Wave Energy Maximum Power Output Design

Pengyu Wu[1, #], Zhengyang Xie[2, *, #], Mengyi Liu[3, #]

[1] School of Mathematics and Statistics, Changshu Institute of Technology, Jiangsu, China, 215500

[2] School of Electronic Information and Engineering, Changshu Institute of Technology, Jiangsu, China, 215500

[3] School of Business, Changshu Institute of Technology, Jiangsu, China, 215500

* Corresponding Author Email: zxc2352859113@163.com

[#]These authors contributed equally.

Abstract. This paper studies the energy conversion efficiency of the wave energy device, analyzes the force of the wave energy device, uses calculus and difference equation to establish the model, takes the sea level as the X-axis to establish the plane cartesian coordinate system, analyzes the force analysis of the float and the oscillator and obtains the coordinates of the float and the oscillator in equilibrium state. MATLAB tool is used to solve the optimal model of wave energy PTO system power conversion, to maximize energy utilization, and find the corresponding float heave displacement, float velocity, oscillator displacement and oscillator velocity.

Keywords: PTO system, difference equation model, optimization model, optimal damping coefficient.

1. Introduction

With the development of economy and society, human beings are facing the double challenge of energy demand and environmental pollution. The development and utilization of wave energy is very important to alleviate the energy crisis and reduce environmental pollution.Wave energy is a kind of renewable clean energy which is easy to be directly used and inexhaustible. It has wide distribution and high energy density, and has considerable application prospects.The energy conversion efficiency of wave energy devices is one of the key issues in the large-scale utilization of wave energy.A wave energy device consists of a float, a oscillator, a central shaft and an energy output system (PTO)[1].Under the action of the wave, the float moves and drives the vibrator to move. Through the relative motion of the two, the linear damper and the rotary damper are driven to do work and output energy. In this paper, the optimal model of the power conversion of the wave energy PTO system is established to maximize the energy utilization[2].

2. Force and motion analysis

2.1. Buoyancy analysis of float

Based on the mechanism analysis of physics and mechanics principles, the internal relationship of the system is obtained, and then the state parameters of the wave energy device system are obtained. Firstly, taking sea level as the X-axis and the center line of the float (the line between the conical apex and the center of the top circle of the float) as the Y-axis, a cartesian coordinate system is established in this paper to discuss the force conditions of the float and the oscillator, as shown in Figure 1. Point P represents the position of the float in the equilibrium state in seawater[3].

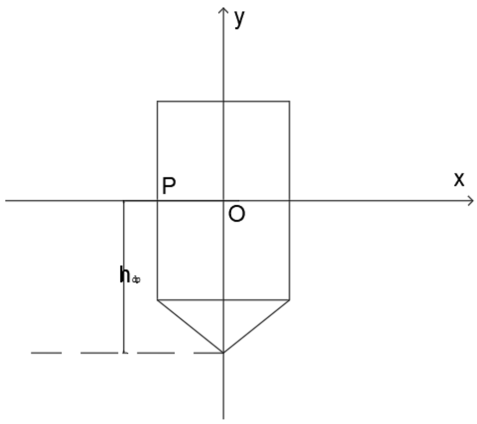


Figure 1. Position of the equilibrium state

The float reaches equilibrium without any external force on the sea surface, where the float touches the water at a depth of h[4]. The float is subjected to its own gravity m_1, the mass of the oscillator inside the float m_2, and the total mass of the whole device m_{total}. The buoyancy force of the whole system is F_{float}.In the equilibrium state, the force balance can be:

$$(m_1 + m_2)g = F_1 \tag{1}$$

Where, the buoyancy formula is:

$$F_{float} = \rho_s V_1 \tag{2}$$

According to the buoyancy formula, buoyancy is only related to the volume of the float immersed in the sea water.A float consists of two parts, a cone and a cylinder. The two parts are connected at the bottom, so the bottom area is the same.According to the volume formula of cone and cone, the buoyancy is only related to the height at which the float is immersed in the sea.In order to find the height of immersion in the sea water, there are two cases.

