Comparison of Performances for Quantum and Conventional Algorithms: Shor's algorithm and Boson Sampling

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Abstract. Quantum computing has been extensive popularity discussed recently. The foundation of quantum study is the investigation of how quantum computing shows its superiority compared with conventional algorithms. On this basis, this paper focuses on the comparison of the performances of quantum and conventional algorithms about the same relevant area. Two classic problems, factorization of large integer and boson sampling, are chosen to show the specific difference of related quantum and conventional algorithms towards the same problem. This paper specifically compares the performance of Shor's algorithm and other advanced traditional integer factorization algorithms for prime factorization problems and compares the processing capabilities of traditional algorithms and Gaussian Boson sampling for sampling problems. According to the analysis, for the same research problem, quantum algorithms show strong advantages over traditional algorithms in many performances, e.g., time efficiency, algorithm adaptability. It leads to a deeper understanding of quantum superiority and a further explanation of the significance of the development of quantum computing.

Keywords: quantum computing; quantum algorithm; Shor's algorithm; Boson sample.

1. Introduction

In the digital era, the scale of data and information has been growing explosively. The conventional computers have limited performance during the processing of tremendous size of data because of the shortcomings in computing power, bits, structures, and programming languages. However, quantum computer signifies its unique advantages when processing storage mess data and parallel computing. Quantum computing is expected to satisfy the powerful computing capabilities required in the areas of digital signal processing, military intelligence and logistics management. Recently, researchers all over the world set about the study of quantum computing. A set of ideas and technological frameworks regarding the essence and processing of information based on the laws of quantum mechanics make up quantum computing and even the area of quantum information. Quantum computing has obvious development, and its expansion extends from the basic research (e.g., the most advanced mathematics and physics, to the cross integration with different engineering disciplines, and even to the exploitation of highly engineered applications).

Three distinct periods can be identified in the development of quantum algorithms, i.e., theory proposition and exploration stage (1980-1994), universal quantum algorithms development stage (1994-2009) and Quantum Algorithm Engineering Stage (Since 2009). In each period, some specific focus points are studied. Primarily, Physicist Feynman first came up with the "concept of quantum computer", and point out that while it is challenging to accurately replicate the evolution of quantum systems using conventional computers, such simulations are possible under the help of quantum computers. In 1985, Deutsch proposed the first quantum algorithm, the Deutsch algorithm [1]. In 1992, Deutsch and Jozsa, extended the early Deutsch algorithm and gave its algorithm under n qubits [2]. It is the first algorithm designed specifically for quantum computers and shows that quantum computers are faster and more effective.

For the second stage, Shor proposed the Shor's algorithm in 1994 [3], which demonstrated that quantum computers can solve the problem of prime factorization in polynomial time. It is the first quantum algorithm exponentially increases the time complexity of the fastest traditional algorithm, which greatly promotes the academic community's attention and investment in quantum computing.

In 1996, Grover proposed the Grover quantum search algorithm [4], which solves the problem of unordered database search. Grover's algorithm is the first quantum algorithm that is completely experimentally implemented and remains one of the benchmark experiments for different quantum computing platforms. In 2009, Harrow, Hassidim and Lloyd proposed the HHL algorithm to find the solution of systems of linear equations, it also has an exponential speedup compared to the conventional algorithm [5]. Systems of linear equations is the core area of many scientific and engineering fields. Hence, the HHL algorithm shows the advantages of quantum computing in many fields of machine learning and data fitting [6].

For the last phase, since 2009, some companies such as Google and IBM bagan to develop the engineering of large-scale quantum computers. During this process, many related quantum algorithms have emerged: some optimized algorithms including Variational Quantum Eigensolver (VQE), Quantum Approximation Optimization Algorithm (QAOA), etc. [7, 8]. Some sampling algorithms, including Boson Sampling, Quantum Walk and other algorithms [9].

Based on above discussion, in order to find the difference of quantum and conventional algorithms, this paper will discuss the performance of different algorithms based on factorization problem and Boson sampling problem. The rest part of the paper is organized as follows. The Sec. 2 will describe the quantum computing. The Sec. 3 will compare the performance of Shor's algorithm and other advanced traditional integer factorization algorithms for prime factorization problems and the Sec. 4 compares the processing capabilities of traditional algorithms and Gaussian Boson sampling for sampling problems. In the Sec. 5, the comparison result is given systematically. The Sec. 6 discusses the limitations and prospects of this paper and the conclusion is given in the final section.

