^c is open there exists r > 0 so that $D_r(w) \subset \Omega^c$.

5. Functions on the complex plane

When you study functions, there are certain properties that are very important, such as continuity and differentiability. The function f is continuous at $z_0 \in$ C if $\lim_{z\to z_0} f(z) = f(z_0)$ exists.

More explicit: For any γ > 0, there exist $\sigma = \sigma(\gamma)$ such that:|f(z) – f(z_0)| < γ when |z - z_0| < $\sigma(\gamma)$. A continuous function f defined on a compact set t τ, then it has it maximum and minimum value. The function f is holomorphic at point $z_0 \in$ C if $\lim_{z\to z_0} \frac{f(z)-f(z_0)}{z-z_0}$ exists and denoted as $f'(z_0)$. More explicit: For any γ > 0, there exist $\sigma = \sigma(\gamma)$ such that:|$\frac{f(z)-f(z_0)}{z-z_0}$ - $f'(z_0)$| < γ when |z - z_0| < $\sigma(\gamma)$. The function f is said to entire if it is holomorphic on C. As for the holomorphic function f and closed path δ in the complex plane, we have: $\int_\delta f$ = 0.When m, n are holomorphic functions defined in set γ, it follows that: m + n is holomorphic in γ and $(m+n)' = m' + n'$.mn is holomorphic in γ and $mn' = m'n'$; when a $\in \gamma$ and n(a) ≠ 0, $\frac{m(a)}{n(a)}$ is holomorphic and $(\frac{m(a)}{n(a)})'$ = $\frac{m(a)'n(a)-n(a)'m(a)}{n(a)^2}$.When function m: a → b and n: b → c is holomorphic, it holds that $(nm)'(z) = n'(m(z))m'(z)$ for all z $\in$ a.

6. Summary

A complex number is what? Today the problem may be easily resolved, but 400 years ago it would have stumped many mathematicians. How was it feasible, for instance, to define the square root of a negative integer, which was considered to be a "impossible" quantity at the time, even if all mathematical values were real? Because of its numerous uses in physics and engineering as well as its linkages to other areas of mathematics, complex analysis is now regarded as a fundamental component of mathematics. Complex numbers were created by resolving some equations with many degrees. Up to the eighteenth century, the majority of mathematicians concentrated on real analysis and physics applications. The first person to put in a significant effort in a difficult test was Cauchy. Using his integral formula, he computed a number of complex integrals, laying the groundwork for defining the functions and traits of complex analysis. The geometrical theory of complex numbers was advanced by Gauss. This essay's first section focuses on the historical discovery of complex numbers' mathematical and geometrical properties. The complex plane's topological characteristics are then thoroughly understood. Finally, based on the topology of the complex field, the lovely properties of complex functions and series are assembled.

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