The Fun of Complex Numbers

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Abstract. By studying the chapter on complex numbers, one found that they are very different from real numbers. Complex numbers can solve problems that cannot be solved by real numbers, such as computation, geometry problems in the complex plane, etc. In this article one will introduce the basic operations of complex numbers, complex conjugates, algebraic and trigonometric forms of complex numbers and their transformations. And how to find the roots of complex numbers is something that one will cover in the text. When one talk about these knowledge points, one will combine the title to help one better explain and readers' understanding. Before writing this article, one read a lot of literature, which helped one learn more knowledge and made one more aware of complex numbers. Therefore, in this article, one will also refer to the literature one refer to and cite them.

Keywords: Complex numbers; geometrical problems; complex conjugates; algebraic and trigonometric forms; the roots of complex numbers.

1. Introduction

If one say that the natural numbers are derived from the description of quantities, the rational numbers are derived from the description of contrast columns, and the irrational numbers are derived from the description of lengths, then the complex numbers are completely artificial and have no actual background in real life.

The complex number is written in the form A + BI, where A and b are real numbers and I is imaginary, satisfying I^2 =-1, which is the solution to the equation x^2= -1. Obviously, the problem is imaginary numbers, because people know by multiplication that when you multiply two numbers, you're going to get a real number. Because of this, although the ancient Greek scholar Diophantus knew that the quadratic equation of one variable had two roots, but one of them was imaginary, he would rather think that the equation was unsolvable. Until the 16th century, mathematicians generally accepted Diophantine's method of dealing with imaginary numbers -- that equations with imaginary roots were unsolvable.

However in fact, the reason people take complex numbers seriously is to solve cubic equations. In one of his published works, the Italian mathematician Cardan discussed algebraic methods for solving cubic equations. He discussed the cubic equation in detail in 13 cases and gave 13 formulas for solving the problem, which are now called *Cardan's formulas*. But an unexpected situation occurs in the solution formula: even if all three roots are real roots, complex numbers will appear in the solution using *Cardan's formula*. Now, does this equation have a solution or does it have no solution?

Imaginary numbers were given their name by Descartes, who could not accept complex roots, and so, in his 1637 book Geometry, explained complex roots by saying "but they are always imaginary". In the history of mathematics, Euler was the first to use the symbol I for √-1. Now that readers have the notation for imaginary numbers, one can define complex numbers, and one can denote by C the set of complex numbers [1].