2. Description of Quantum Computing

In this section, the fundamental of quantum computing is discussed. The principle of quantum computing can be explained by two parts. One part is the physical principles and physical realization of quantum computers; the other part is quantum algorithms. Classical computers use bits as their fundamental data units. A bit is a physical system having two states, which are represented by 0 and 1. Quantum bits (Qubits) serve as the fundamental data unit in quantum computers. To represent the 0 and 1 states, qubits use the two quantum states |0> and |1>. Qubits have different characteristics of existence from bits. They exist as a superposition of two logical states, which means that the corresponding quantum states of 0 and 1 are superimposed on the two states.

One fundamental idea in quantum physics is the superposition of states. Every possible motion within a system is referred to as a state. In contrast to the determined motion state of the macroscopic system, the quantum motion state in the microscopic system is statistical and cannot be determined. A tiny system's state is referred to as a quantum state. A superposition state is the result of combining many quantum states of a quantum system in a normalized linear fashion. Quantum computers can save a lot of memory units and can be quite efficient. The powerful computing power of quantum computing comes from the principle of quantum parallel computing. Quantum algorithms are algorithms specifically applied to quantum computing. Different from conventional computer algorithms, quantum algorithms can utilize the enormous parallel processing capabilities of quantum computers. This is also an important reason why quantum algorithms need to be greatly developed.

3. Performance based on Shor's algorithm

Shor's algorithm focuses on a classic mathematical problem, finding prime factors of an integer. This problem is shown as follows. Given a composite number N and N, one has unique prime factorization P1 and P2, i.e., $N = P1 \times P2$, but P1 and P2 is unknown. Then, one needs to find P1 and P2. Many different algorithms are committed to solve the problem. As the most popular encryption algorithm, such as RSA, assumes that large integer factoring is computationally challenging. Obviously, this assumption is valid for classical computer. Shor's algorithm, however,

reveals that factoring integers is efficient on an ideal quantum computer, indicating that building a big quantum computer may be possible to defeat RSA. In this section, this study compares the difference between one of the most efficient classic algorithm and the Shor's algorithm.

3.1. Operation Process

3.1.1 Classic Algorithm

Owing to the popularity of the factorization problem, there are many different algorithms to solve it based on classic computer. In this part, several different factorization algorithms are discussed. One of the most traditional algorithms is trial division and it is the least intelligent and efficient algorithm. Comparatively, the simplest classic algorithm is the Pollard's rho and Pollard's P-1 algorithm [10]. These two algorithms significantly improve the efficiency of prime factorization by simplifying the number of random numbers that need to be verified in trial division method. Whereas, it's worth noting that Pollard's P-1 algorithm is based on the condition that as for the target factor p of N, there can be a B found which is large than every factor of p-1 and B is a number chosen by users. So, Pollard's p-1 algorithm is not available for all circumstances [11]. There is also algorithm of factorization based on elliptic curve and decomposition algorithm based on perfect square proposed to solve the problem and both methods have efficiency improvement. So far, GNFS (General Number Field Screening method) is one of the most efficient algorithms to factor large integer [12]. The latest large number decomposition theory and technology includes five basic steps: polynomial extraction, sieve method, filtering, large-scale sparse linear equation solving and algebraic square root solving.

3.1.2 Shor's algorithm

The factorization problem is split into two subroutines using Shor's algorithm: a classical subroutine and a quantum subroutine. The classical subroutine can be quickly completed on a classical computer and reduces the original issue to the issue of order-finding. The order-finding problem is solved using a quantum algorithm in the quantum component. In the classic subroutine, one needs to simplify the problem into a problem related to periodic problem. Firstly, a random integer number a less than N is chosen and the greatest common divisor K=gcd(a,N) of a and N is computed by Euclidean algorithm. If K≠1, K is the non-trivial factor of N, the problem is done directly. If K=1, one turns to quantum subroutine to figure out the unknown period r of $f(r)=a^r \bmod n$. In other word, r is the smallest integer to make x+r and x share the same value of function f. If the final r is not even, return to the first step and choose another integer number. Otherwise, r is even; $(a^{r/2}-1)(a^{r/2}+1)=0 \bmod N$. If $a^{r/2}+1=0 \bmod N$, turn back to the first step either. Otherwise, the final non-trivial factors of N are acquired: $\gcd(a^{r/2}-1, N)$ and $\gcd(a^{r/2}+1, N)$. In the quantum subroutine, for each choice of N and each choice of a used in $f(x)=a^x \bmod N$, find $Q=2^q$ such that $N^2 \leq Q < 2N^2$, which means $Q/r > N$. Two different registers are needed in this part, register 1 holds values from 0 to Q-1in the form of superpositions, while register 2 store the result $f(r)=a^r \bmod n$. The state of the two registers is shown as follow:

$$|\text{Register1}\rangle|\text{Register2}\rangle = |r\rangle|f(r)\rangle \tag{1}$$

