Study on the Self-Aligning Torque of BMW and Chevrolet During Driving and Its Influence on Vehicle Dynamic Performance

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Abstract. The existence of self-aligning torque makes the wheel direction of the car consistent with the driving direction, which is the key to ensure the stable straight-line driving of the car. In this paper, we will mainly apply Python to establish the force and motion models of BMWM3 and Chevrolet under different driving modes, and work through the function of the self-aligning torque through the classical calculation model of the self-aligning torque. In our calculation, we examine the variation of self-aligning torque of BMW and Chevrolet with the change of time under various driving conditions. Then, we evaluated the dynamic mechanical properties of the vehicle during driving, and provide reference for the design of vehicle parameters. We also proved their relationship between self-aligning torque and slip angle for the two cars and found the maximum torque values according to the slip angle. Use such model, it becomes possible to calculate the real-time self-aligning torque only with the data measured by python programming.

Keywords: Self-aligning torque, BMWM3, Chevrolet, python simulation.

1. Introduction

In the modern car industry, self-aligning torques can be a useful tool in many ways. It could return control steering wheel angle (SWA) from an outward to the central position, which evolves a nonlinear control of SWA to bound its tracking error so that a huge improvement of the controller tracking becomes possible [1]. While vehicles are in rolling motion, there are certain levels of unstableness in vehicles' motion due to tire slip. In order to analyze such loss of normal forces, self-aligning torques can be crucial, especially for novel electric and racer vehicles [2, 3]. In the aspects of steering, sometimes, the empiricism of the driver can be somehow misleading due to the existence of self-aligning torques. Thus, understanding the principles directly help car factories and drivers to design and gain better controls of those vehicles by improving the control algorithm of the current Electric Power Steering system [4]. Also, devices like real-time self-aligning torque (SAT) can estimate the real-time self-aligning torques from tires and their effects on cars' motion as well as on the steering system [5, 6]. Since the modern traffic system requires vehicles to become safer and safer, this technique can substantially prevent dangerous movements of vehicles. Sometimes it's also useful to go through simulation with a scaled test vehicle to validate blowout impacts more accurately since the model is valid in different scales. There are many variables that can affect self-aligning torques, like velocity, turn type, slip angle, centre of mass and so on [7]. In our model, we will thoroughly examine them. Besides, tire model is also important as the models for forces and torques are closely related to the idiosyncrasies of the tire. Elements like deformation of tire rubber, the in-plane belt deflection, and out-of-plane sidewall rotation are well-worth considering [8]. If we hope to research further on the contact between the tire and the road, the road adhesion coefficient is a great aspect to look at. This is also an identical benefit of analysing self-aligning torques. Vehicle active safety control system could use the more accurate parameter values calculated with tire self-aligning torque distribution to make sure the safety [9, 10]. By changing tire structures, car factories can change the performance of the tires to meet the requirements [10].

In this paper, we will examine BMWM3's and Corvette's self-aligning torques and their effects on the two types of turning of the two cars, which are U-turns and lane-changes. First, we discuss the tire and vehicle model we are interested in. Then, we build up physical force models and run data analysis through python, as well as visualizing our discoveries. With the help of rigorous mathematical model, real-time self-aligning torques can be calculated with similar models. It is also possible to verify through more difficult measurements with decent equipment. Finally, we compare our results and discuss how make the turns safe and effective, applying the rules of self-aligning torques.

2. Method

2.1. Establishment of vehicle model

A car is made up of thousands of parts, which makes its internal structure very complex. In order to study the dynamic behavior of the vehicle, we simplify the vehicle model and only consider the structural parameters that determine the dynamic performance of the vehicle. The main structural parameters of BMW and Chevrolet are shown in Table 1.

Table 1. Main structural parameters of BMW and Chevrolet

	BMW M3	Chevrolet
Front wheel	a=1.36m	a=0.98m
Back wheel	b=1.37m	b=1.66m
Mass	1549kg	1187kg
Yaw moment	2886kg*m^2	1928kg*m^2

In addition, we make the following three assumptions to ensure that we can use a concise physical model to describe the dynamic behavior of the vehicle:

(1) The car runs smoothly without acceleration or braking;

(2) Only consider the effect of gravity and the force on the front and rear wheels on the car;

(3) The force on the left side of the car is equal to that on the right side.

