The discovery and development of Imaginary number

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Abstract. The discovery of imaginary numbers and the establishment of complex systems took a long time. These nonexistent numbers have been studied in great detail by mathematicians thousands of years ago, including in Tartaglia's arithmetic competition. The general solution of Girolamo's cubic equation with one variable. All reflect the wisdom of the ancients. In modern times, as more and more mathematicians use more sophisticated methods to explore imaginary numbers, which are rarely studied, theories and systems are becoming more and more robust. Some famous mathematicians such as Euler, Gauss, Newton, Leibniz, and Descartes have made indelible contributions. These research areas also laid the foundation for the future development of science and technology physics. This review was focused on the mathematicians' outstanding discovery of complex system.

Keywords: complex system; Imaginary number; cubic equation.

1. Introduction

With the development of mathematics, mathematicians found that the real roots of some cubic equations must be expressed by the square root of negative numbers. Moreover, if the square root of a negative number is admitted, the problem of whether an algebraic equation has any roots can be solved, and a satisfactory result will be obtained: an equation of degree N has N roots. In addition, the square root of a negative number is operated according to the algorithm of numbers, and the result is correct. The first discovery of imaginary numbers was 500 years ago. Before the system of imaginary numbers was fully established. There were many stories about that. "Is it possible to find a square root of a negative number? "This question seems not to make sense in the real world. It's like finding a length of a rectangle with negative area. It was not until centuries later that more mathematicians took note of this seemingly absurd problem. Mathematicians who study the solutions of different solid state equations notice something deeper - at the scale of the impossible "impossible". The fact that the concept was so alien convinced them that they were onto something different.

2. Finding imaginary number by solve equations-Tartaglia

In essence, mathematicians work with a "language barrier". There were neither letters or symbols for mathematicians who discovered these "unbelievable" things to use. The first mathematician Tartaglia claimed to have mastered the solution of cubic equation of one variable. This announcement opened the "Battle of Venice for cubic Equations"[1-19]. There was a mathematician named Ferro who thought that Tartaglia, a self-taught boy, would not be able to compete with him. So, the two sides agreed to hold a mathematical contest in Milan on February 22, 1535[1-19]. Each side would give the other 30 problems to do, and the winner would be decided within two hours. Whoever can solve the problem the most and the fastest wins. Tartaglia is self-taught so nervous before the race. He worked through various combinations of cubic equations in his head and was thrilled to find a new method eight days before the race. So, I used the eight days to familiarize myself with my method repeatedly and constructed 30 cubic equations that could only be solved by this new method. On the day of the match, the city of Milan was abuzz with people wanting to see who the winner of this special match was. When the game began, Tartaglia had her fingers in the air. And Ferro frowns, unable to do anything, the final zero to 30 defeats. After his victory, Tartaglia, after further exploration, finally found a general solution to the cubic equation, which he had kept secret.

3. General solution of cubic equation-Girolamo

To trace the trace of the imaginary number, we should relate the process of the appearance of the real number relative to it[1-19]. We know that real numbers correspond to imaginary numbers, including rational numbers and irrational numbers, that is to say, they are real numbers.

The term "imaginary number" was coined by the famous mathematician and philosopher Descartes in the 17th century, because the idea at the time was that it was a real number that did not exist. Later it was found that imaginary numbers can correspond to the vertical axis of the plane, as real as the real numbers on the horizontal axis of the corresponding plane.

It was established that algebraic equations were intractable regardless of whether rational and irrational numbers were used. There is no answer to even the simplest basic quadratic equations, like x squared + 1=0, inside the realm of real numbers. The eminent Indian mathematician Bhaskar believed that it was impossible to analyze this equation in the twelfth century. He held that a positive number and a negative number both contain a positive square root, so that the square root of a positive number is double; negative numbers do not have a square root, hence they are not squares. This is analogous to dismissing the validity of the equation's negative square root.

Later, Tartaglia confidentially shared the solution of cubic equation with his good friend Girolamo Cardano. Cardano betrayed his good friend Tartaglia in the book that they published after their competition. In his published book " Ars Magna", He is said to be giving credit to Italian hypocrite Ferro, who also developed the sunken dome jersey. Cardano, however, does not give Tartaglia the credit. He got ideas from him.. Cardano expressed his idea in 3D picture. Compared to the Tartaglia's method, he used more vivid expressions to explain the method of solving cubic equations. He developed the Gerolamo Cardano's "Cube" (Figure 1) to explain the process

According to his rule, the formula of "special cases" of cubic equations was developed.