Since the 20th century, complex function theory has been widely used in theoretical physics, elastic theory and celestial mechanics. In various abstract space theories, complex function theory often provides new ideas and new models for them [2]. The chapter "Trigonometric Representation of Complex Numbers" is a required part of high school mathematics in China, including the calculation of complex numbers, trigonometric representation of geometric meaning and so on. The author thinks that the content of this chapter is very important and teachers need to teach students, but some teachers think it is not necessary. In this article, Koryo Geum (the author) explains why she finds it important [3].In the solution of complex problems, the method should be flexible, mathematical thought method is conducive to quickly find the specific solution ideas, so as to crack the target problem smoothly. The thought method of classification and integration in dealing with some mathematical problems sometimes involves more than one possibility, then it is necessary to carry out a specific analysis of each possible situation, and then summarize [4]. Chinese high school mathematics clearly requires that the core literacy of mathematics is the starting point of mathematics teaching. The teaching goal is not only to acquire knowledge, but more importantly to promote students' active learning and cultivate students' core literacy of mathematics. This research is a study of complex numbers under the core literacy of mathematics. Firstly, through literature review, related researches on core mathematical literacy, unit teaching design and plural teaching are sorted out to provide theoretical basis for the next research. Secondly, through the interview and questionnaire survey to understand the teaching status of high school mathematics teachers' multiple teaching and unit teaching, through the test research to study the learning status of high school students' multiple knowledge. It is found that high school mathematics teachers and high school students do not pay enough attention to the pluralistic knowledge, and the research level of the pluralistic teaching and learning problems stays at the shallow level. Finally, on the basis of the above theoretical research and empirical investigation, it is believed that the combination of complex knowledge and unit teaching can design a more complete and logical teaching design, which can make teachers and students realize the value of diversity from a new perspective. Based on the research of mathematical literacy and the core of the "ADDIE" model, the complex teaching design, based on the unit teaching design, designs specific unit-class teaching cases, so as to provide a reference for other mathematics teachers. The research shows that the multi-unit teaching design oriented teaching can provide students with a holistic thinking environment, diversified learning concepts, improve teachers' classroom teaching objectives, and cultivate students' mathematical literacy. By improving the lack of relevance in the teaching of complex numbers, teachers and students can be promoted and get the value of math education [5].Complex number is the main content in the course of sets and algebra. In the content of "The expansion of number system and the concept of Complex number", people try to make students realize the necessity of establishing the concept of complex number gradually from the history of number system. In THE process and method OF EXPANDING from real number to complex number system, people may use mathematical language to describe, abstract mathematics, and integrate it into mathematical thinking, so as to pursue the unified mathematical spirit [6]. Complex numbers have an important application in high school mathematics learning. This study adds a more specific overview on the concept of complex numbers to further study the problem of mathematical complex numbers [7].Different versions of textbooks have different characteristics and values. This study selected four high school mathematics books from four countries, from their structure, content, title and other parts of the comparative study. Literature analysis, content analysis and comparative analysis were used in this study. Some research conclusions are drawn through the analysis. China and South Korea [8]. In the mathematical competition, complex problems like to appear in the form of complex recurrence relation. Solving this kind of problems often need to use the concept of complex numbers, properties and operation rules and other knowledge, but also need to use the structure of recursive sequence characteristics and related problem-solving skills, comprehensive relatively strong. This paper will take the test questions as examples to explore the common solutions [9]. The content of the textbook is very important. The curriculum of Chinese textbooks is diversified, and Britain and Australia are the countries with very basic educational fields. The paper makes a comparative study on the plural content of mathematics textbooks in China, Britain and Australia [10]. Mathematics culture is one of the hot spots in mathematics education research and plays a very important role in mathematics teaching. Many researchers have introduced mathematical culture into pluralistic teaching. However, most of the introduced mathematical culture is superficial. In view of this situation, this paper selects the relevant contents of mathematical thought and mathematical culture in the history of mathematical beauty, and tries to put these contents into the multi-trigonometry theory in the process of mathematical culture. This study aims to answer the following two questions through classroom observation, questionnaire survey and teacher interview :1. Through the implementation of the case, students' attitude towards mathematics culture will change, how to change? 2. Through the implementation of the case, explore the cultural fragments that students are more interested in mathematics and the way of integration. Based on the analysis of the data, a conclusion was drawn [11]. Complex is a very important factor in high school mathematics knowledge. One of its teaching objectives is to make students realize the importance of the formation stage of multiple concepts and realize rational thinking, innovative spirit and mathematical culture. And some high schools for this part of the teaching at the present stage is to cope with the test, do not let students feel the culture of mathematics in the classroom. It is one of the effective ways to realize the teaching goal to integrate the multiple development process into the middle school classroom. This paper is divided into four parts, centering on the process of pluralistic development and the process of pluralistic teaching. In the case teaching of mathematics, the process of mathematical research and research results are transformed into practical teaching, so that students can experience the process of mathematical knowledge generation and cultivate mathematical culture [12].

2. Method

2.1. Complex numbers

Complex numbers can be defined as ordered pairs (x, y) of real numbers, which are interpreted as points in the complex plane, or they can be expressed in terms of coordinates [13].

It is common to denote by z the coordinates z (x, y) of the complex number, the X-axis the real part of Z, and y the points on the imaginary axis. And if you draw it graphically, you can draw it just like Figure 1.

The vertical axis is called the real axis, x is the real number; The vertical axis is called imaginable axis, and y is imaginary. So the complex number z can also be written as z=x+iy or y=a+bi (a, b$\in$ R).

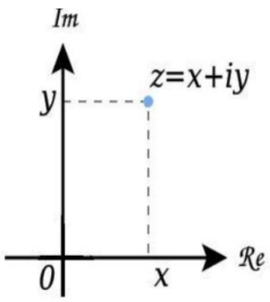


Figure 1. Imaginary numbers represent graphs on coordinate axes

2.2. The calculation of complex numbers

Unlike real numbers, there is no size relation in the set of complex numbers, that is, two complex numbers cannot be compared in size. Complex numbers are not numbers in the sense that numbers are symbols that can be arranged from small to large. This is not unusual, since no pair of numbers (including vectors) can compare sizes in the usual sense. However, the set of complex numbers contains the set of real numbers, because you only need to set the coefficient in front of the imaginary number I to zero in the complex number. One can define operations on complex numbers [13].

