The Residue Theorem

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Abstract. Cauchy's Residue Theorem, commonly known as the residue Theorem, is a key theorem in complex variables because it enables people to determine the enclosed curve line of integrals for functionalities. Real integrals and infinite series may also be calculated using it. Our ability to handle complicated analysis is improved by the residue theorem. By connecting the internal Ehrhart polynomials to the closures, we illustrate these tetrahedron Ehrhart-Macdonald reciprocity laws. We calculate the Earhart coefficient of the codimensions to demonstrate our methodology. We conclude by demonstrating how to use our ideas to locate any paste applied with a convex lattice.

Keywords: Cauchy's residue Theorem Ehrhart Polynomial Integration, Lattice point counting, Riemann surface.

1. Introduction

Because it enables individuals to assess the range for integrals of integrals in shuttered curves, Cauchy's Residue Theorem, also known as residue Theorem, is an application of specific in complex variables. Real integrals and exponentials may also be derived using it. The residue theorem enhances our capacity for handling intricate analysis. The residue theorem may be used to describe the numbers of lattice points in an expanded, n-dimensional tetrahedron with vertex at lattice points on each coordinate line and the origin, which is referred to as the Ehrhart polynomial. As we show, it is polynomials in t, where t is the integral dilatation parameter. We prove the Ehrhart-Macdonald equivalence rule for these tetrahedral by establishing a connection between the interior Ehrhart's polynomials and the closure of the tetrahedron. To illustrate our approach, we compute the coefficient claimed by Ehrhart for similarities. We conclude by showing how our ideas may are applied in order to determine Ehrhart's polynomials for any built-up with a convex lattice at last.

1.1. Complex-value functions

Definition 1.2.1 Continuous function

The function f is continuous at $z_0 \in$ C if $\lim_{z\to z_0} f(z) = f(z_0)$ exists.

More explicit: For any γ > 0, there exist $\sigma = \sigma(\gamma)$ such that:

|f(z) – f(z_0)| < γ when |z - z_0| < $\sigma(\gamma)$

Theorem 1.2.2

A continuous function f defined on a compact set t τ, then it has it maximum and minimum value.

Definition 1.2.3 Holomorphic (Differentiable) function

The function f is holomorphic at point $z_0 \in$ C if $\lim_{z\to z_0} \frac{f(z)-f(z_0)}{z-z_0}$ exists and denoted as $f'(z_0)$.

More explicit: For any γ > 0, there exist $\sigma = \sigma(\gamma)$ such that:

$|\frac{f(z)-f(z_0)}{z-z_0} - f'(z_0)| < \gamma$ when |z - z_0| < $\sigma(\gamma)$

Definition 1.2.4 Entire function

The function f is said to entire if it is holomorphic on C.

Definition 1.2.5 Contour integration

As for the holomorphic function f and closed path δ in the complex plane, we have: $\int_\delta f = 0$.

Proposition 1.2.6

When m, n are holomorphic functions defined in set γ, we have:

a) m + n is holomorphic in γ and $(m+n)' = m' + n'$

b) mn is holomorphic in γ and $mn' = m'n'$

c) when a $\in \gamma$ and n(a) ≠ 0, $\frac{m(a)}{n(a)}$ is holomorphic and $(\frac{m(a)}{n(a)})' = \frac{m(a)'n(a) - n(a)'m(a)}{n(a)^2}$.

d) When function m: a → b and n: b → c is holomorphic, we have:
$(nm)'(z) = n'(m(z))m'(z)$ for all z $\in$ a.

Theorem 1.2.7 Cauchy-Riemann condition

f be the function on set γ and f can be represented in Cartesian form:
$f(x+iy) = U(x,y) + iV(x_0+y_0)$
Assume f is holomorphic at $z_0 = x_0+iy_0$, then
$U_X(x_0,y_0)$, $U_y(x_0,y_0)$, $V_X(x_0,y_0)$, $V_y(x_0,y_0)$ exist
$U_X(x_0,y_0) = V_y(x_0,y_0)$
$U_y(x_0,y_0) = -V_X(x_0,y_0)$

Definition 1.2.7 Integration along path

As for the continuous function f, the integration along the path γ from a to b shown as:
$\int_\gamma f = \int_a^b f(\gamma(t))\ \gamma'(t)dt$
When a = b, then the path γ is a **closed path**.

Definition 1.2.8 Primitive

If g(z) holomorphic and satisfies $g'(z) = f(z)$, then g(z) is called a primitive of f.

