Discovery of Complex numbers

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Abstract. A branch of mathematical analysis called complex analysis studies the functions of complex numbers. It is also sometimes referred to as the theory of functions of a complex variable. Complex analysis has been used extensively in mathematics, physics, and engineering over the years, particularly in the areas of algebraic geometry, fluid dynamics, quantum mechanics, and other related fields. It dates back to the 16th century, when Italian mathematicians Girolamo Cardano and Raphael Bombelli first noticed complex numbers while attempting to solve a particular algebra, and was later developed by Cauchy and Riemann in the 19th century. The development of complex numbers has a lengthy history. Mathematicians have advanced the discipline of mathematics significantly after thousands of years of development. During this time, mathematicians also found a great deal of previously unknown mathematical information and proved formulae and phenomena that had previously been impossible to verify. And it covers complex numbers as well as some of the mathematics related to them. In this paper, the complete discovery of complex numbers from cubic equation to topology of complex numbers is detailedly revealed.

Keywords: Complex numbers, cubic equation, complex set and topolgy.

1. Introduction

The history of Complex Number goes back a long way. After thousands of years of development, mathematicians have made great progress in the study of mathematics. Mathematicians also discovered a lot of undiscovered mathematical knowledge during this period, and successively proved the formulas and phenomena that could not be proved before. And that includes complex numbers and some of the mathematics associated with complex numbers. So what is a complex number? When were complex numbers discovered and proved to exist? The square root of a negative number cannot, under normal circumstances, be specified by the definition of a square root. It wasn't until around 300 years ago that this inquiry received an answer [1-15]. These numbers appear often in the study of mathematical fields, whether or not they make sense in everyday life.

Mathematicians examine complex numbers along the way. Research in fields like function roots, Cartesian coordinate systems, and intermediate algebraic equations is strengthened by starting with delayed consensus, showing the existence of complex systems, and concluding with the proof of complex numbers. Complex analysis is being studied by an increasing number of individuals since it is universally acknowledged as a mathematical reality. As a result, the field of complex analysis was created to investigate complex numbers. It is therefore obvious that the creation of complex numbers, made possible by the extension of rigorous proof, gave rise to new and enlarging viewpoints from which to approach the many fields of mathematics.

Italian cubic equations include the oldest references to imaginary numbers. Nicolo Tartaglia must be noted in this situation. Because of his linguistic difficulties as a youngster as a result of the murder in his community, he is a math prodigy and an intriguing figure who is frequently referred to as "the stammerer." One of his most notable contributions was the invention of a "secret" technique for resolving a certain kind of cubic problem. Despite being absolutely exemplary in style, Tartaglia's approach to algebra was revolutionary. Instead than using numerical examples, this strategy relies on abstract reasoning to answer the frequently asked question, "Does this work for any number?" In order to depict his scenario, Tartaglia created numerous equations using the Diophantine approach[1-15].

2. Main works

2.1. Cubic equation and birth of complex numbers

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Rafael Bombelli, a famous Renaissance European engineer. He was also a brilliant mathematician. He was the first to put forward the concept of complex numbers. His book Algebra, published in 1572, discussed the square root of negative numbers. The book had a wide influence in Europe [1-15].

Bombelli believed that the quantities appearing in Cardano's method were real. This is also a potential for new discovery for mathematics, not just a symbol of partially ineffective scholarship. Because he was the first to have this idea, he basically built it from scratch, so he decided to rename the content. Because symbolic notation was not widely used at the time to convey mathematics, especially since Pombelli's notation for imaginary numbers had not yet been created. Thus, at the time this method of representing mathematical expressions of the unknown by using the Bombelli language would be very different from the symbolic notation used today.