It is known that the order of operation of real numbers is multiplication, division, addition and subtraction. Complex numbers work the same way, you add the real parts, and then you add the imaginary parts. One may consider the numbers

$$Z_1=a_1+ib_1 \qquad Z_2=a_2+ib_2 \tag{1}$$

So people can get:

$$Z_1+Z_2=a_1+ib_1+a_2+ib_2=(a_1+a_2)+i(b_1+b_2) \quad (2)$$

$$Z_1-Z_2=a_1+ib_1-(a_2+ib_2)=(a_1-b_2)+i(a_1-b_2) \quad (3)$$

$$Z_1*Z_2=(a_1+b_1)*(a_2+b_2)=a_1a_2+ia_1b_2+ia_2b_1-b_1b_2 \quad (4)$$

Here is another example as follows.

(3+i)+(3-i)

According to (2), one obtains

=3+3

=6

(3+2i)-(4i-15)

According to (3), one obtains

=3+2i-4i+15

=18-2i

(3+i)(3-i)

According to (4), one obtains

=3*3-3*i+3*i-i*i

=9-3i+3i-(-1)

=9+1

=10

2.3. Complex conjugate of imaginary numbers

2.3.1 The vector

In mathematics, a vector is called a quantity that has both magnitude and direction. For example, in physics, force, velocity, acceleration, displacement, impulse, momentum, and so on are vectors [14].

2.3.2 The plural mode

One call the norm of a vector $\overrightarrow{OZ}$ (see Figure 2, where imaginary numbers represent graphs on coordinate axes) the norm or absolute value of the complex number z=a+ bi (a, b∈ R) [14].

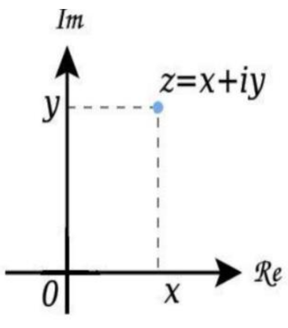


Figure 2. Imaginary numbers represent graphs on coordinate axes

2.3.3 Complex conjugates

When the real parts of two real numbers are equal and the imaginary parts are negative of each other, the two complex numbers are said to be complex conjugates of each other. Two complex conjugate numbers whose imaginary parts are not zero are also called conjugate imaginary numbers [14].

$$\bar{Z}=\mathrm{a}-bi \quad (5)$$

The point (x,-y) stands for the number $\bar{z}$, which is the reflection in the real axis of the point (x,y) representing z. Note that [11]

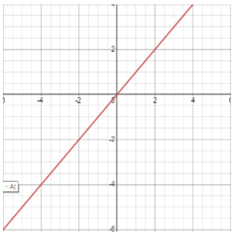
$$|Z|=|\overline{Z}| \qquad \text{and} \qquad Z=\overline{\overline{Z}}$$

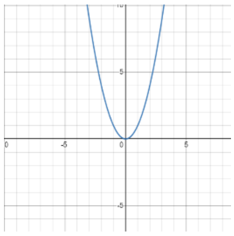
One can do a couple of examples here, like 5+6i , 5-6i are complex conjugates of each other i , -i are conjugated imaginary numbers. Complex numbers can be represented in coordinates in the complex plane. (Already mentioned in 2.1)So how do one compute the coordinates of the points in the complex plane of the complex number 6i+5 conjugate? One may write out that the complex conjugate of 6i+5 is 5-6i.One may write out the coordinates of the complex number 5-6i in the complex plane (5,-6).

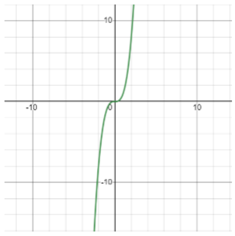
Therefore, the coordinates of the points in the complex plane of the complex number 6i+5 are (5, -6).

2.4. The same and difference between complex function Z and real function y

Complex functions z=x+iy lie in the complex plane and real functions lie in the real number plane, but they can both be expressed in coordinates (The previous section already mentioned this in 2.1). (see Figure 3. y=x, Figure 4. y=x2, Figure 5. y=x3)







(**Figure 3**. y=x) (**Figure 4**. $y=x^2$) (**Figure 5**. $y=x^3$)

The X-axis of the complex plane represents the real part of the complex number, so people call it the real axis, and the Y-axis represents the imaginary part of the complex number, so people call it the imaginary axis. The horizontal and vertical axes of the real number plane have different meanings than the complex plane. The horizontal axis represents a real number x, which is the independent variable, and the vertical axis y is a real number that varies with the value of x, which is the dependent variable.

In the complex plane, the distance between a point on a complex function and the origin or the distance between a point on a coordinate axis and the origin is called the magnitude of the complex number. In the real number plane, the distance between a point on the number line and the origin is called the absolute value of the number.