As for the continuous function f defined on the set U and has its primitive g. Suppose there is a path γ from points a to b in Up, we have: $\int_\gamma f = g(b) - g(a)$

The holomorphic function f defined in C and $f' = 0$, then f is constant.

As for the continuous function f defined on open set U has its primitive. The integration along a closed path γ in U as: $\int_\gamma f(z)dz = 0$

Proof:

Since the function f has its primitive on set U, denote $g' = f$ and the integral along the path γ from starting point a to ending point b as: $\int_\gamma f = g(b) - g(a)$.

As for the closed path γ , thus a = b.

In such case, $\int_\gamma f = g(b) - g(a) = g(a) – g(a) = 0$. So that $\int_\gamma f(z)dz = 0$.

Theorem 1.2.10 Goursat's theorem:

The holomorphic function f defined on open set U. The integral along closed triangle path $\gamma \subset U$ as: $\int_\gamma f = 0$.

Decompose the rectangle path $\gamma^{(0)}$ into 4 triangles as $\gamma_1^{(1)}$, $\gamma_2^{(1)}$, $\gamma_3^{(1)}$, $\gamma_4^{(1)}$ by bisecting each side and its length $l^{(0)}$ and width $w^{(0)}$. Then it is obvious that: $\int_\gamma f = \sum_{i=1}^4 \int_{\gamma_i^{(1)}} f$.

By triangle inequality $\int_\gamma |f| \leq \sum_{i=1}^4 |\int_{\gamma_i^{(1)}} f|$

Consequently, there exists j $\in$ {1,2,3,4} such that: $|\int_{\gamma_j^{(1)}} f| \geq \frac{1}{4}\int_\gamma |f|$

$\int_\gamma |f| \leq 4|\int_{\gamma_j^{(1)}} f|$

Then, we decompose the specific rectangle $\gamma_j^{(1)}$ into 4 rectangles as $\gamma_1^{(2)}$, $\gamma_2^{(2)}$, $\gamma_3^{(3)}$, $\gamma_4^{(4)}$ and its diameter $d^{(1)}$ and perimeter $p^{(1)}$ by bisecting each side and so on.

Iteratively, the conclusion would be: $\int_\gamma |f| \leq 4^n|\int_{\gamma_j^{(n)}} f|$ and $l^{(0)} = 2^n l^{(n)}, w^{(0)} = 2^n w^{(n)}$ after decomposing n times.f(z) is differentiable or holomorphic at z_0 if $lim_{z\to z_0}(f(z) - f(z_0))/(z - z_0)$ exists. If the limit exists, we denote it by $f'(z_0)$. More explicitly, $\forall\varepsilon > 0, \exists\delta(\varepsilon)\ such\ that\ 0 < |z - z_0| < \delta(\varepsilon) \Rightarrow |(f(z) - f(z_0))/(z - z_0) - f'(z_0)| < \varepsilon$. If f is holomorphic on D, then f is infinitely differentiable on D. Moreover, for $\underline{D_r}(z_0) \subset D$, if $C_r(z_0)$ is a positively oriented circle, then$f^{(n)}(z) = \frac{n!}{2\pi i}\int_{C_r(z_0)} \frac{f(\xi)}{(\xi - z)^{n+1}}\, d\xi$,for all $n \in N_0$ and $z \in D_r(z_0)$.

(Residual formula)

Let U be an open set, and γ a closed chain in U such that γ is homologous to 0 in U. Let f be analytic on U except at a finite number of points z_1, …, z_n. Let m_i = W(γ, z_i). Then $\int_\gamma f = 2\pi i \sum_{i=1}^{m} m_i\, Res_{z_i} f$.

Lemma 1.3

Let f have a simple pole at z_0, and let g be holomorphic at z_0. Then

$$Res_{z_o}(fg) = g(z_0) * Res_{z_0}(f)$$

Suppose f(z_0) = 0 but $f'(z_0) \neq 0$. Then $\frac{1}{f}$ has a pole of order 1 at z_0 and the residue of $\frac{1}{f}$ zt z_0 is $\frac{1}{f'(z_0)}$.

2. METHOD

2.1. Applications of the Residue Theorem in geometry and dynamics fields by Jesu´s Mucin˜o–Raymundo Carlos Valero–Vald´es

2.1.1 Use the cut and paste method to deal with complex analysis

In differential geometry, the cut-and-paste technique is practical and straightforward for building appealing manifolds. The summary of this is given in the sentences that follow. Choose a few appropriate geometric forms and cut them into smaller pieces first. Use the suitable paste strategy to reassemble the components. Add as many of the previously cut pieces as you can. Based on the above-mentioned methodology, the paper's authors pose various queries. Since it is unclear how many manifolds will develop, it is essential to investigate the many types of flat polyhedral structures and the laws that cause them to appear.