Bombelli tackles the cube's irreducible situation before obtaining the complex number's cube root. Additionally, he created his own technique for resolving cubic problems. He bravely acknowledges the reality of negative square roots, or in other words he assumes that there are negative numbers in square roots. So Bombelli pioneered a new approach. Bombelli's method nicely marks the beginning of the plural. The claim is that Bombelli stopped calling them numbers. Instead, he sees them as "a new kind of connected radical." Bombelli considers the development of research on triple irreducible instances. Consequently, a cubic has three actual roots. He begins by demonstrating how Cardano's formula is irreducible according to Tartaglia and Cardano since it makes it difficult to identify these roots. He then filled in a gap in his study by demonstrating how a combination of fictitious roots may result in a real number. That is, to think differently and creatively. This experiment also supports Cardano's methodology. Here is Bombelli's explanation of how real numbers are created from complex numbers. x^3= 15x + 4, x = $\sqrt[3]{\sqrt{-121} \pm 2}$, x = $\sqrt[3]{2 \pm \sqrt{-121}}$.

Where there should be three roots, only one exists, according to Bombelli, and none of these roots. Bombelli started his own mathematical studies in 1560. He really demonstrated how to modify the fictitious phrase provided by Cardano technology to generate a real number. Here is an illustration from the source that shows how actual value can be drawn out. The two expressions in the next equations, according to Bombelli, merely change in sign. He initially applied his creative interpretation to these formulae by using the numbers A and B. As he puts it, there doesn't appear to be much of a difference between the two phrases on the surface alone—there is just one different sign.In common notation, there are $\sqrt[3]{2+\sqrt{-121}} = a + b\sqrt{-1}$; $\sqrt[3]{2-\sqrt{-121}} = a - b\sqrt{-1}$. Now, check the first expression above. 2 + $\sqrt{-121}$ = $(a + b\sqrt{-1})^3$ = $a^3 + 3a^2b\sqrt{-1} + 3ab^2(\sqrt{-1})^2 + b^3(\sqrt{-1})^3 =$ $\mathrm{a}(a^2 - 3b^2) + b(3a^2 - b^2)\sqrt{-1}$. And then it can compute the solution that satisfies these two equations, u=2, v=1. When these amounts are added to the a and b equations above, The equation can be written as $\sqrt[3]{2+\sqrt{-121}} = 2 + \sqrt{-1}$; $\sqrt[3]{2-\sqrt{-121}} = 2 - \sqrt{-1}$. Because the unintentionally revealed real numbers are likewise complicated, Bombelli's notion is incredibly original and startling. In essence, Bombelli provided more empirical support for the idea that any real number may be expressed as a complex number. A mathematical innovation in this area is Bombelli's theory. The organizational definition of the set of real numbers and the close connection between the set of real numbers and imaginary numbers provide significant depth to the development of set theory. Bombelli also gave later mathematicians a fresh perspective on how to approach this area of mathematics if they want to pursue it further.

Bombelli's research is the foundation of algebra, and he is considered the founder of new algebra. This is a great achievement, because Bombelli first recognized imaginary numbers, and was able to let those mathematicians in later generations understand and see algebra from a new perspective. As those mathematicians know. Later, this view was also confirmed and accepted, because his bold argument was more like a more concrete proof idea. Simply put, he takes something that is currently known to exist and shows why there is a misunderstanding of it, rather than formulating rules from calculations. His research and discovery of complex numbers also provided mathematicians with the opportunity to expand the thinking of cubic functions and study algebra from new aspects [1-15].

2.2. Complex number and the complex plane.

Complex numbers are defined as the set of of $\mathbb{C} := \{\mathrm{a} + \mathrm{ib} : \mathrm{a}, \mathrm{b} \in \text{all real number}\}$, where $\mathrm{i} = \sqrt{-1}$ (that is, i^2=1). For $\mathrm{z} = \mathrm{x} + \mathrm{iy} \in \mathbb{C}$ are defines. The real part is Re(z):=x . And the imaginary part is Im(z):= y . The absolute value is $|z| := (x^2 + y^2)^{1/2}$. For the operations on $\mathbb{C}$. $\mathbb{C}$ admits the operations of Additions: $(x_1 + iy_1) + (x_2 + iy_2) = (x_1 + x_2) + i(y_1 + y_2)$; Multiplication: $(x_1 + iy_1) \times (x_2 + iy_2) = (x_1x_2 - y_1y_2) + i(x_1y_2 + x_2y_2)$;Complex conjugation: $\overline{x + iy} = x - iy$.