$$x = \sqrt[3]{\sqrt[2]{\left(\frac{d}{2}\right)^2 + \left(\frac{c}{3}\right)^3} + \frac{d}{2}} - \sqrt[3]{\sqrt[2]{\left(\frac{d}{2}\right)^2 + \left(\frac{c}{3}\right)^3} - \frac{d}{2}} \tag{1}$$

In that time, Cardano did not realize the situation that d and c could be negative[4] . In that case, the equation would incorporate the square root of negative numbers. For more than 300 years, mathematicians explored for such roots because they were not acknowledged as a real solution, but they ultimately found they were practically impossible to uncover simply because they were only employing real-number approaches. In consequence, he figured for the generic version of cubic equations. He was so confused because he could only solve the equation in singular path: $ax^3 + bx^2 + cx + d = 0$. In additional to hitting a brick wall, he essentially "missed" the concrete answers that fulfilled the cubic. Thus, it appeared that his commentary on a subject he didn't fully comprehend backfired. Rafael Bombelli would eventually add the culmination to his solutions [1-19].

4. Bombelli and Cardino named new discover things " imaginary number"

Bombelli came up with language for unknown things. Due to he created those terms based on nothing, he wanted to give these type of number a name according to their property. Bombelli's terms are the first representation of imaginary number. He called the square root of negative one " plus of minus. Similarly, He called negative square root of negative one"plus of minus"[1-19]. the mathematicians of the time would like to use word to describe things rather than use symbols. When they encounter the imaginary number, symbol representation became essential because they did not exist in the reality.

$$x^3 = 15x + 4 \tag{2}$$

Solving this via the Cardan method yields the solution

$$x = \sqrt[3]{\sqrt[2]{-121} + 2} - \sqrt[3]{\sqrt[2]{-121} - 2} \tag{3}$$

$$x = \sqrt[3]{2 + \sqrt[2]{-121}} + \sqrt[3]{2 - \sqrt[2]{-121}} \quad (4)$$

The problem of Cardino’s method was he had no idea about how to deal with square root of negative number. when the new symbol for unknown things was developed, the equation could be solved in totally different way. His imaginary number could transform to real numbers by using some algebra process.

Around the 18th century, mathematicians were looking for the new way to explore the complex numbers. They were accepted by more and more mathematicians gradually. During 17century, mathematicians studied them so well that explore many property of imaginary number .The further development of complex number was illuminated by Gottfried Wilhelm Leibniz,. he noticed the equation below could have a real solution. On the other hand, real solution made people believe the imaginary number more.

$$\sqrt{2 + \sqrt{-5}} + \sqrt{2 - \sqrt{-5}} \quad (5)$$

Squaring this quantity, one obtains

$$(2 + \sqrt{-5} + 2\sqrt{2 + \sqrt{-5}}\sqrt{2 — \sqrt{-5}} + 2 - \sqrt{-5} = 10 \quad (6)$$

Thus, we can say that

$$\sqrt{2 + \sqrt{-5}} + \sqrt{2 - \sqrt{-5}} = 10 \quad (7)$$

It was not until the early 19th century, when Gauss systematically used the J symbol and advocated the use of even numbers (a and b) to represent a+ bi, called complex numbers, that imaginary numbers gradually gained popularity[6].

When imaginary number entered the field of numbers, people knew nothing about its practical use, and there seemed to be no quantity expressed by complex numbers in real life. Therefore, for a long time, people had all kinds of doubts and misunderstandings about it. What Descartes meant by "imaginary numbers" was that they were false; Leibniz thought: "Imaginary number is a wonderful and strange hiding place of the gods, it is almost both exist and do not exist amphibian." Although he used imaginary numbers in many places, Euler said, "All mathematical expressions of the form √-1,√ -2 are impossible, imaginary numbers, because they represent the square root of a negative number. Of these numbers we can only assert that they are neither nothing, nor more than nothing, nor less than nothing; they are purely illusory."

5. Euler further developed complex system in 1700s

He established, many essential rules for the imaginary numbers such as “natural congregate” and the general form for all complex number, a+bi. The greatest contribution of Leibniz was that he gave type of number a name “imaginary”

Euler also play a essential role in developing complex system. His idea almost embedded in every single area of mathematics [1-19]. In the area of complex number, he created more “modern language” for the complex system. He was the first person who use letter “I” to represent square root of negative 1. Euler coined i to concise the expression of imaginary number. Although the expression for imaginary number was more brief and more accurate, people still thought the imaginary number was not a real thing. They were not part of the who number system.