The dynamics of the meromorphic vector field X is an additional instrument. Consider f(z) has a true tangent vector range in M at z and define Re(X) = X + (X) as the related real vector field. We can create a dynamical system called R × M→ M, using the classical theory of (real) differential equations. This dynamical system can only be defined practically everywhere in R × M for generic X. Very intriguing characteristics can be found in mixing the vector field R(X) with the polyhedral structure g. For instance, the geodesics of g are the trajectory solutions of Re(X). The merging of the vector field and the polyhedral structure g has several extremely intriguing characteristics. For instance, the Geodesics of g make up the trajectory solutions of Re(x). Another essential feature is that singularities in falt polyhedral structures originate from the analytic functions vector field's decimal digits and polarity, adding a great deal of flexibility and richness. Simple zeros of a meromorphic vector field, in particular, provide birth to sources, centers, or sinks of Re from a dynamical perspective (X).

According to the traditional residue theorem, every meromorphic differential from won a compact Riemann surface has a residue sum of zero. Be aware that information on the w residues shifts to information on the linear parts at the X zeros for simple singularities. The following reduce some geometrical and dynamical flavor:

When the Riemann surface is compact, let x be a continuously differentiable unit vector (M, J) having only simple zeros with linear parts. Consider Re(X) its natural associated vector field, having an empty set 1 as time-1 flow, and g the individual flat on M, the Riemannian metrics-as before, zeros and X poles. After that

1., 1.- 1 leaves g invariant.

2.- The signed g-area leaving (and entering, respectively) a source pi (and entering, respectively) a sink is () 2Re a1, j, where the related differential form's residue at pi, a1 = j, is j.3.- Map 1 on M fulfills the g-area Conservation Law, where pi is a source and [g-area coming from pi] + [g-area falling in pj] = 0 [g-area coming from pi] is a source. Following is a definition of a source pi under 1's g-area. Assume that it is transverse to Re and considered a suitable little simple loop enclosing pi with no additional singularities of Re(X) (X). The annulus is then bounded by 1(), and its g-area is equal to the desired value, as shown in Figure 1. Be aware that the area is positive for a source and unfavorable for a sink. The Conservation Law and the Classical Residue Theorem are equivalent, as shown in Section 4. The previous theorem's opposite is as follows: Given the numbers a1..., as in Cl, s 2, M should be a compacted orientable C 2-manifold. These numbers must all be such that a1 = 0. Then, a complicated construction A continuously differentiable vector yield in M and J both exist with linear components aj = X′ and simple zeros in s distinct positions p1..., ps (PJ). It is provided here. See [3] p. 52 for definitive proof. According to [1], [5], and [12], as mentioned earlier, flat polyhedral structures are an essential topic in quadratic differential theory, a well-developed and practical field in complex analysis. Using holomorphic functions as a starting point, we believe that simple polyhedral structures can help to illustrate certain fundamental concepts in complex analysis. Here, we begin working with meromorphic differential forms in order to lessen the paper's background, which is necessary.

2. Flat metrics, complex integration, and American football

Proof: Using a suitable holomorphic shift of coordinates in a power series, X is again reduced to its standard form. For the explicit computations, see [2] or [12] p. 27–37. The relationships of meromorphic vector fields, divergent forms, and polynomial differentials on Riemann surfaces are definitively proven. Thus, to apply the theory in [12], the vector fields f(z) _ can be converted into the meromorphic quadratic differential. The question of whether the integral value is entirely genuine or entirely imaginary for which actual trajectories begin at p0 is natural. Assign the real and imaginary components of the linked meromorphic vector field X= to the natural vector fields: The question of whether the integral's value is entirely real or entirely imaginary for which actual trajectories are beginning at p0 is a natural one. √ ∂

Consider the real and imaginary components of the associated meromorphic vector field X = (u+ 1v) z, which are known as the real vector fields: Here, the conjugate is denoted by x. They characterize the dynamical system X defines in terms of its real and hypothetical times. in other words, suppose... are local non-singular trajectories for Re(X) and (x), with alpha (0) =beta (0) =p0, respectively.

We determine the range of Re(x) and (X). Follow the re(X) path via p0 for time t, starting at point p0. Then trace the trajectory of (x) for time s and the reverse of (X) for time t. Finally, for time -s, follow the trajectory of (X) backwards.

If the trajectory, as mentioned earlier, y, then is a rectangle-based Euclidean distance.