Addition and multiplication are simply the natural extensions of the corresponding operations on all real numbers along with the rule that $i^2 = -1$. Furthermore, these operations make $\mathbb{C}$ a field: addition and multiplication are associations, commutative, admit identities (0 and 1, respectively), recogenize inverses -z and $\frac{1}{2}$ (for z $\neq 0$), and satisfy the distributive laws that exercise these factual persuasion. One also has $\overline{h+k} = \overline{h} + \overline{k}$; $\overline{hk} = \overline{h} \times \overline{k}$ $and\ Re(h) = \frac{1}{2}\left(h + \overline{h}\right)$; $Im(\mathrm{h}) = \frac{1}{2i}\left(h - \overline{h}\right)$; $|h| = (h \times \overline{h})^{\frac{1}{2}}$. For all $\mathrm{h}, \mathrm{w} \in \mathbb{C}$. The last identity shows $\frac{1}{h} = \frac{\overline{h}}{\lceil h \rceil^2}$ (after checking $\mathrm{h} \neq 0$ if and only if $\lceil h \rceil \neq 0$). This can also derived tag “conjugating the denominator”: $\frac{1}{s+it} = \frac{1}{s+it} \times \frac{s-it}{s-it} = \frac{s-it}{s^2+t} = \frac{\overline{s+it}}{|s+it|^2}$. Using the above identities, it shows $|hk|^2 = (hk) \times \left(\overline{hk}\right) = hk$; $\overline{h} \times \overline{wk} = |h|^2 \times |k|^2 \rightarrow |hk| = |h||k|$.

Also, h ∈ C is real if and only if $\overline{h} = h, and\ h\ is\ imaginary\ if\ and\ only\ if\ \overline{h} = -h$. Let α = a + bi be a complex number, $\overline{\alpha}$ = a – bi is called the conjugate of α. $\alpha\overline{\alpha} = a^2 + b^2 = |\alpha|^2$. Let α, β be complex number, then $\overline{\alpha\beta} = \overline{\alpha}\overline{\beta}$, $\overline{\alpha + \beta} = \overline{\alpha} + \overline{\beta}$, $\overline{\overline{\alpha}} = \alpha$. What is Visualizing C. C should be visualized as a coordinate plane the square of the all real number where a=p+iq is ploy is the pair (p,q). In this case, the coordinate plane is referred to as the complex plane, and the x-axis and y-axis are referred to as the real and imaginary axes, respectively. Using the square of all real integers as the distance formula, it is known that $|z|$ is the distance from z to 0 on the complex plane. More generally, $|z - w|\ is\ the\ distance\ from\ z\ to\ w$.

Note that addition matches vector space addition on full real squares and can therefore be visualized in this way. To visualize the multiplication. Recall for the first time that every point in the square of all real numbers can be described by polar coordinates and converted to C : for any z ∈ C\{0}. $\left|\frac{z}{[z]}\right| = |z| \times \left|\frac{1}{|z|}\right| = |z| \times \frac{1}{|z|} = 1$. Thus these exists θ ϵ all real number so that $\frac{z}{[z]} = \cos\theta + i\sin\theta$. If it write $e^{i\theta} = \cos\theta + i\sin\theta$. Definition of absolute value of complex number is given as for a given complex number $\alpha = \alpha_1 + i\alpha_2$ ($\alpha_1, \alpha_2 \in R$), $|\alpha| = \sqrt{{\alpha_1}^2 + {\alpha_2}^2}$. Notation that will be justified by discussion of power series in sectiona above, then $z = [z]e^{i\theta}$.

Define for z ∈ C, $z = |z|e^{i\theta}$ is said to be the polar form of z. The angle θ ϵ all real number is said to be the argument of z, and is denoted average z. Using trigonometric identities, one can check that $e^{i\theta 1}e^{i\theta 2} = e^{i(\theta_1+\theta_2)}$. Thus for z, $zw = |z|e^{iavg\omega} \times |w|e^{iavg\omega} = |zw| \times e^{i(avg(z)+avg\omega)}$. complex conjugation $\overline{x + iy} = x - iy$ corresponds to reflection over the real axis. Since the absolute value corresponds to distance in the complex plane, one obtains the triangle inequality: $|h + k| \le |h| + |k|$. This further implies the triangle inequality: $\big||h| - |k|\big| \le |h - k|$

Indeed from the triangle inequality we have so that $|h| - |k| \le |h - h|$. Also $|k| = |k - h + h| \le |k - h| + |h|$. So that $-(|h| - |k|) = |k| - |h| \le |k - h| = |h - k|$. Hence it can get solutions $|\mathrm{Re(h)}| \le |h|$ and $|Im(h)| \le |h|$.