① When the seawater is not completely immersed in the conical part, the seawater immersed part is analyzed. The similarity of triangles is used to obtain:

$$r_c = (\frac{h_{dp}}{0.8})r_p = \frac{h_{dp}}{0.8} \tag{3}$$

The r bring into the bottom area of the cone to obtain:

$$S_c = \pi r_c^{\ 2} = \pi\left(\frac{h_{dp}}{0.8}\right)^2 \tag{4}$$

Put the above to obtain: S_C V_c

$$V_c = \frac{1}{3}S_c h_{dp} = \frac{1}{3}\pi\left(\frac{5}{4}\right)^2 2h_{dp} = \frac{25}{48}\pi h_{dp}^{\ 3} \tag{5}$$

The distance from the top of the cone to the center of its partition is 0.8m, and the value range is m h_{dp} $h_{dp} \in (0,0.8]$

② Seawater is immersed in the cone part, and the seawater is immersed in the cylinder part is analyzed, and the data can be obtained as follows [5]:

$$r_{c1} = 1m \tag{6}$$

$$S_{\mathrm{cl}} = \pi r_{\mathrm{cl}}^{\ 2} = \pi \tag{7}$$

The height of the cylinder in contact with the water surface can be obtained by subtracting the height of the cone. The height of the cylinder in contact with the water surface can be denoted by: h_{dp} $\mathrm{h}_{dp} - 0.8$ S_{cl} V_{cl}

$$V_{\mathrm{cl}} = S_{cl}(h_{dp} - 0.8) = \pi h_{dp} - 0.8\pi \tag{8}$$

To sum up, the relationship between buoyancy and the change of water inlet depth of the float can be obtained as follows:

$$F_{\mathrm{float}} = \begin{cases} \rho_s g \pi \dfrac{25}{48} h_{dp}^{\ 3} \left(0 < h_{dp} < 0.8\right) \\ \rho_s g \pi (h_{dp} - \dfrac{8}{15}) \left(0.8 < h_{dp} < 3.8\right) \end{cases} \tag{9}$$

The water entry depth and buoyancy change are shown in Figure 2.

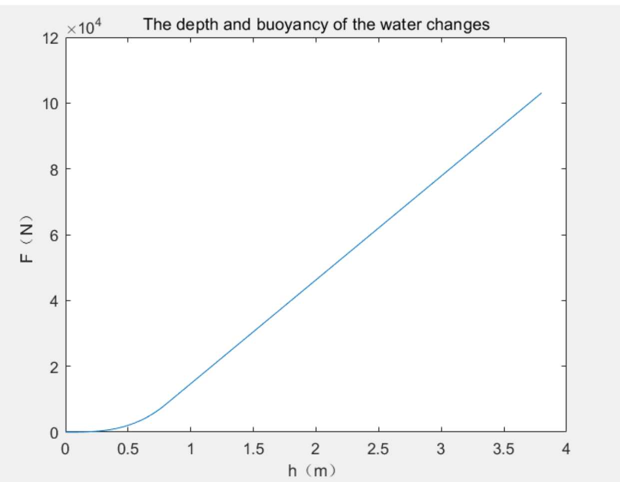


Figure 2. Water entry depth and buoyancy change

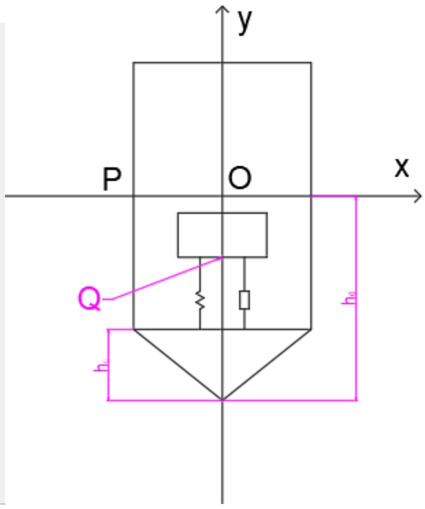


Figure 3. Cartesian coordinate diagram

2.2. Hydrostatic balance force analysis

① When the static water is in balance, the force analysis of the float (gravitational acceleration) is as follows: $F_1 = \mathrm{m}_{total} g$ g

