

Re-estimation of the Hubble's Constant Based on Analyzing Other Cosmological Parameters and Intergalactic Parallax Ranging

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Abstract. The Hubble's constant is one of the most important cosmological parameters in the theoretical framework of the widely accepted Λ CDM model, which is key in determining many other cosmological parameters (e.g., the age, ultimate fate, and observable radius of the universe). However, the value of the Hubble's constant is disputed between studies employing different methods to estimate the value of the Hubble's constant, causing a crisis in cosmology and also casting doubt on the compatibility of the Λ CDM model. Therefore, this study aims to propose new methods to re-estimate the value of the Hubble's constant at the present epoch with greater reliability and accuracy. Specifically, 2 methods are proposed for the re-estimation of the Hubble's constant: (1) deriving an equation from pre-established formulae in cosmology that describes the mathematical relation between the Hubble's constant and other known cosmological parameters; (2) sending a medium-sized space telescope to interstellar space to enable parallax ranging of other galaxies. The former method yielded a result of $66.49_{-1.13}^{+1.16} (km/s)/Mpc$, which is comparable to results of other studies. Using mathematical deduction, this study verified the estimations for the value of the Hubble's constant by previous attempts. Moreover, this study also proposes an explanation for the yet unexplained discrepancy in the Hubble's constant's value between different methods of measurement. The results of the study provide new outlooks to the methods of estimating the Hubble's constant, and possibly inspiring projects to send space telescopes for intergalactic parallax ranging.

Keywords: Hubble's Constant; Cosmology; Λ CDM model; Friedmann equations.

1. Introduction

In 1929, astronomer Edwin Hubble and Georges Lemaître proposed the Hubble-Lemaître law based on observations suggesting a linear positive correlation for the recession velocity, which is determined by measuring redshift, and distance of galaxies from the Milky Way galaxy [1]. The Hubble-Lemaître law describes such a correlation as:

$$v = H_0 D \quad (1)$$

Where, v represents the galaxy's recession velocity from the Milky way, D represents the distance of the galaxy from the milky way, and H_0 is a constant describing the correlation between distance and recessional velocity [1]. Based on the cosmological principal, which is the assumption that the universe is, on a large scale, isotropic and homogeneous, the Hubble-Lemaître law applies universally and the Hubble's constant remains consistent regardless of the location of the observer. From the perspective of every point in the universe and at every direction, galaxies are receding from the observer, implying the uniform and isotropic expansion of the universe. Moreover, due to the Hubble's constant's role as a description of the correlation between recession velocity and distance of a galaxy, the Hubble's constant is an indicator of the rate of the universe's expansion. Consequently, possessing the precise value of the Hubble's constant is crucial in resolving many important problems in cosmology, including the deduction of the age of the universe, t_0 , which can be estimated with the inverse of the Hubble's constant H_0^{-1} , investigation of the future expansion and evolution of the universe (which depends on the rate of the expansion of the universe), and estimation of the radius of the observable universe $R_{obs} = ct_0$ [2, 3].

Ever since the proposal of the Hubble-Lemaître law, repeated attempts had been made to estimate the value of the Hubble's constant [3]. Early attempts resulted in unusually high numbers [3]. By using Einstein's field equations to analyze observational data of the distances (by measuring the brightness of cepheid variables as a standard candle) and receding velocities (by observing the degree of doppler's redshift in emission and absorption spectra) of galaxies provided by Edwin Hubble using the Mount Wilson Telescope, Georges Lemaître, who proposed the Hubble-Lemaître law in his 1927 paper, estimated the value of the Hubble's constant as approximately 625 ± 50 (km/s)/Mpc [4]. On the other hand, in 1929, Edwin Hubble, using his own observational data, estimated the Hubble constant to be 500 ± 50 (km/s)/Mpc [1]. Such results contain large error bars, and no consensus across the field of astronomy was reached between different investigations of the true value of the Hubble's constant [1, 3]. In the 1960s, most estimates of the Hubble's constant fell below 100 (km/s)/Mpc, as more precise measurements lead to the realization that Edwin Hubble's observational measurements of the luminosities of Cepheid variables were the luminosities of entire star clusters instead of individual stars [3]. In the following years, the value for the Hubble's constant yielded from different investigations varied between 50-90 (km/s)/Mpc [3]