The above is explained in mathematical terms in the following outcome.

The inverse function theorem states that h = (dz/f) is locally an invertible map and that we are in the initial point p0. Re(X) and Im(X), commute between the two vector fields.

The aforementioned items also serve as an eigenvector framework for the Riemannian metrics function g, where g(Re(X), Re(X)) = g(Im(X), Im(X)) = 1 and g(Re(X), Im(X)) = 0.

A metric with a commutent orthonormal frame is flat, and the curvature of g is zero is a notable finding. Let us use an illustration.

A diagrammatic explanation of the flat metric. Think back to a football game. The field is an IR2 Euclidean rectangle with a family of parallel lines with an x value of constant. The players attempt to carry the ball across these lines in each play. Concerning the trajectory of the player holding the ball, only the horizontal advance matters. In particular, a closed trajectory has zero progress. A horizontal advance is one play in a mathematical synthesis using the route integral. γ

The game is generalized in the case of complex integrals. Locally, we have a rectangle decorated with Re(X) trajectories and Im(X) as horizontal and vertical lines. The players attempt to carry the ball across these lines on each play. Additionally, major horizontal and vertical advancements are currently being made. They rely on the start and finish points and are independent of the trajectory. in graphical. The reader's imagination will have to fill in the details of field goals and scores. The aforementioned is explained in the following result in mathematical terms.

Let C be a domain with local holomorphic coordinate change:

1. If w and dz are equivalent, then po is a common value of a meromorphic differential form.

2. x is identical to if p0 is a regular value of the meromorphic vector field with the formula x=f(z) on.

i) Select the Euclidean plane (Cl) as your subject matter.

ii) Use the material to cut rectangles with the standard horizontal and vertical trajectories as the borders.

iii) Describe the pasting technique as follows: gluing rectangles to obtain topological two-manifolds and gluing the sides in the boundary of the rectangles by isometries to ensure well-defined horizontal and vertical trajectories over the border of the rectangles. In both instances, h(p)= provides the holomorphic change of coordinates. Here we apply the flat metric to 2k-2 replicas of flat elliptic sectors. The standard situation is the ordinary flat metric in a duplicate of (in the vicinity of the location at infinity) where k=2 is _ in a neighborhood of 0 C. under proper stereographic projection. Additionally, the vector field's precise trajectories are its typical horizontal trajectories, with alpha and w-limits infinity corresponding to 0C under the map z. At 0C, they provide two elliptic sectors. When a trajectory wraps counterclockwise around 0 and requires much difficulty, that time is essentially zero. This residue is the desired linear portion of the vector field's inverse. The measurements are g=compliant. The vector field is produced by each cylinder's end (PJ), and the cone point (q) has an order of zero (s-2). The specifics of how the polygon degenerates are left to the reader's attention. Case 3. M is from the category g >=1. Consider a vector area with meromorphism (Y) with a particular residual (S2) on the sphere. Then, by taking into account the total of the equidistant connections of g copies of one torus, we raise the genus of the double manifold. We cut a flat torus C/A along a geodesic line l of finite length g. We cut S2 along the geodesic line (S2, g) that has length g and does not overlap the vector fields Y and 0. We cut S2 along the geodesic (S2, g) length l, which does not cross the even and 0 points of the vector field Y.Finally, evenly spaced glue is applied between the torus's cuts of S2. It should be noted that the initial vector field Y expands in a meridian pattern on the ring surface. The origin of the new vector field X's simple poles can also be found by connecting the endpoints in the notch and S U C/A. Higher genus cases are clear after adding appropriate torus copies. Be aware that the concept, as mentioned earlier, only creates some Riemann surfaces. As stated in the mathematical proof, see [3] p.52, we are not even sure if it's feasible to extract every complex structure from the C infinite field M.

2.2. Lattice point counting using the residue theorem

2.2.1 Initialization

Assume that Zn Rn is a complicated-dimensional integer lattice and that P equals x1to xn and put 1 in Rn:n, and all xk > 0 are the outcomes that we can obtain.