Firstly, one initializes the two registers with 0 qubit. For register 1, one applies quantum Fourier transform as follow:

$$U_{\text{QFT}} = \frac{1}{\sqrt{Q}} \sum_{x=0}^{Q-1} |x\rangle|0\rangle. \tag{2}$$

Then, registers 1 holds superposition of values from 0 to Q-1. One constructs f(x) as a quantum function and applies it to the above state. The linear U_{QFT} to the two registers is given as follow:

$$U_{\text{QFT}}{}' = \frac{1}{\sqrt{Q}} \sum_{x=0}^{Q-1} |x\rangle|f(x)\rangle. \tag{3}$$

The quantum entanglement of the two registers is acquired. Next, measure register 2. A periodic superposition generates within register 1. Then, $|f\rangle \rightarrow f(x_0)$ and $|x\rangle \rightarrow x'$ when $x' = x_0+np$, n=1, 2, ..., (N/t-1)r. Measurement M are shown as follow:

$$M = \sqrt{\frac{r}{Q}} \sum_{i=0}^{\frac{Q}{r}-1} |x_0 + ir\rangle |f(x_0)\rangle. \quad (4)$$

Applying Fourier transform to register 1,then one can measure it as:

$$\sqrt{\frac{r}{Q}} \sum_{i=0}^{\frac{Q}{r}-1} |x_0 + ir\rangle \rightarrow M = \sqrt{\frac{1}{r}} \sum_{i=0}^{r-1} \left| i\frac{Q}{r} \right\rangle \emptyset i\,. \quad (5)$$

The result of measurement is a multiple of Q/r. Going back to the first step, getting enough distinct results and computing gcd of the retrieve Q/r will be the final step [13].

3.2. Performance Comparison

Many different characters of an single algorithm reflect on the performance of a algorithm, whether they be the largest integer size can be processed, time complexity etc. As for the factorization, the most important index to evaluate the ability of a algorithm is the speed to calculate the final factors. In this part, the integer sizes and time complexity applicable of different algorithms is discussed in this part. The other performance application features have been discussed in the previous section. Firstly, one needs to consider the performance of the traditional algorithms. Applicable integer size of an algorithm means the means the maximum number of digits of an integer this algorithm can handle. The specific performance of each algorithm is shown in the Table 1. The time complexity analysis of traditional algorithms involves sophisticated mathematical theoretical derivation, time complexity comparisons are listed in Table. 2.

Table 1. Applicable Integer Size of Different Algorithm

Algorithm	Maximum Integer Size
Trial Division	<110bit
Pollard's Rho algorithm	<110bit
Pollard's p-1 algorithm	<110bit
Elliptic curve decomposition algorithm	<110bit
Algorithm based on perfect square	>110bit
GNFS	>110bit

Table 2. Applicable Integer Size of Different Algorithm

Algorithm	Time Complexity
Trial Division	$O(N^{\frac{1}{2}})$
Pollard's rho algorithm	$O(\sqrt{p}(\log n)^2)$
Pollard's p-1 algorithm	$O(B\log B(\log n)^2)$
Elliptic curve decomposition algorithm	$O(e^{(1+o(1))(2\ln p \ln\ln p)^{\frac{1}{2}}})$
Algorithm based on perfect square	$O(e^{(\frac{64}{9})^{\frac{1}{3}}(\ln p)^{\frac{1}{3}}(\ln\ln p)^{\frac{2}{3}}})$
GNFS	$O(e^{(1.92+o(1))(\ln n)^{\frac{1}{3}}(\ln\ln n)^{\frac{2}{3}}})$

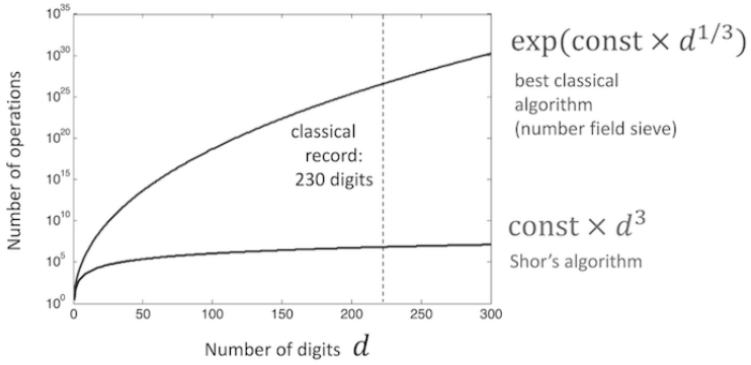

Fig. 1 The number of classical algorithms and quantum Shor's algorithm operations required to calculate the number of natural d bit.