Three variables x, y and χ are used to describe the motion state of the car, in which x and y describe the position of the car and χ describes the direction of the car. As shown in Figure 1, two vehicle coordinate systems are established to describe the motion of the vehicle, namely stationary frame and car-fixed frame. We will use stationary fixed frame to write the equations of motion. However, it is easier to express the forces in a car-fixed frame.

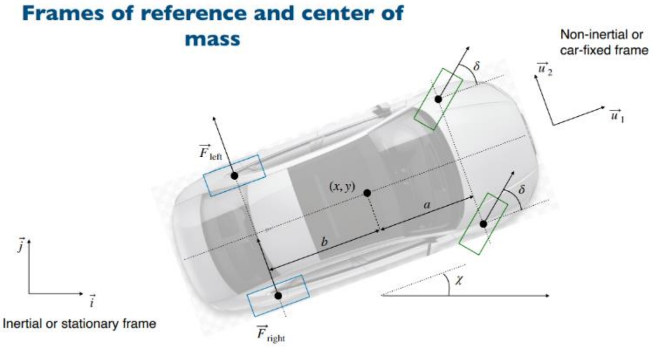

Figure 1. Coordinates and parameters of the vehicle mode

2.2. Mathematical description of vehicle driving process

In the actual driving process, due to the change of road conditions, the motion trajectory of the car also presents different shapes. In this work, we establish two typical vehicle trajectory models, namely U-turn and lane change. In the process of driving, the driver will control the rotation of the steering wheel to control the angle of the front wheel, and then control the trajectory of the vehicle. For the two driving tracks of U-turn and lane change, we establish the change function of angle with time to describe the motion of the vehicle, as shown in the following formula.

For U-turn, we have:

$$\delta = \begin{cases} 0, & if\ t < t_{start} \\ \frac{\delta_0}{2}(1 - cos\left(\frac{\pi(t-t_{start})}{t_0}\right), & if\ t_{start} \le t < t_{start} + t_0 \\ \delta_0, & if\ t_{start} + t_0 \le t < t_{end} \\ \frac{\delta_0}{2}\left(1 - \cos\left(\frac{\pi(t_{end}+t_0-t)}{t_0}\right)\right), & if\ t_{end} + t \le t < t_0 + t_{end} \\ 0, & if t_{end} + t_0 \le t \end{cases} \tag{1}$$

Where $\delta_0 = \frac{\pi}{24}\ rad,\ t_{start} = 0.1\ s,\ t_0 = 1\ s,\ t_{end} = 5.45\ sec.$

For lane change, we have:

$$\delta = \begin{cases} 0, & if\ t < t_{start} \\ \delta_0 \sin\left(\frac{t-t_{start}}{t_0}\right), & if\ t_{start} \le t < t_{start} + 2\pi t_0 \\ 0, & if\ t_{start} + 2\pi t_0\ \le t \end{cases} \tag{2}$$

$\delta_0 = \frac{\pi}{50}\ rad,\ t_{start} = 0.1\ s,\ t_0 = 0.5\ s,\ t_{end} = t_{start} + 2\pi t_0.$

Equation (1) describes the change of angle with time in the process of U-shape, while equation (2) describes the change of angle with time in the process of lane change. The change curve of angle with time under the two driving modes is shown in Figure 2.