$$F_1 = \left(\mathrm{m}_1 + m_2\right) g = 71530.2N \tag{10}$$

According to Figure 2, when the static water equilibrium is reached, the height of the float's equilibrium point P and the conical tip of the float is between(0.8,3.8]. Therefore, the relationship between the equilibrium point is as follows[6]:

$$\rho_s \mathrm{g} \pi (h_0 - \frac{8}{15}) = m_{total} g \tag{11}$$

② When the oscillator is balanced, the force analysis of the oscillator can obtain: $F_{\mathrm{t}} = m_2 g F_t = m_2 g$

According to Newton's third law, the direction of the spring should be up, the spring is in a compressed state, is the ordinate of the equilibrium state Q point, According to Hooke's law:

$$F_{float} = -\lambda(h_0 + y_2(0) - y_1(0) - h_c - L_0) \tag{12}$$

Two forms of bipolar:

$$\begin{cases} F_t = m_2 g \\ F_t = -k(h_0 + y_2(0) - y_2(0) - h_0 - L_0) \end{cases} \tag{13}$$

2.3. Float movement process

According to the above analysis of the force in the equilibrium state, the state parameters of the system in the equilibrium state have been obtained.On this basis, the state quantities of the system in the moving state are analyzed.Under the action of waves, the device only makes an idealized heave motion, so only the motion of the device along the Y-axis needs to be considered[7].Then, the force of the float in the process of movement is analyzed. Under the action of external force, the float is subjected to its own gravity and the gravity of additional mass, which can be summed up as:

$$(\mathrm{m}_e + \mathrm{m}_1)\ \mathrm{g} \tag{14}$$

The buoyancy of the float by the sea water, indicating the volume of the float immersed in the sea water, substituting into the following:

$$F_{\mathrm{float}} = \rho_{\mathrm{s}} g V_1 \tag{15}$$

$$F_{\mathrm{float}} = \rho_{\mathrm{s}}\mathrm{g}[\frac{1}{3}\pi\, \mathrm{r}^2\mathrm{h_c} + \pi\, \mathrm{r}^2(\mathrm{h}_{dp} - \mathrm{h}_c - \mathrm{y}_{float})] \tag{16}$$

Here refers to the depth of the whole device below the sea level, is to be calculated to represent the relative displacement of the float, device radius r=1m, =0.8m, =2.8m substitute into the formula to obtain[8].At the same time, the wave motion will also provide energy for the device and provide upward excitation force for the device (in this case, the idealized model only considers heave motion). According to the following equation:

$$F_{\mathrm{j}} = \mathrm{fcoswt} \tag{17}$$

W = 1.4005, f = 6250. s^{-1} $N \bullet M$ Therefore, we have figured out the force of the device under the action of external force.But there is still oscillator motion inside the device, so it is necessary to analyze the internal force.

In the process of up and down reciprocating motion of the device, inertia will have an impact on the internal vibration of the system. For example, the device moves up under the excitation of waves because the inertial oscillator moves down along the Y-axis (the direction of motion is not absolute) [9].At the same time, the spring will also deform under the drive of the oscillator to produce elastic force. Here, the elastic force produced by the spring is regarded as a whole, and the relative displacement of the float movement and the oscillator movement has an impact on the spring stretch.

$$F_{\mathrm{t1}} = \lambda\ (\mathrm{y}_2 - \mathrm{y}_1 - \mathrm{L}_0 + \mathrm{h}_0 - \mathrm{h}_c) \tag{18}$$

λ represents spring stiffness, x represents the original length of the spring.