The n-dimensional complementary triangular prism with the vertex from 0 a1, a2 to 0, where a1, and a have all been greater than 0. The lattice point count in the extended output tP and its occlusion are represented, correspondingly, by L(P,t) and L(P,t), where t is the time in seconds. According to N. Ehrhart ([17]), L(P,t) and L(P,t) are also both equations in t of degree n. He also determined the constant and the two dominating factors. 50% of the surface of P, standardized in relation to the sub-lattice on each side of P, is represented by the corresponding component, 1/2 Vol(P). Moreover, it seems to be the name of the initial coefficient, whereas Vol is the name of the secondary coefficient (P). The consistent coefficient is similar to the Euler characteristic of P. (P). Unclear is the existence of the other L(P,t) and L(P,t) coefficients. In reality, a technique for figuring out these coefficients wasn't known until recently (as shown in [12], [15], [16], [18], [19]). Here, we apply the residue theorem to give a straightforward procedure to help us discover P with Ehrhart's polynomials. Then, we decide to establish the Ehrhart's reciprocal law in order to solve these n-dimensional tetrahedral structures. After that, computing the first fairly significant coefficient help us illustrate our approach, cn2. We conclude by showing the process that our ideas work out o determine the Ehrhart's polynomials in any yield with a convex lattice.

2.2.2 The key concept

First, let's think about L(P,t), a closure of the expanded tetrahedron. We propose that the formulation of ak with the condition of ak has been neglected. In this situation, we can suggest that the function L(P,t) = card from m1 to mn Zn: all mk 0 are k=1 and all mk t are card from m1 to mn, Zn+1: mk,m 0; nk=1mkAk + m=tA.

It is possible to read L(P,t) since Taylor's coefficient of ztA for the equation 1+zA1 .L(P,t)=Res (1zA1)(1zA2)(1zAn)(1z),z=0 is equivalent. We just need to calculate some other residues of ft(z) if this formula counts the numbers of lattice sites and apply the residue theorem to the sphere C. L (P, t) = Res (ft(z), z = 0).

Also, the residue theorem claims the first part of results that plays a significant role mainly since Res (ft,z =) = 0.

Mathematical software like Mathematica or Maple can be used to simplify computation in higher dimensions. It is apparent that Res(ft(z), z = 1) is polynomials in which the variables in the range from a1to and has rational expressions. Secondly, computing the residual points in the interval range of the unity-in-distance roots is generally more challenging. As we shall show in section 4, they lead to Dedekind-like summation and their higher-dimensional feature equivalents. One aspect of these residues, however, stands out immediately away: their dependency on the dilation parameter t. Additionally, this proves Corollary 3 below, saying that function L, which contains the point(P,t), is a polynomial.

To prove the previous statement, we let B'th component of union to exist in the sum of all ak. Then we make z-tA to its power series and as should be the center of the range of z. By choosing the appropriate branches of the logarithm to make B exp 1 log (1) = and setting z = wB = exp 1 log w, we may add a new variable to this situation. The derivative of the functions z-tA/B, approximated at z = 1, are the terms dependent on t in the power series of z-tA Bthus, which demonstrates that the equations in t act as the ztA power series' coefficients. Finally, the assertion demonstrates that L(P,t) represents the total amount of all of various points.

3. Conslusion

Cauchy's Residue Theorem allows us to compute real integrals and infinite series. We illustrate that the residue theorem and the Provides a useful corresponding rule may be used to compute the Ehrhart polynomials for any yield with a convex lattice. To build appealing manifolds, use the cut-and-paste technique. Cut the components into smaller pieces, and then reassemble the components using the suitable paste strategy. Based on the above-mentioned methodology, the paper's authors pose various queries and investigate the many types of flat polyhedral structures and the laws that cause them to appear. The authors use the classical theory of (real) differential equations to create an IR M M dynamical system.

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A metric with a commutent orthonormal frame is flat, and the curvature of g is zero. A diagrammatic explanation of the flat metric is given using a football game. The game is generalized in the case of complex integrals, and significant horizontal and vertical advancements are currently being made. The reader's imagination will have to fill in the details of field goals and scores. A differential geometric interpretation of a flat elliptic sector is shown for (2). The flat metric is a flow box in the differential equation’s language. The vector field's precise trajectories are its typical horizontal trajectories, with alpha and w-limits infinity corresponding to 0C under the map z.

A double manifold of genus g >= one can be created by cutting a flat torus along a geodesic line of finite length g, then applying evenly spaced glue between the torus's cuts of S2. Ehrhart demonstrated that the closure of a dilated tetrahedron is an n-degree polynomial in t whose steady coefficient approaches the Euler property of the tetrahedron. The residue at z=1 is simply ezft(z = 0),z = 1 (1eA1z)(1eA2z)(1eAnz)(1ez), and we get the first part of our primary outcome measure from the residue theory for the sphere C.Mathematical software like Mathematica or Maple can simplify computation in higher dimensions. Let B'th root of unity be, and translate z-tA into its power series centered on z =. The polynomials in t serve as the coefficients of the z-tA power series. Thus, L(P,t) simply represents the sum of the various residues.

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