By the 1990s, most of the newly published results yielded at around 70 (km/s)/Mpc, though a precise value for the Hubble's constant is still unprovided [3]. While most of the previous investigations involves the measurement of the distance galaxies from Earth using the measurement of the luminosity standard candles (e.g., Ia supernovae and Cepheid variables), and the measurement of the recession velocity of the galaxies by examining the doppler's redshift of chemical spectra, the introduction of the Λ CDM cosmological model, coupled with the precise measurements of Cosmic Microwave Background (CMB) allowed the estimation of the Hubble's constant through the examination of anisotropies in the observed CMB [3]. With the Λ CDM model, such estimations using data of the anisotropies of observed CMB yielded results that are around 70 (km/s)/Mpc [3]. Meanwhile, estimates using data from optical surveys yielded similar results [3].

By the 21st Century, while measurements became more accurately using previously used methods as instruments of detection became more sophisticated, new methods to measure the Hubble's constant began to emerge [3]. The Planck mission and the Hubble Space Telescope (HST) provided more precise data for the estimation of the Hubble's constant [5, 6]. Moreover, the detection of gravitational waves also enabled the measurement of the distance to galaxies by examining the gravitational waves emitted by neutron star mergers in such galaxies, which yielded a result of $73.3^{+5.3}_{-5.0}$ (km/s)/Mpc [7, 8]. Adding on, another reported method employs the use of the TRGB (Tip of the Red-Giant Branch) as standard candles to measure the distance of galaxies from the Milky Way, which yielded a result of 69.8 ± 1.9 (km/s)/Mpc [9]. Recently, more and more precise values had been discovered, however, still, no consensus of the precise value of the Hubble's constant was reached between different studies. The resulting value of the Hubble's constant from measurements done through examining the anisotropies of CMB are mostly around 67 (km/s)/Mpc, while the resulting value of the Hubble's constant from optical observations are mostly around 71 (km/s)/Mpc. Such differences between these values continue to be an unsolved problem of cosmology as of the publishing of this study [10].

The Hubble's constant is regarded as one of the most important parameters in Λ CDM model, as it can be used for the calculation of the age, size, fate, energy density, and cosmological constant, Λ , of the universe [2, 3, 11]. However, no consensus on the precise value of the Hubble's constant has been reached. As error bars are reduced and measurements for the Hubble's constant become more and more precise, instead of yielding increasingly similar results, different methods of measurements seem to result in precise yet different values for the Hubble's constant. The value of the Hubble's constant from measurements resulting from examining the anisotropies of CMB is 67.3 ± 1.2 (km/s)/Mpc, while the result from optical observations made by the HST is 72 ± 8 (km/s)/Mpc [2, 3, 6]. These disagreements among different measurements for the value of the Hubble's constant is causing a "cosmological crisis", as described by Don Lincoln [12].

Owing to the importance and the uncertainty of the Hubble's constant's value, this study aims to propose a new method to estimate the Hubble's constant's value. Moreover, the study also aims to explore the possible causes of the difference in the results for the value of the Hubble's constant from the examinations of anisotropies in the CMB and the observations done by the HST. Therefore, the objectives of the study are: (1) outlining the pre-existing situation in the investigation of the Hubble's constant; (2) proposing a new method for providing observational data to the estimation of the disputed value of the Hubble's constant; (3) discussing the possible causes of the difference in value of the Hubble's constant measured by different methods.

However, attributed to the universe's accelerating expansion, the Hubble's constant is not consistent with the passing of time [2, 13]. Thus, the Hubble's constant, H_0 , is only applicable in describing observations made in the present using the Hubble-Lemaître law [2]. The result of this re-estimation of the Hubble's constant will only apply for describing present phenomenon with the Hubble-Lemaître law.