In addition, the inertia of the damper will also follow the reciprocating motion of the oscillator, which will also affect the whole device.In question (1), given the damping coefficient 10000, the damping force can be obtained as follows:

$$F_{\mathrm{z}} = K_{\mathrm{z}}\frac{\mathrm{d_{rel}}}{\mathrm{dt}}, \mathrm{d}_{rel} = \mathrm{dy}_2 - \mathrm{dy}_1 \tag{19}$$

The oscillator is in continuous heave motion. The heave damping coefficient of 656.3616 is obtained in Annex 3. It can be seen that:

$$F_{cz} = K_z \frac{d_{rel}}{dt} \tag{20}$$

So far, all force analysis has been completed, as shown in figure 4.

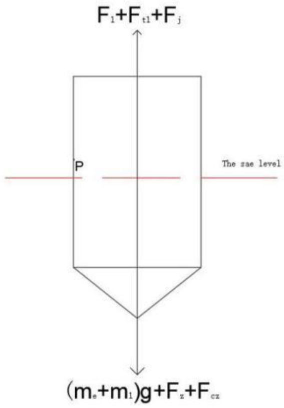


Figure 4. Force analysis diagram

Newton's second law is used to find the net force after the graphic analysis:

$$F_{t1} + F_1 + F_j - (m_1 + m_e)\ g - F_z - F_{cz} = (m_1 + m_e)\ a = (m_1 + m_e)\frac{d^2 y_1}{dt^2} \tag{21}$$

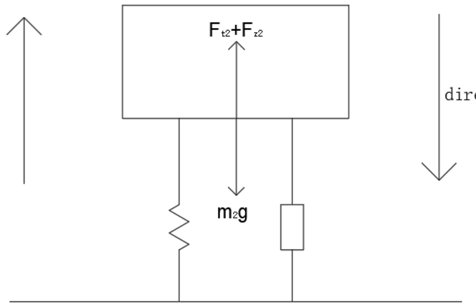
Here a is the acceleration of the float, since the double integral of the acceleration of the float can obtain the displacement of the float, so:

$$\frac{d^2 y_1}{dt^2} = a \tag{22}$$

Further splitting the equation yields:

$$\begin{aligned} &\lambda\ (y_2 - y_1 - L_0 + h_{dp} - h_c) + \rho_s g[\frac{1}{3}\pi r^2 h_c + \pi r^2 (h_0 - h_c - y_1)] + f\cos wt - (m_1 + m_e)\ g \\ &- K_z \frac{dy_2 - dy_1}{dt} + K_{cz}\frac{dy_2 - dy_1}{dt} = (m_1 + m_e)\frac{d^2 y_1}{dt^2} \end{aligned} \tag{23}$$

2.4. Motion process of oscillator



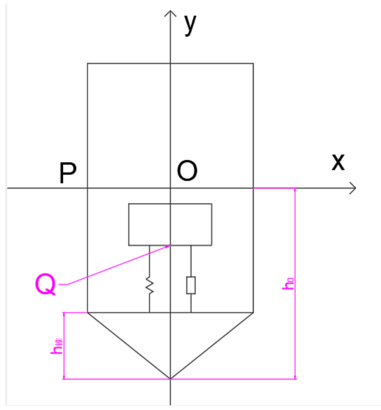


Figure 5. Schematic diagram of force **Figure 6.** Schematic diagram of resultant force

Oscillator in the system could be used as an independent individual analysis, also only do idealized prolapse swing motion, as shown in figure 5 and figure 6, known in the oscillator moves up and down, respectively, by the spring force, damper resistance as well as its own weight, here in oscillator downward movement, for example get oscillator motion when the resultant force equation, differential equation for solving the vibration displacement of the child[10].The oscillators are subjected to their own gravity as $m_2 g$ .In this state of motion, the oscillator is also pulled by the spring F_{t2} .