Now, one focuses on the performance of Shor's algorithm. It is a quantum algorithm, so in theory, the integers it can handle can be very large, much larger than 110bit. Shor's algorithm takes $O((\log n)^2(\log\log n)(\log\log \log n))$ steps. It has the ability to factor a integer in polynomial time. Hence, it has a huge speed improvement [14]. The number of operations classical algorithms and Shor's algorithm required to calculate the number of natural d bit is shown as Fig. 1. The version of the algorithm described below, requires roughly 10d qubits, and has runtime roughly d^3 [15].

4. Performance based on Boson Sampling

For the quantum algorithm engineering stage, there have been many quantum algorithms applied to specific engineering area. Instead of other quantum algorithms just focused in the field of complex mathematical calculation, they have shown incomparable advantage in many performances except time complexity. A classical problem in engineering physics, sampling problem, is discussed.

4.1. Boson Sampling

The generalized sampling problem refers to obtaining a sample of a specific distribution. Thus, the properties of the distribution behind the sample can be obtained from the results of multiple independent sampling. In the quantum sampling problem, under the specified measurement basis, the final quantum state after a quantum process can be regarded as a specific distribution of fundamental vectors, which means that one measurement of a quantum state corresponds to one sampling. Boson sampling is a quantum process built on optical systems [16]. A common Boson sampling injects a photon into each of the first *n* modes at the input of the linear optical network, the component coefficients of the components of the linear optical network are random to some degree, and the number of photons is sampled at the output [17].

Table 3. Performance of Different Conventional Sampling Algorithm

Algorithm	Number of Sampled Photons	Time Performance
Metropolis Independent Sampling Algorithm [18]	30	laptop
	50	supercomputer
Clifford's Exact Sampling Algorithm [19]	n	$O(n^{2n})$
Wu's Algorithm [20]	n	$O(m \sin h^2 r) + O(\text{poly}(n)2^{\frac{8n}{3}})$
Quesada's Classical Simulation Algorithm for Thresholded GBS Models [21]	20	Huawei Kunlun Server
	20	56CPU cloud

4.2. Performance of Conventional Algorithms

Aaronson and Arkhipov proposed that the prototype of Boson sampling needs a linear optical network device. Because the prototype needs to prepare many high-quality single photons. To realize this process experimentally is difficult either. They proposed that the distribution probability of n-photon Boson sampling is proportional to the modulus of the n-dimensional matrix product and sum formula. From the perspective of computational complexity, the difficulty of solving the product-sum formula is #P-hard. Obviously, to solve #P-hard problem the current classical algorithm requires $O(n^{2n})$ steps, and as the number of photons rises, the number of solution steps also grows exponentially. For such a classically computationally #P-complete hard problem, conventional computers fail even in the small data scales. Afterwards, the classical simulation of Bose sampling and its derived problems has also made some progress in recent years. Some typical sampling algorithms and their corresponding photon performances are shown in Table 3:

4.3. Superiority of Quantum Algorithms

Boson sampling is a method of calculating how a linear optical circuit outputs results with multiple inputs and outputs. Only one photon enters the circuit in parallel and encounters beam splitters and other optical components [20]. On account of boson property, when two photons enter the beam splitter simultaneously, they take the same path. Ascribed to this feature, even with an adequate number of input photons and output channels, it is very challenging for conventional computers to calculate the output of a circuit. While Bose sampling is hard to implement and requires advanced quantum optics, it performs much better than the most powerful supercomputers in theory. As shown in the previous sections, under some specific circumstance, the quantum random circuit are suitable and the number of processed photons is moderate, the conventional algorithm can deal with the sample process. However, when the scale and accuracy of quantum random circuits reach a certain threshold, it will no longer be possible to find algorithms that can be efficiently simulated on a classical computer in a few hours or even years. Recently, the Gaussian Bose sampling realized by Pan's team on the experimental platform based on optical devices has not been effectively simulated by any classical computers, and fully shows the progress of quantum superiority.

Pan utilized a similar method known as "Gaussian boson sampling," which substitutes a single-mode squeezed light state for the single photon [22]. Gaussian boson sampling provides a comparable quantity instead of revealing the circuit's constant state, which is very challenging to compute with standard traditional computers. Pan's team used more suitable light sources, interference devices and photon detectors to obtain the output of 76 photons. Thousands of samples were sampled only in 200 seconds. Bose sampling process was completed, and the probabilities under various conditions were obtained. It’s estimated that Sunway TaihuLight, the most efficient supercomputer in China, will spend over 2.5 billion years to complete the calculations.