2. Previous Estimations of the Hubble's Constant

There are many methods that are currently employed to estimate the value of the Hubble's constant [3]. The two most common yet conflicting methods are the cosmic distance ladder method and the CMB anisotropies method [2, 3]. The cosmic distance ladder method has many variants, varying with the type of method for determining the distance to galaxies, which includes using Cepheid variables, TRGBs, and Ia supernovae in other galaxies as standard candles, and the use of gravitational wave-emitting neutron star merger events in other galaxies as standard sirens.

According to Eq. (1), to gain the recession velocity of a galaxy, the Doppler's Redshift of the galaxy's emission/absorption spectrum is measured [2, 4-6, 8]. With the amount of Doppler's Redshift in hand, the recession velocity of the galaxy can be deduced. However, the observed recession velocity of different galaxies at the same distance from the Milky way may not be isotropic, as spatial expansion is not the only factor affecting the recession velocity of galaxies. Therefore, observations usually take the recession velocities of multiple galaxies into account to eliminate such errors [2, 4-6, 8].

The greatest challenge in estimating the Hubble's Constant is the precise measurement of the distance from the Milky way to the receding galaxies. One of the most common methods in the cosmic distance ladder for measurement is to use Cepheid variables as a standard candle [3]. The variation in the luminosity of Cepheid variables in distant galaxies have an inverse square relation with the distance of the observer from the galaxy. With the absolute luminosity of the Cepheid variable known, one can measure their distance to the galaxy containing the Cepheid by measuring the apparent luminosity of the Cepheid variable [2, 3, 5, 10]. However, the measurement of the distance to the galaxy comes with great uncertainty, as the precise value of the absolute magnitude of Cepheid variables is currently subject to further investigations [3].

With the distance to the galaxy and the receding velocity of the galaxy known, one can estimate the Hubble's constant. Based on observational data from the HST, the estimated value for the Hubble's constant, by astronomer Wendy L. Freedman, is 72 ± 8 (km/s)/Mpc [2]. However, large error bars exist within this value due to the unreliability of data of the absolute magnitude of Cepheid variables. Moreover, TRGBs are also used as standard candles [9]. Using TRGBs from other galaxies as standard candles, research made by Freedman yielded a value of 69.8 ± 1.6 (km/s)/Mpc for the Hubble's constant. The result yielded from using TRGBs as a standard candle is 3.1% lower than the result yield from using Cepheid variables as a standard candle, which is caused by a significant difference in the measured distance to far galaxies by using Cepheids and TRGBs as different standard candles, which in turn is a yet unexplained difference [9].

The final type of standard candles used for the measurement of the extragalactic distances is the supernova [5, 15]. Ia supernovae are commonly used standard candles for measuring large distances, due to their high absolute luminosity, making them visible from hundreds of millions of parsecs away,

and that Ia supernovae all explode with the same energy, making all of them equally luminous [15]. The measured result from combining data on the distance to galaxies measured by using Ia supernovae as a standard candle and the doppler's redshift of galaxies resulted in the value of 73.04 ± 1.04 (km/s)/Mpc for the Hubble's constant [5]. This result is closer to the result from the Cepheid variable method than the TRGB method [5, 9] (seen from Table. 1).

Table. 1 Estimated values of the Hubble's constant of different methods [1-3, 5, 6, 8-10, 15, 16].

Year of Estimation	Estimated value of H_0	Estimating scientist or organization	Method of estimation
1927	625	Lemaître	Cepheid Variable
1929	500	Hubble	Cepheid Variable
1958	75	Sandage	Cepheid Variable
1968	79	Racine	GCLF
1973	40	Branch	Ia Supernovae
1975	59	Sandage & Tammann	Cepheid Variable
1979	61	Aaronson & Huchra	IRTF
1982	71	Visvanathan	ITF
1984	75	Refstal	Gravitational Lens
1987	67	Dressler	$D_n - \sigma$
1989	67	VandenBergh	Cepheid Variable
1994	55	Birkinsaw & Hughes	CMB anisotropies
1995	70	Freedman	Cepheid Variable
1996	50	Sandage & Tammann	Cepheid Variable
2000	65	Fillipenko & Riess	Ia Supernovae
2000	72	Freedman	Multiple
2003	71	WMAP	CMB anisotropy
2008	71	WMAP	CMB anisotropy
2010	63	Sandage & Tammann	Multiple
2013	67	Planck mission	CMB anisotropy
2017	70	LIGO Scientific collaboration & Virgo collaboration	Standard Siren
2019	70	Freedman	TRGB
2020	67	Planck mission	CMB anisotropy
2021	70	Freedman	Ia Supernovae
2022	73	Riess	Cepheid-Ia Supernovae