$$F_{t2} = -\lambda\,(y_2 - y_1 + h_0 - h_c - x) \tag{24}$$

h_0 is the depth of the float in the sea water at equilibrium.Continue to solve the vibration under the action of the damper, by the action of the damper.

$$F_{z2} = -K_z \frac{d_{rel}}{dt}, d_{rel} = dy_2 - dy_1 \tag{25}$$

Therefore, all the force analysis of the oscillator has been analyzed, and the resultant force can be obtained, as shown in figure 6.Newton's second law is used to obtain the following equation:

$$-F_{z2} - F_{t2} - m_2 g = m_2 g \tag{26}$$

Further derivation:

$$-\lambda\,(y_2 - y_1 + h_0 - h_c - L_0) - K_z \frac{d\,(y_2 - y_1)}{dt} - m_2 g = m_2 \frac{d^2 y_2}{dt^2} \tag{27}$$

Now we have obtained the differential equation of float and oscillator respectively. This differential equation is a bivariate second-order differential equation and requires the relative displacement and velocity of 0.2s every interval. Here, it is regarded as the grid division of this second-order equation, and the continuous change region of independent variable t is replaced by a finite set of discrete points.Therefore, the difference method is used to solve the system of equations:

$$\begin{cases} -\lambda(y_2 - y_1 + h_0 - h_c - x) - K_{z2} \dfrac{d(y_2 - y_1)}{dt} - m_2 g = m_2 \dfrac{d^2 y_2}{dt^2} \\ \lambda(y_2 - y_1 - x + h_{dp} - h_c) + \rho_s g[\dfrac{1}{3}\pi r^2 h_c + \pi r^2 (h_0 - h_c - y_1)] + f\cos wt - (m_1 + m_e)g \\ -K_z \dfrac{dy_2 - dy_1}{dt} + K_{cz} \dfrac{dy_2 - dy_1}{dt} = (m_1 + m_e) \dfrac{d^2 y_1}{dt} \end{cases} \tag{28}$$

Let the substitution simplify to: $\begin{cases} \dfrac{dy_1}{dt} = v_1, \\ \dfrac{dy_2}{dt} = v_2, \end{cases}$

$$\begin{cases} -\lambda\,(y_2 - y_1 + h_0 - h_c - L_0) - K_z(v_2 - v_1) - m_2 g = m_2 \dfrac{dv_2}{dt} \\ \lambda\,(y_2 - y_1 + h_0 - h_c - L_0) + \rho_s g[\dfrac{1}{3}\pi r^2 h_c + \pi r^2 (h_0 - h_c - y_1)] + f\cos wt - (m_1 + \\ m_e) - K_z(v_2 - v_1) + K_{cz}(v_2 - v_1) = (m_1 + m_e) \dfrac{dv_1}{dt} \end{cases} \tag{29}$$

The derivative of difference method is used to obtain:

$$\begin{cases} y_1^{k+1} - y_1^k = v_1^k \tau \\ y_2^{k+1} - y_2^k = v_2^k \tau \\ \dfrac{v_1^{k+1} - v_2^{k}}{\tau} = \dfrac{-\lambda\ (y^k{}_2 - y^k{}_1 + h_0 - h_c - L_0) - K_z(v^k{}_2 - v^k{}_1) - m_2 g}{m_2} \\ (m_1 + m_e)\dfrac{v_1^{k+1} - v_2^{k}}{\tau} = \lambda\ (y^k{}_2 - y^k{}_1 + h_0 - h_c - L_0) + \rho_s g[\dfrac{1}{3}\pi r^2 h_c + \\ \pi r^2 (h_0 - h_c - y^k{}_1)] + f\cos wt - (m_1 + m_e) - K_z(v^k{}_2 - v^k{}_1) + K_{cz}(v^k{}_2 - v^k{}_1) \end{cases} \tag{30}$$

3. Model solving

Calculate h0=2.8m, , we know the target equation, $y_2(0) = -1.8$,spring stiffness λ =50000, the water depth distance of float in equilibrium state h_0 =2.8m, damping coefficient is $10000\ N \cdot m/s$,oscillator mass m_2 =2433kg, gravitational acceleration, $g = 9.8m/s^2$,the height of the conical part of the float, $h_c = 0.8m$ the original spring length, $L_0 = 0.5m$ $\rho_s = 1025\,kg/m^3$ the radius of the bottom of the float $r = 0.5m$, and damping coefficient of floating traveling wave. $K_{cz} = 656.3616\ N \cdot s/m$

Matlab is used to solve the difference equation to obtain Table 1, where the change function is shown in FIG. 7.