6. John Wallis explain imaginary numbers in geometry

In 1673, Wallis invented a simple way to represent imaginary numbers as points on a plane. He introduced another number line perpendicular to the real number line, and placed imaginary numbers on this number line. The real numbers form one number line on the plane, and the imaginary numbers form another. Other points on the plane correspond one to one with the complex number, which consists of two parts, one real part and the other imaginary part. In Cartesian coordinates, we measure the real part along the real axis and the imaginary part along the direction parallel to the imaginary axis. This is similar to Descartes' algebraic approach to plane geometry, using a number line. For example, 3+2i is 3 to the right of the origin and 2 to the top. Wallis's idea solved the problem of the meaning of imaginary numbers, but no one cared. But slowly, his ideas were accepted by the people. Most mathematicians no longer fret that square roots of complex numbers have no place on the real number line.

7. Gauss expanded imaginary number into real applications

In 1831 Gauss found the complex system was not well known enough in the area of mathematics, though many theorems were published over 200 years, and the following year he published a memorandum that established complex system[1-19].

Gauss used this set of real numbers to represent complex numbers and established several actions on complex numbers, so some actions on complex numbers can also be "algebraic", such as real numbers. He also coined the term "complex number" for the first time in 1832 and combined two different ways of representing the same point in the plan, i.e. Cartesian method and polar method. It is combined into algebraic and angular shapes that represent the same complex numbers and achieve one-to-one matching between points on the numeric axis and real numbers

Complex numbers attracted the attention of famous mathematicians, including Kumer (1844), Kronecker (1845), Scheffler(1845, 1851, 1880), Bellavitis(1835, 1852), George Picuk (1845), and de Morgan (1849). Mobius published numerous short essays on complex geometry, and Johann Peter Dirichlet extended many concepts of real numbers, such as prime numbers, to complex numbers [1-19].

8. Modern applications of complex system

After the time of John Wallis, Mathematicians began to separate real and imaginary numbers from geometric and algebraic perspectives, and to discuss them in their own categories. In the seventeenth and eighteenth centuries there were significant developments not only in imaginary numbers [1-19], but also in algebra, which was astounding, and many important formulas and identities such as Euler's formula were invented at that time. This learning coincided with the development of a new system of imaginary numbers, which led people to believe that imaginary numbers were real things and not just imaginary numbers like names.

After Euler, the Norwegian surveyor Wiesel proposed to express the complex number (a+ bi) in terms of points on the plane. Later Gauss put forward the concept of complex plane, which finally gave the complex number a foothold and opened up the way for its application. At present, complex numbers are generally used to represent vectors (oriented quantities), which are widely used in hydrology, cartography and aeronautics, and imaginary numbers increasingly show their rich content.

After the long and unremitting efforts of many mathematicians, from irrational numbers to real numbers, from imaginary numbers to complex numbers, irrational numbers are not "unreasonable", imaginary numbers are not "imaginary", so the set of real numbers expanded to the set of complex numbers, and finally the theory of complex numbers was developed.

With the progress of science and technology, complex theory has become more and more important, not only for the development of mathematics itself has extremely important significance. For example, complex numbers are widely used in electricity. In electricity, there is a concept called impedance, which hinders alternating current. But impedance is not completely resistance, it is composed of resistance and reactance. The resistance must be expressed in real numbers, and the reactance must be expressed in imaginary numbers. Together they form a complex expression [1-19].

And it plays an important role in proving the basic theorem of wing lift force. Complex numbers are used when studying rocket launches, calculating atomic bombs, and calculating the yield of CPU wafers. It shows its power in solving the problem of dam seepage and provides an important theoretical basis for building huge hydropower station.

Einstein's special theory of relativity introduces imaginary numbers when describing the distance in four-dimensional space-time, which is more acceptable. It is easier to understand the imaginary number in the field of electromagnetism and communication. Of course, you don't have to introduce imaginary numbers to explain these theories. However, quantum mechanics has to introduce imaginary numbers. According to Heisenberg's uncertainty principle, the position of a single atom is uncertain without observation. After introducing imaginary numbers, the probability of electron position is calculated, and the probability distribution is represented by wave functions with complex values.

9. Summary

Many years later, these theories were accepted by the public. The imaginary number and complex number system also evolved into a special discipline in mathematics - complex analysis. These numbers that seemed impossible a few hundred years ago, imagined numbers have become the necessities of rocket science and physical circuits today. The contribution of Cartesian coordinate system to the whole complex analysis is indelible, and his perfect integration of imaginary numbers into the common XY coordinate system. Euler also wrote Euler's formula according to Taylor series, which played an important role in complex analysis, and also laid a solid foundation for the later Cauchy integral formula and Laurent series. In the future, based on foundation of this paper, more comlicated techniques regarding complex numbers will be reported.

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