3. Problems for the state-of-art Evaluation

In the previous decades, the Λ CDM model had been successful in explaining many aspects of observed phenomena in the universe, e.g., the structure of CMB, distribution of matter-energy, gravitational lensing, and the observed expansion of the universe. The Λ CDM model has been verified in multiple investigations conducted by a range of cosmologists across several decades, and it is now among the most supported physical cosmological model [2, 6, 9, 10, 12, 16, 18]. However, it has been noted that the Λ CDM model has unexplained discrepancies with observed phenomenon, one of which is the estimation the value of the Hubble's constant [2, 6, 9, 10, 12, 16, 18]. Current estimations of the Hubble's constant vary greatly across different methods of measurement. The results acquired from the distance ladder methods and the CMB anisotropies method have a difference of approximately 7%, which is a difference far to large to be acceptable. Such errors could imply that the expansion of the universe is expanding faster than its predicted rate by the Λ CDM model. This unexplained discrepancy has provided supporting arguments for non-standard cosmological models. The discrepancy within the value of the Hubble's constant could indicate that the Λ CDM model will

soon be deemed as incompatible in many aspects of cosmology, thus providing possibilities for the origins of new cosmological models [10]. Owing to the far implications of the solution of this discrepancy on the future developments of cosmology, astrophysicist Don Lincoln called this situation a “modern cosmology in crisis”, implying the cruciality of the current situation of resolving the discrepancy problem [12].

The current situation of the discrepancy is that while more new methods emerge with the advancement of more precise tools for measurement, such as the standard siren with the beginning of gravitational wave astronomy and the TRGB method with the HST, leading to the measurements resulting in more and more precise values for the Hubble’s constant, the new results for the measurement of the Hubble’s constant are, however, concentrated around two, but not one, values [10, 12], as shown in Table 1. The results from all distance ladder methods, including the Cepheid Variable, TRGB, Ia Supernovae, and the standard Siren method, all yielded values for the Hubble’s constant as ~ 72 (km/s)/Mpc [2, 5, 10, 12]. On the other hand, other estimations of the Hubble’s constant through the analysis of kinematic Sunyaev-Zeldovich effects yielded the resulting value for the Hubble’s constant to be around ~ 67 (km/s)/Mpc [2, 10, 12, 16]. The two values have an unexplained difference of 7% and a standard deviation of 5-6 σ [10]. Which has been called a “cosmological crisis” [12, 18].

Several attempts had been made to explain the discrepancy [9, 10, 12]. Since the discrepancy of the value of the Hubble’s constant happens between the value yielded from the distance-ladder method, which can provide an estimation of the value of the Hubble’s constant based on optical observations of nearby observable galaxies, and the CMB anisotropies method, which, through the kinematic Sunyaev-Zeldovich effect, can provide an estimation of the value of the Hubble’s constant with information of the motion of charged baryonic matter from the entire observable universe, thereby leading some to speculate the existence of a difference between the local expansion and the overall expansion of the universe [2, 6, 10, 12, 16]. If this speculation is proven, then it could topple one of the basic assumptions in the Friedmann equations, which is the postulation that, on large scales, the universe is homogeneous [11, 17, 19]. The Friedmann’s equations form the foundations for the Λ CDM model, so if the discrepancy in the Hubble’s constant is proven to be caused by heterogeneity in the expansion of the universe, then it could overthrow all the standard model of cosmology [11, 17, 19].

4. Proposals for Re-estimation

The Hubble’s Constant is a cosmological parameter of critical importance in the standard model of cosmology, yet despite continued attempts acquire the precise value of the Hubble’s constant, there is no consensus across different studies on the Hubble’s constant [10, 12]. Therefore, this study presents two new methods to estimate the value of the Hubble’s constant through a global survey of CMB to derive the Hubble’s constant using pre-established mathematical equations in the Λ CDM cosmological model, and to use the more precise method of parallax to measure the distance to receding galaxies to provide data to estimate the Hubble’s constant with the Hubble-Lemaître Law. Moreover, this study also attempts to provide an explanation for the discrepancy in the Hubble’s constant’s value between the results from distance ladder method and the analysis of CMB anisotropies.