2.5. The trigonometric form of a complex number

In general, any complex number z=a+bi can be expressed in terms of r (cosθ+isinθ). r (cosθ+isinθ) is called the trigonometric representation of the complex number z=a+bi, or the trigonometric form. a+bi is called the algebraic representation of complex numbers, or the algebraic form for short [14].

In Figure 6, the triangular form of the complex number r (cosθ+isinθ), θ starts with the negative half of the X-axis and ends with the $\overrightarrow{OZ}$, which is called the argument of the complex number z=a+bi. One may specify that the value of the argument θ in the range $0\leq\theta\leq 2\pi$ is called the principal value of the argument, usually denoted as arg z [14].

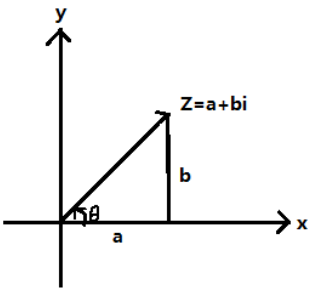


Figure 6. It shows that the arg of a complex number

The principle value of arg z, donated by Arg z, is the unique value Θ such that $-\pi < \Theta \leq \pi$. Evidently, then,

$$\arg z = \operatorname{Arg} z + 2n\pi \qquad (n=0, \pm 1, \pm 2, \ldots). \tag{6}$$

One can do a couple of examples here, like

If θ =45°please denote it by arg and Arg

One may write out that

Arg45°=tan θ =tan45°= π /2

One may write out that

arg45° =Arg z+2n π = π /2+2n π

One can do another couple of examples here, like

z=i/-1-i find the principle value of Arg z

One may write out that

arg z=arg i- arg (-1-i)

Since

Arg i = π /2 and Arg(-1-i)=-3 π /4

One may write out that

Arg (i/-1-i)=(5/4) π -2 π =(-3/4) π

2.6. Euler's formula

The symbol $e^{i\theta}$, or exp($i\theta$) , is defined by means of Euler's formula as

$$e^{i\theta} = \cos\theta + i \sin\theta \tag{7}$$

where θ is to be measured in radians. It enables us to write the polar form z=r(cos θ +i sin θ) more compactly in exponential form as follows [13].

$$z = r^{i\theta} \tag{8}$$

2.7. Application of complex numbers

Given that the complex numbers corresponding to the three vertices of a parallelogram in Figure 7 are 2,4 +2i and -2+4i, find the complex numbers corresponding to the fourth fixed point.

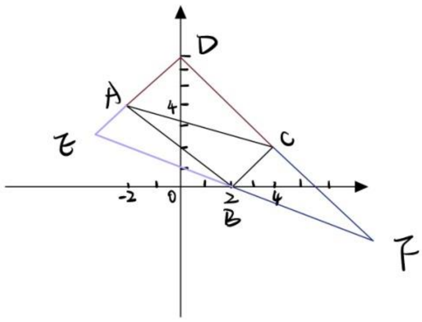


Figure 7. It shows a parallelogram

The fourth point of the parallelogram can have three places, namely E,D and F, as shown in Figure 6. When AC is hypotenuse, the fourth point is at D; When AB is hypotenuse, the fourth point is at E; When CB is hypotenuse, the fourth point is at F. Call the three points one know A,B, and C. So A (2,4), B (2,0), C (4,2). And AB= $4\sqrt{2}$, BC= $2\sqrt{2}$, AC= $2\sqrt{13}$. Let us propose the classification discussion.

(1) When AC is hypotenuse, since BC is the hypotenuse of a 2*2 triangle, AD is also the hypotenuse of a 2*2 triangle. The opposite side of the parallelogram is parallel, so AD/ /BC, so D (0,6), the corresponding complex number is 6i.

(2) When AB is hypotenuse, since BC is the hypotenuse of a 2*2 triangle, AE is also the hypotenuse of a 2*2 triangle. The opposite side of the parallelogram is parallel, so AE//BC, so E (4,2), the corresponding complex number is -4+2i.

(3) When BC is hypotenuse, since AC is the hypotenuse of a 6*2 triangle, AF is also the hypotenuse of a 6*2 triangle. The opposite side of the parallelogram is parallel, so AC//BF, so F (8,-2), the corresponding complex number is 8-2i.

3. Conclusion

This paper introduces the concept of complex number, calculation method, vector and express complex number by vector, express complex number by coordinate, conjugate complex number, the similarities and differences between imaginary number equation and real number equation, the trigonometric form of complex number and the application of complex number and other knowledge points, involving the characteristics of Euler formula and parallel quadrilateral. Use flexible thinking and methods to solve problems related to complex numbers. The plural is widely used in our life. When readers learn the plural, they should be enthusiastic and curious about it.

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