Table 1. Difference Equation 1 Table

Time (s)	10	20	40	60	100
Float heave displacement (m)	0.420	0.160	0.255	0.335	0.420
Oscillator swing displacement (m)	0.448	0.176	0.273	0.360	0.449
Float speed (m/s)	0.0076	0.5915	0.4718	0.3649	0.0785
Oscillator speed (m/s)	- 0.1.5	0.6276	0.4964	0.3819	0.0766

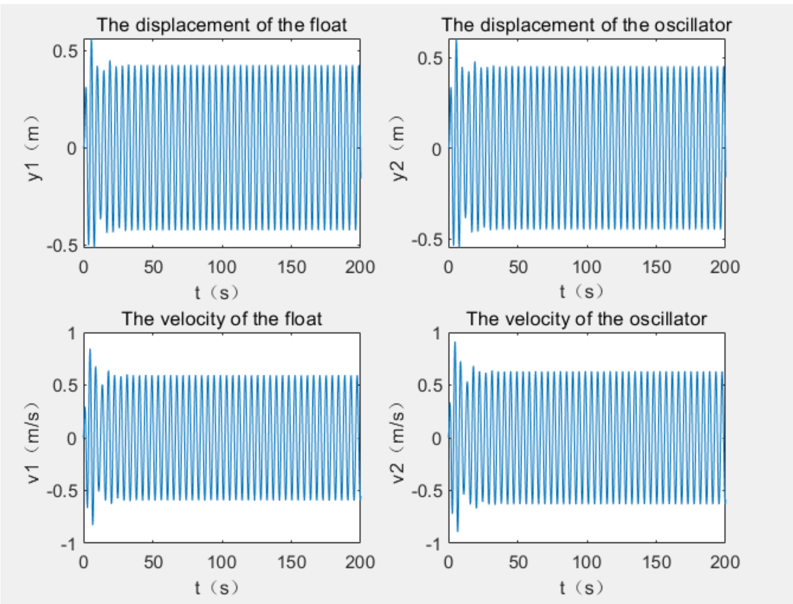


Figure 7. MATLAN solution result 1

Matlab was used to solve the difference equation to obtain Table 2, where the change function is shown in Figure 8:

Table 2. Difference equation 2 Table

Time (s)	10	20	40	60	100
Float heave displacement (m)	0.447	0.145	0.207	0.334	0.433
Oscillator swing displacement (m)	0.479	0.159	0.217	0.358	0.464
Float speed (m/s)	0.337	0.915	0.334	0.527	0.173
Oscillator velocity (m/s)	0.381	0.990	0.344	0.565	0.185