According to the Λ CDM cosmological model, the Hubble’s constant’s reciprocal, in theory, is equal the current age of the universe, as presented below [2, 3]:

$$t_0 = H_0^{-1} \quad (2)$$

If equation (2) can perfect describe the relation between the age of the universe and the Hubble’s constant, then the Hubble’s constant can be deduced if the age of the universe is known. However, such a relation is only possible for an empty universe that does not contain any mass nor dark energy, in which every part of the universe can expand uniformly despite the passing of time [3, 4]. In reality,

mass and dark energy distort spacetime of the universe, causing a nonuniform expansion of spacetime in the universe [3, 4, 13]. As shown in Figure (1), the density parameters, Ω , of the universe can influence the course of the universe's expansion. Therefore, the Hubble's constant is also influenced by other cosmological parameters, and the age of the universe, at present, is more accurately described as [11, 17]:

$$t_0 = H_0^{-1}F(\Omega) \tag{3}$$

Here, Ω is the density parameters [4, 17, 19]. The most significant density parameters in the universe are Ω_m , and Ω_λ , which are the mass density parameter and the cosmological constant density parameter. Other density parameters, such as radiation density, which was once considered in the original Friedmann equations, are deemed to be negligible and will not be considered further in this study [4, 17, 19]. Therefore, equation (3) can be rewritten as:

$$t_0 = H_0^{-1}F(\Omega_m, \Omega_\lambda) \tag{4}$$

Here, the factors of Ω_m , and Ω_λ are the influential factors in the relation between the age of the universe and the Hubble's constant at present. These two factors are the density parameters of the universe, which can describe the overall shape of spatial curvature in the universe, and can thus describe the expansion of spacetime of the universe. However, equation (4) only gives an overview of the cosmological parameters involved in the relation between the H_0 and t_0 . The full expression of the formula should be the equations as follows [11]:

$$t = H_0^{-1} \int_0^R \frac{dR}{\sqrt{\left(\frac{\Omega_{m,0}R_0^3}{R} + \Omega_{\lambda,0}R^2 + \Omega_{K,0}R_0^2\right)}} \tag{5}$$

Where R is the universe's length dimensional scale factor, and Ω_k is the spatial curvature density parameter dependent on the other density parameters [11]. Considering only the density parameters of Ω_m and Ω_λ , Ω_k is defined by one of Friedmann's equations as [19]:

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{m,0}a^{-3} + \Omega_{k,0}a^{-2} + \Omega_{\lambda,0} \tag{6}$$

In which a is defined as [19]:

$$a = \frac{R}{R_0} \tag{7}$$

To estimate the Hubble's constant at the present epoch, H_0 , using available observational data, all of the input values for the factors in equation (5) must all be in the present epoch. Therefore, equation (5) must be formatted as:

$$t_0 = H_0^{-1} \int_0^{R_0} \frac{dR}{\sqrt{\left(\frac{\Omega_{m,0}R_0^3}{R} + \Omega_{\lambda,0}R^2 + \Omega_{K,0}R_0^2\right)}} = H_0^{-1} \frac{\sqrt{\Omega_{m,0} + \Omega_{\lambda,0} + \Omega_{k,0}}}{2\Omega_{\lambda,0} - \Omega_{m,0}} \tag{8}$$