The definition of the convergence is a sequence $(z_n)_{n\in\omega} \in C\ converges\ to\ \omega \epsilon C$ if $\lim\limits_{n\to\infty}|z_n - \omega| = 0$, in which case it shows $\lim\limits_{\mathrm{n}\to\infty} z_n = \omega$ and call to the limit of the sequence. More explicating, $(z_n)_{n\in\omega}$ converge to ω if ∀ ε > 0; ∃n ∈ n, so that $\forall_{n\ge N}, |z_n - \omega| < \varepsilon$.

Since $|Re(z)|, |Im(z)| \le |z| = (Re(z)^2 + Im(z)^2)^{\frac{1}{2}}$. One has $\lim\limits_{\mathrm{n}\to\infty} z_n = \omega \leftrightarrow \lim\limits_{\mathrm{n}\to\infty} \mathrm{Re}(z_n) = \omega\ and\ \lim\limits_{\mathrm{n}\to\infty} \mathrm{Im}(z_n) = \omega$. Thus this notice of convergence agrees with the usual one on the square of all real number. The other definition is $(z_n)_{n\epsilon\omega} < C$ is a Cauchy sequence if $\forall_{\varepsilon>0} \ni \mathrm{N} \in \mathrm{N}$ So that $\forall_{n,m\ge 0}\ are\ has\ |z_n - z_m| < \varepsilon$. To put it another way, the terms in a Cauchy sequence finally get as near as they are willing to. Cauchy sequences are those that gradually approach both the limit and one another. It turns at the converge is true: C is complete: all cauchy sequences converge. This illustrates that all real number is complete, so the real and imaginary parts converge to some x, y ∈ all real number, respectively. Define ω $:= x + iy. Then\ \lim\limits_{n\to\infty}|z_n - \omega| = \lim\limits_{n\to\infty}((Re(z_n) - x)^2 + (Im(z_n) - y)^2)^{\frac{1}{2}} = 0$.
$Thus\ (z_n)_{n\epsilon N}\ converge\ to\ \omega$.

2.3. Complex sets and Topology

Complex sets and Topology establish some notation and terminology for subsets C.

Define for $z_0 \in C\ and\ r > 0$ the open disc of radius r centered at z_0 is $\ni_r (z_0) := \{z \in C: |z - z_0| < r\}$.The closed disc of radius r centered at $z_0\ is\ \overline{\ni_r}\ (z_0) := \{z \in C: |z - z_0| \le r\}$.The circle of radius r centered at $z_0\ is\ C_r(z_0) := \{z \in C: |z - z_0| = r\}$. Finally, it can be reserved the following notation for the unit disc all real number $:= \ni_I\ C_o = \{Z \in C: |z| < 1\}$.

Let's discuss the topological characteristics of complex numbers now. These characteristics are only a translation from the square of all real numbers to complex numbers since the complex plane may be associated with the square of all real numbers and includes certain notations of distance and convergence.