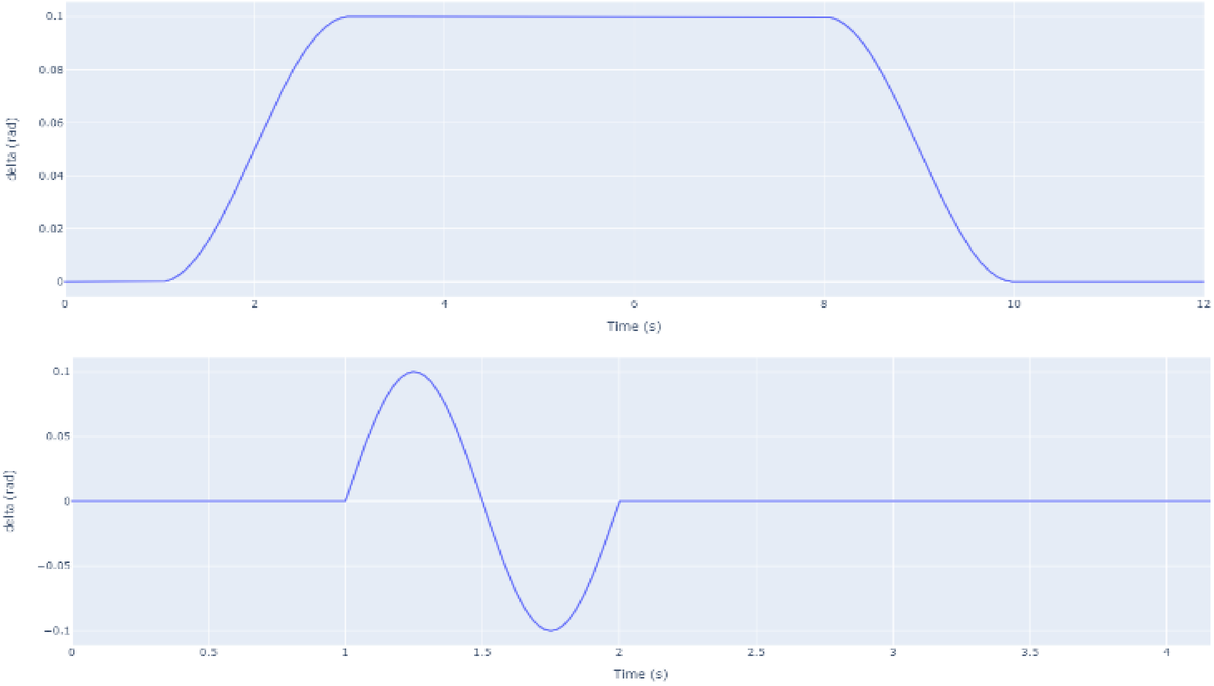

Figure 2. Delta vs. time for U-turn and lane change

2.3. Establishment of vehicle dynamic model

According to Newton's second law, it is necessary to obtain the function of the force of the vehicle with time to obtain the function of the angular velocity and angular acceleration of the vehicle. In the linear model of vehicle lateral force calculation, the slip angle of the vehicle needs to be calculated first, which is defined as the angle between the actual driving direction of the vehicle and the wheel direction. According to the definition, the slip angle of the vehicle can be calculated by the following formula:

$$a_f = \delta - \sin^{-1}\left(\frac{\overrightarrow{v_f}\cdot\overrightarrow{u_2}}{\|v_f\|}\right) \tag{3}$$

$$a_r = -\sin^{-1}\left(\frac{\overrightarrow{v_r}\cdot\overrightarrow{u_2}}{\|u_2\|}\right) \tag{4}$$

Formula (3) is the calculation function of front wheel slip angle, while formula (4) is the calculation function of rear wheel slip angle. In the formula, v_f is the moving speed of the front wheel and v_r is the moving speed of the rear wheel.

After obtaining the calculation model of slip angle, use the linear model to calculate the lateral force on the wheel, as shown in the following formula:

$$F = Ca \tag{5}$$

Then, we calculate the acceleration and angular acceleration of the vehicle through the lateral force of the wheel, as shown in the following formula:

$$x" = \frac{1}{M_G}\left[-F_r\sin(\chi) - F_r\sin(\chi+\delta)\right] \tag{6}$$

$$y" = \frac{1}{M_G}\left[F_r\cos(\chi) + F_f\cos(\chi+\delta)\right] \tag{7}$$

$$\chi" = \frac{aF_f\cos\delta - bF_r}{I_Z} \tag{8}$$

In order to calculate other motion parameters of the car, a system of ordinary differential equations is established to establish the relationship between various motion parameters. Using Python’s function to solve the ordinary differential equations, we can get other motion parameters of the car, such as speed, position and angular position.