Since all values employed in this study are values of cosmological parameters at the present epoch, according to equation (6), at the present epoch, a is equal to 1. Therefore, equation (6) can be rewritten as:

$$1 = \Omega_{m,0} + \Omega_{k,0} + \Omega_{\lambda,0} \tag{9}$$

Merging equations (8) and (9), one results in:

$$t_0 = H_0^{-1}(2\Omega_{\lambda,0} - \Omega_{m,0})^{-1} \tag{10}$$

To fulfill the objective of this study, i.e., to estimate the Hubble's constant, equation (10) can then be rewritten as:

$$H_0 = t_0^{-1} (2\Omega_{\lambda,0} - \Omega_{m,0})^{-1} \quad (11)$$

Based on equation (11), one can estimate the Hubble's constant if the age, the cosmological and mass density parameters of the universe are known. According to the observations done by the Plank satellite in 2018 of the properties of CMB, the current age of the universe, t_0 , lies at about 13.787 ± 0.02 Ga. The same study measured the value of $\Omega_{\lambda,0}$ as approximately 0.6889 ± 0.0056 and the value of $\Omega_{m,0}$ as approximately 0.3111 ± 0.0056 [16]. With such data in hand, the Hubble's constant, H_0 is calculated to be:

$$H_0 = 66.49_{-1.13}^{+1.16} (km/s)/Mpc \quad (12)$$

In comparison to the resulting values of the Hubble's constant from previous studies, the result from this study for the Hubble's constant is lower than the resulting values of the Hubble's constant determined by most other studies [3]. In comparison to the results of the Hubble's constant estimated from the distance ladder method, which is around ~ 72 (km/s)/Mpc, the result of $66.49_{-1.13}^{+1.16} (km/s)/Mpc$ is closer to the result gained by the CMB anisotropy method, which is around ~ 67 (km/s)/Mpc [3, 5, 16]. A comparison between the resulting value from equation (12) and the resulting values from previous studies of the value of the Hubble's constant in cosmology can be seen in Table. 2.

A second possible new method to measure Hubble's constant is a variant of the distance ladder method. While most distance ladder methods employ standard candles, such as Cepheid variables, TRGBs, and Ia supernovae, and the standard siren to measure intergalactic distances, great uncertainties lie in the absolute magnitude of each of such standard candles. In some situations, the closest standard candles objects could be too far from Earth to accurately determine the absolute magnitudes of such objects, some types of standard candles could have too much of a variance in their absolute luminosity between different individual standard candles, and non-luminous dust and gas clouds can obscure light from standard candles and cause errors in the perceived and actual value of the distance to the galaxies. The error rate in using standard candles is as high as 2.29% for using TRGB as standard candles and 1.42% for using Ia supernovae as standard candles to estimating the Hubble's constant. Standard sirens, on the other hand, yield results with great uncertainties, due to the uncertainty of the masses of the massive compact bodies that produce the gravitational waves. The standard siren method has an error rate of as high as 13.89% for estimating the Hubble's constant. Therefore, standard candles and standard sirens cannot provide reliable highly accurate results.

To obtain more accurate results with more reliable methods, the parallax method can be employed to measure the distance to galaxies. However, currently the HST, one of the most advanced telescopes as of the writing of this paper, only has an accuracy of 20-40 μ as in measuring the position of distant bodies. The furthest measurable distance using parallax measures the distance to an object which, from the midpoint between the two points of observation, perpendicularly intersects the straight line between the two points of observation. In such a situation, the maximum measurable distance is described as:

$$d = \frac{a}{\sin p} \quad (13)$$

Here, d is the distance to the observed body, a is the distance between the two points of observation, and p is the parallax angle. Based on Eq. (13), it can be determined that the furthest measurable distance using the HST is $d \approx 25000pc$

Hence, the parallax method is impossible to provide accurate values for distances to objects on the intergalactic scale even using the HST. Moreover, the HST still may not be able to give a reliable precise value of the distance to an object at such distance. Therefore, parallax can only be applied to measuring intergalactic distances and the estimation of the Hubble's constant if the parallax method

is improved. On one hand, the precision of telescopes can be improved to reduce the value of p and increase the value of d , which could require the development of more precise instruments of measurement with the development of technology. Otherwise, the value of d can also be increased if a is increased, which would require a farther point of observation. This can be done by sending a medium-sized optical space telescope to interstellar space, like the previously launched Pioneer 10, 11, Voyager 1, 2, and New Horizons spacecraft. The size and mass of such a space telescope would be comparable to that of the Gaia space telescope launched by the ESA, and the journey to the outskirts of the solar system would be similar to the voyager spacecrafts. If the space telescope reaches the distance of 150AU from Earth, then it would be possible to use the parallax method to measure distances to galaxies within a 3.75 Mpc radius from Earth, which would encompass most galaxies of the local group and parts of other galaxy groups. Though it would take decades for the space telescope to reach such distances, the distance of the telescope from Earth would be continuously increasing, thus allowing it to provide more and more accurate values of distances to other galaxies from Earth as it travels further out into interstellar space.