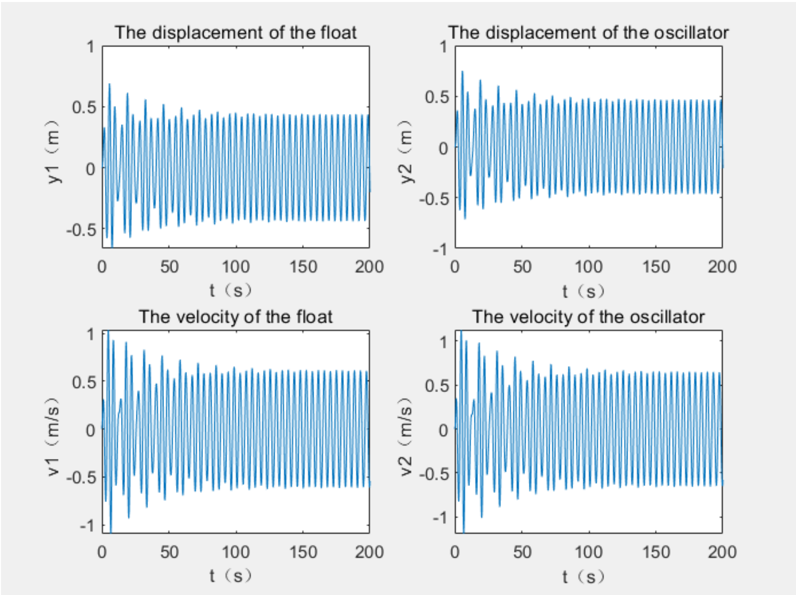


Figure 8. MATLAN solution result 2

4. Conclusion

The forward difference method is used to solve the bivariate quadratic differential equation, and the approximate solution of the differential equation is obtained by solving the difference equation. It is the discretization of the continuous problem, and our objective problem is exactly a discretization result.This model fully considers the force of each object, in the case of considering the rigorous simplification of the problem, as a simplified model it can not calculate the corresponding wave device parameters for the complex sea state, but as an idealized design model to meet the design needs of the idealized case.

References

[1] Li Lei, Cai Yue, Li Wenjun, Liu Tingting, Wan Huijun. Moment of inertia of rigid bodies of elliptical rings and disks in polar coordinates [J]. College Physics, 2022,41 (04): 14-16+26. DOI: 10.16854/j.cnki.1000-0712.210273

[2] Lin Yanjia. Improvement of measuring rigid body's moment of inertia by compound pendulum method [J]. China New Technology and New Product, 2021 (08): 59-61. DOI: 10.13612/j.cnki.cntp.2021.08.019

[3] Yan Min, Dai Yuqin, Yuan Jun, Wang Xiaoxiong. An improved scheme for the verification experiment of the parallel axis theorem of moment of inertia [J]. College Physics, 2020,39 (05): 66-69. DOI: 10.16854/j.cnki.1000-0712.190392

[4] Yin Zichao, Li Ming, Chen Bo, Li Yinghui. Study on the vibration characteristics of atomizer shaft [J]. Journal of Dynamics and Control, 2021,19 (01): 47-54

[5] Fu Meihuan. Calculating the Moment of Inertia of a Homogeneous Rigid Body by Summing Sequences [J]. Journal of Nanjing Institute of Industry and Technology, 2018,18 (02): 33-35. DOI: 10.15903/j.cnki.jniit.2018.02.008

[6] Fang Zifan, Zuo Xinqiu, Xiong Fei, Wang Jiajia, Xie Xueyuan. Design and research on hydraulic conversion system of wave energy multi chamber oil cylinder [J]. Hydraulic and Pneumatic, 2022, 46 (10): 40-49

[7] Chen Xinhui, He Hongzhou, Sun Pengyuan. Design and optimization of twin screw inner rotor wave energy power generation device [J]. Journal of Guangzhou Institute of Navigation, 2022,30 (03): 45-51

[8] Wang Chunxiao, Yu Huaming, Li Songlin, Guan Hao, Ge Jingjing. Wave energy resource analysis based on wave reanalysis data [J]. Journal of Solar Energy, 2022,43 (09): 430-436. DOI: 10.19912/j.0254-0096. tynxb. 2020-1381

[9] Liu Huabing, Zhang Qinghe, Liu Yanjiao, Peng Aiwu. Research on maximum power tracking strategy of wave energy based on magnetohydrodynamic generator [J]. Journal of Solar Energy, 2022,43 (09): 402-409. DOI: 10.19912/j.0254-0096. tynxb. 2021-0122

[10] Liu Jiahao, Jiang Peiyi. Overview of wind wave energy complementary power generation technology development [J]. East China Science and Technology, 2022 (09): 54-56