The absolute value of a complex number satisfies the following properties. If α, β are complex numbers, then |αβ| = |α|*|β|; |α+β| ≤ |α| + |β| (triangle inequality). Let θ, φ be complex numbers, then $e^{i(\theta+\varphi)} = e^{i\theta}e^{i\varphi}$. Let α, β be complex numbers, then $e^{\alpha+\beta} = e^{\alpha}e^{\beta}$. The expression $re^{i\theta}$ is called the polar form of the complex number x + iy. The number θ is called the angle, or argument of z, and we write: θ = argz. Let α ∈ S, we say that f is continuous at z_0 if $\lim_{z\to z_0} f(z) = f(z_0) = w_0$. More explicit: For any $\varepsilon > 0$, there exists $\delta = \delta(\varepsilon)$ such that:If |z - z_0| < $\delta(\varepsilon)$, then |f(z) – f(z_0)| < ε. We say that f is differentiable at z_0 if $\lim_{z\to z_0} \frac{f(z)-f(z_0)}{z-z_0}$ exists. We denote the limit by $f'(z_0)$. More explicit: For any e > 0, there exist $\delta = \delta(\varepsilon)$ such that: If 0 < |z - z_0| < $\delta(\varepsilon)$, then $|\frac{f(z)-f(z_0)}{z-z_0} - f'(z_0)| < \varepsilon$. Let s ⊆ C subset, if a point α (not necessarily contained in s) such that ∀D(α, -) centered at α contained both points of s and points are not in s.

Let u ⊆ C subset, u is called open if for ∀α ∈ u, there exists r ∈ R^+ and such that D(α, r) ⊆ u.Let u ⊆ C subset is called closed subsets if C\u is open in C. Or Let u ⊆ C subset is called closed if u contains all its boundary points. Let α ⊆ C is called an interior point of s if $\exists D(\alpha, r)$ with r ∈ R^+ and D(α, r) ⊆ s. At last, in order to prove power of complex numbers, two question solved by complex numbers techniques are given.

Take z = 1 and n ϵ z^+ fixed, there are exactly n different numbers such that w^n = z = 1.Prove: Let $\theta = \frac{2\pi}{n}$ $w_1 = 1*e^{i\frac{2\pi}{n}} = e^{i\theta}$, $w_2 = 1*e^{i\frac{4\pi}{n}} = e^{2i\theta}$…$w_n = 1*e^{i2\pi}$ = 1 are all satisfy w_i^n = 1 for i = 1…n. Let n be a prime bigger than three , then $\left(1+2cos\frac{2\pi}{n}\right)\left(1+2cos\frac{4\pi}{n}\right)\left(1+2cos\frac{6\pi}{n}\right)\ldots\left(1+2cos\frac{2k\pi}{n}\right) = 3$. Let w =$e^{\frac{2\pi i}{n}}$. That is w^n =1, $w^{-\frac{n}{2}} = e^{-xi}$ = -1, 2 cos$\frac{2k\pi}{n} = w^k = w^k + w^{-k}$. $\prod_{k=1}^{n}\left(1+2cos\frac{2k\pi}{n}\right) = \prod_{k=1}^{n}(1+w^k+w^{-k}) = \prod_{k=1}^{n} w^{-k}(w^{2k}+w^k+1) = w^{-\frac{x(n+1)}{2}} \times 3\prod_{k=1}^{n-1}\frac{1-w^{3k}}{1-w^k} = (-1)^{n+1} \times 3\prod_{k=1}^{n-1}\frac{1-w^{3k}}{1-w^k}$. Take n = 5 as an example to illustrate such an elegant identity. $(-1)^{n+1} \times 3\prod_{k=1}^{n-1}\frac{1-w^{3k}}{1-w^k} = (-1)^6 \times 3\prod_{k=1}^{4}\frac{1-w^{3k}}{1-w^k}$.

3. Summary

Complex analysis is a subfield of mathematical analysis that focuses on the operations of complex numbers. The theory of functions of a complex variable is another name for it. In particular, algebraic geometry, fluid dynamics, quantum mechanics, and other related sciences have made substantial use of complex analysis in mathematics, physics, and engineering over the years. Girolamo Cardano and Raphael Bombelli, two Italian mathematicians, initially recognized complex numbers while attempting to solve a particular algebra in the 16th century, while Cauchy and Riemann further improved it in the 19th century. Complex numbers have a long history of development. After thousands of years of growth, mathematicians have made enormous advancements in the field. In addition, a considerable lot of previously undiscovered mathematical knowledge was discovered during this time, and previously unprovable formulas and phenomena were proved. Additionally, some of the mathematics associated with complex numbers and their use are covered. This paper presents a comprehensive account of the discovery of complex numbers, including everything from the cubic equation to their topology.

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