A possible explanation for the difference between the value of the Hubble's constant from the distance ladder method and the CMB anisotropy method is that the results from observations are largely reliable, but the accelerating expansion of the universe causes a difference in the observed global and local rate of expansion. Using the distance ladder method to observe galaxies to estimate the Hubble's constant, only the observed galaxies that are usually relatively close to Earth are being surveyed, due to the difficulty to obtain detailed information on distant galaxies. However, the CMB anisotropy method considers the kinematic Sunyaev-Zeldovich effect done on CMB radiation by all matter in the observable universe. The observed kinetic Sunyaev-Zeldovich effect by more distant matter are done on CMB in the more distant past, as it takes time for the affected light to reach Earth from distant points in the universe. Nevertheless, since the expansion of the universe is and always had been accelerating, therefore the Hubble's constant is lower in the distant past, when distant galaxies made the Sunyaev-Zeldovich effect to CMB radiation. In reality, though, the distant matter are actually receding faster away than their perceived receding velocity, and that they are also further than their perceived distance, causing the Hubble-Lemaître law to be incompatible in describing perceived phenomenon on such distances. By estimating of the Hubble's constant by measuring of the anisotropies in CMB, instead of measuring only the Hubble's constant of the present epoch, past values of the Hubble's constant, which is less than its value at the present epoch, is also a contributing factor to the final result estimated by the CMB anisotropies method. In comparison, the distance ladder method only measures nearby galaxies, which have an observed distance and receding velocity similar to that of their actual distance and receding velocity. Therefore, the Hubble-Lemaître law is mostly compatible in describing observed phenomenon using the distance ladder method, and the result gained from the distance ladder method will be more similar to the actual value of H_0 . As a result, the value of the Hubble's constant, which is higher in the present than in the past, is observed to be larger from results by the distance ladder method than from the CMB anisotropies method. The effects are shown in Fig. 1.

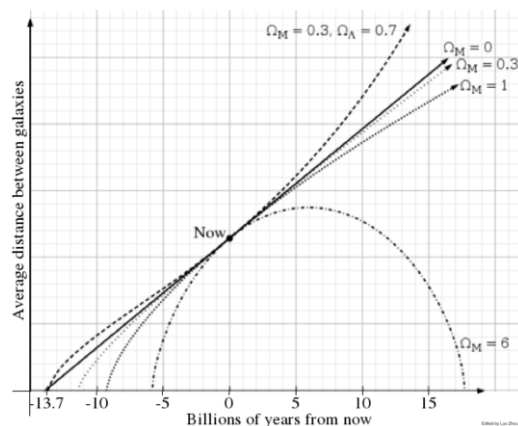


Fig. 1 The effects of the values of the cosmological density parameters of Ω_Λ and Ω_m on the course of the expansion and the ultimate fate of the universe.

Table 2. The value of the Hubble’s constant estimated using different methods.

Method of Estimation	Median Value
Cepheid variables	72
TRGB	69.8
Ia Supernovae	73.04
Standard Siren	70
CMB anisotropies	67.3
Cosmological parameters derivation	66.49

5. Limitations and Future Outlooks

This study proposed 2 methods to re-estimate the value of the Hubble’s constant, which is to deduce the Hubble’s constant with other cosmological parameters, and to perform stellar parallax distancing on the galactic scale. Moreover, the study also proposed an explanation for the disagreement between the values of the Hubble’s constant between the distance ladder method and the CMB anisotropies method, which had been described as a “cosmological crisis”. However, limitations exist within the methods as proposed above, causing uncertainties and casting doubts in the value of the Hubble’s constant from the estimation done above.