5. Comparison

Quantum computing has been developed for several years, and applications of quantum algorithms have been applied in various fields. Two specific problems are discussed in this paper and represent two important stages of quantum algorithms. Obviously, the application of quantum algorithms is likely to be based on conventional algorithmic research about this problem. The comparison of quantum and traditional algorithms to some extent reflects the scientific significance of quantum computing and the development of quantum computers. In the universal quantum algorithms development stage, many quantum algorithms emerged and have been developed. At this stage, the research of quantum algorithms focuses on improving the algorithm time complexity of a certain problem and solving many problems that are very complicated or even impossible to solve on traditional computers.

The third section details the performance of Shor's algorithm and conventional algorithms on the problem of factoring prime factors. In terms of processing data length, there are only a few traditional

algorithms have a better performance, such as Algorithm based on perfect square which can handle integers of more than 110 bits, but the calculation time is not ideal. Most algorithms cannot handle the prime factorization of integers with very large digits, which makes them not capable of attacks such as RSA algorithm. In terms of processing time, Sec. 3 points out that the time complexity of the fastest algorithm among traditional algorithms, GNFS, is still as high as $O(e^{(1.92+o(1))(\ln n)^{1/3}(\ln \ln n)^{2/3}})$. It is almost impossible for traditional computers to process this algorithm. Shor's algorithm successfully reduces the time complexity to polynomial time, which makes the problem of factoring large numbers into a mathematical problem that can be done quickly. In fact, the research of other quantum algorithms at this stage (e.g., Grover's algorithm and HHL algorithm) all have the same advantages compared to the traditional algorithms in their research problem area. They also optimize the time complexity of a particular problem, most of which are classical complex mathematical problems, and most of them can be improved exponentially.

Subsequently, the development of quantum computing step into a new stage, in which the development of quantum algorithms entered the engineering stage. More and more quantum algorithms focus on specific engineering problems to improve the performance of the problem. The fourth chapter of this paper takes the Boson sampling as an example. Traditional algorithms or sampling processes relying on traditional computers can only deal with the sampling of a certain number of input photons, and the time complexity of many algorithms reached an exponential level. Pan's team established an experiment to prove quantum superiority. In optical systems, increase the number of particles (e.g., the number of photons in interferometers and the number of bits in superconducting circuits) to the results that classical simulations were not feasible. The superiority of quantum computing may not only be reflected in the complexity of the algorithm itself or the optimization of the algorithm, but also in the superiority of the quantum computing device itself compared to traditional computers.

6. Limitations & Prospects

Compared with conventional algorithms, quantum algorithms have superior performance. However, influenced by the development of quantum computers, the specific implementation process of these quantum algorithms is not easy. The hardware technology route of quantum computing includes superconductivity, photon quantum, ion trap, etc. [23]. However, due to the disadvantages of each technical route in realizing quantum computing applications, the hardware to support quantum computing is seriously insufficient. Meanwhile, scene adaptability of quantum algorithms is not ideal than that of classical computing. The logic of classical computing is simple, but it can be applied in various problems easily. Nevertheless, quantum computing has extremely strict requirements on the type of operation, and the migration ability between different algorithms is very weak. However, it is undeniable that quantum algorithms bring great development prospects. It is generally predicted that quantum computing is expected to land earlier in many scenarios.

One of them is cryptanalysis. Encrypting and breaking ciphers is an ongoing theme throughout history. The development of Shor's algorithm made it possible to decipher public key systems such as RSA-based system. It has important practical value to carry out code breaking. In the future, the research on other means against quantum computing's attack on communication security, quantum secure communication, will play a key role, such as quantum key distribution and quantum direct communication. Another possible development is to simulate quantum phenomena. Gaussian boson sampling, as a new mode of boson sampling, not only provides an efficient method for large-scale implementation, but also has potential applications in graph and quantum chemistry-based problems. Besides, quantum computing in other fields can provide powerful tools for protein structure simulation, drug research, new material research, and new semiconductor development.

7. Conclusion

In conclusion, this paper investigates comparison of the performances between Quantum and Conventional Algorithms based on two representative quantum algorithm of different quantum developing stage, i.e., Shor's algorithm and Boson Sample. According to the analysis, Shor's algorithm shows an obvious advantage on the factorization problem in terms of the bit of target integer and time complexity. Meanwhile, Gaussian Boson sampling exhibits the superiority of quantum computing compared with conventional sampling algorithm. In the future, quantum computing is expected to lead to excellent develop on the related area because of the advantage of quantum algorithms.

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