Firstly, the estimation above employed equation (11), which is derived from other formulae in cosmology, to mathematically deduce the Hubble’s constant using preexisting observational data. While the derivation of equation (11) is mathematically logical and will yield accurate results for the Hubble constant, the values of t_0 , $\Omega_{\Lambda,0}$, and $\Omega_{m,0}$ are all obtained from the 2018 analysis of observational results of the properties of CMB by the Planck Satellite. The greatest concern in the data provided by the Planck mission is that the production of the results for t_0 , $\Omega_{\Lambda,0}$, and $\Omega_{m,0}$ are mathematically derived with the inclusion of the value of the Hubble’s constant resulting from the Planck mission’s estimation, which is a possible explanation for the similarity between the values of the Hubble’s constant derived from this study and the results from the Planck mission in 2018. Another concern about the final result for the value of the Hubble’s constant in equation (11) is that it’s difference with the distance ladder method is greater than the difference between result yielded by the CMB anisotropy method and the distance ladder method.

The original aim of estimating the value of the Hubble’s constant in equation (11) is to yield a result independent of any observational bias or other possible errors, but the proposed explanation for the discrepancy in the resulting value of Hubble’s constant from the CMB anisotropy method and the distance ladder method in the previous section states that: “the result gained from the distance ladder method will be more similar to the actual value of H_0 ” and: “by estimating of the Hubble’s constant by measuring of the anisotropies in CMB, instead of measuring only the Hubble’s constant at the present epoch, the Hubble’s constant in the past, which is less than its value at the present epoch, is also a contributing factor to the final result estimated by the CMB anisotropies method”. This implies an inconsistency between the results in equation (11) and the proposed explanation for the discrepancy between the results of the value of the Hubble’s constant estimated by the distance ladder method and the CMB anisotropy method. The possibility that the Planck mission’s published results are dependent on their estimation of the Hubble’s constant is the possible cause of this inconsistency, as the values of t_0 , $\Omega_{\Lambda,0}$, and $\Omega_{m,0}$ in equation (11) are all dependent on the Sunyaev-Zeldovich effect on CMB, which is proposed in this study to be an unreliable source of data, and that the process in equation (11) is but an inverse calculation for the 2018 results published by the Planck mission from its other derived cosmological parameters in the same study. Secondly, the second proposed method to re-estimate the Hubble’s constant using stellar parallax on an intergalactic scale may not be economically profitable, as launching a spacecraft with a launch mass of more than 1500kg may require more powerful launch systems not viable as of the writing of this paper.

In the future, as more work is done on to investigate the Hubble's constant and as more attempts are being made to provide an accurate value for the Hubble's constant to resolve the discrepancy for the value of the Hubble's constant between the results from different studies, there could be 3 likely possible scenarios for the future development of the current "cosmological crisis" on the Hubble's constant and the related Λ CDM model: either that the currently perceived discrepancy problem on the value of the Hubble's constant is eventually explained by future observations that revealed previously unknown features of the expansion of the universe, thus solving the "cosmological crisis", determining the most reliable value between the currently disputed values, and thus providing supporting arguments for the Λ CDM model. Besides, the Hubble's constant is being estimated by a newly proposed method with the advent of technologies used in deep-space astronomy, and the resulting value is convincingly accurate to be widely accepted by most cosmologist, thus providing supporting arguments for the Λ CDM model; or that newly made observations provide disproving evidence to the Λ CDM model, starting a new era in cosmology in which a new cosmological model gives a more accurate description of observed phenomena.

6. Conclusion

In conclusion, this paper investigates possible new methods to re-estimate the disputed value of the Hubble's constant based on extending pre-existing methods of estimating the value of the Hubble's constant. Specifically, the value of the Hubble's constant from other cosmological parameters using pre-established formulae in cosmology are mathematically deduced. Additionally, the possibility to conduct parallax ranging of distances on the intergalactic scale are discussed. Besides, this study attempted to provide a possible explanation for the currently perceived discrepancy in the resulting values for the Hubble's constant across different studies. An overview of previous work in the realm of cosmology is also summarized, which was carried out to estimate the Hubble's constant. Furthermore, brief overviews of the possible scenarios of the future development of the Hubble's constant are also given. Overall, these results offer a guideline for any future estimations of the Hubble's constant.

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