2.4. Establishment of calculation model of self-aligning torque

Through strict assumptions and derivation of material mechanics, the relationship between dimensionless slip angle and self-aligning torque can be obtained, as shown in the following formula:

$$\frac{M}{\mu W} = \frac{1}{6}\psi - \frac{1}{6}\psi^2 + \frac{1}{18}\psi^3 - 162\psi^4 \tag{9}$$

$$\psi = \frac{K}{\mu W}\tan\beta \tag{10}$$

Where M is the self-aligning torque, μ is the coefficient of friction between the tire and the ground, W is the vertical force,

According to the above formula, when the lateral stiffness of the vehicle, the contact length between the wheel and the ground, the friction coefficient between the wheel and the ground and the vertical force are determined, the value of the self-aligning torque can be calculated given the slip angle.

3. Result

3.1. Analysis of self-aligning torque of vehicle during U-turn

By solving the ordinary differential equation in the program, all the kinematic parameters of the vehicle in the turning state are obtained. Taking BMW as an example, the X and Y coordinates of the vehicle during turning are drawn on the same picture, and the trajectory of the vehicle during U-turning is obtained, as shown in Figure 3.

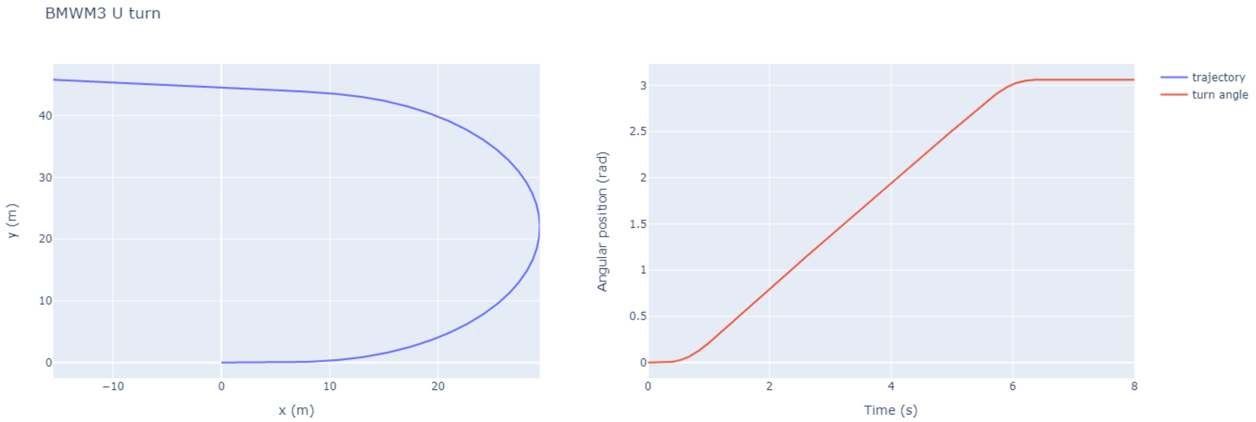

Figure 3. Trajectory and turn angle plot for M3 in U-turn

It can be seen from the picture that the car starts from point (0,0) at zero time, turns a U-shaped corner counterclockwise and continues to drive in the speed direction opposite to the initial speed direction. This result conforms to the definition of U-turn and the correctness of dynamic model. Then, draw the change curve of the angular position of the vehicle with time, as shown in the Figure 4. It can be seen that the angular position of the vehicle increases smoothly from 0 to about 180 degrees, which is consistent with the driving track of the vehicle. After obtaining the dynamic parameters of the vehicle, through the calculation model of vehicle lateral force and slip angle, the relationship images between them with time are obtained respectively. It can be seen from Figure 4 that in the process of U-turn, the lateral force starts from zero with the passage of time, first increases with a large slope, remains stable when it increases to about 4778N, and finally drops to zero quickly. The change trend of slip angle is exactly the same as that of lateral force, because in the linear model of lateral force calculation, the lateral force is equal to the product of slip angle and vehicle turning stiffness. Since the turning stiffness of the vehicle is given by the manufacturing process of the vehicle, it can be regarded as a constant in the actual movement process, and the lateral force and slip angle should be directly proportional in the movement process. This is the same as the trend shown in the figure. When the car turns, the driver will turn the steering wheel to make the direction of the tire and the driving direction of the vehicle form a certain angle. At the same time, the forward direction of the car will also change along the rotation direction of the tire, but the rotation angle lags behind the rotation angle of the tire. The two will form a certain angle, that is, the slip angle. At the beginning of cornering, due to the large tendency of tire rotation, the angle between the forward direction of the vehicle and the tire direction increases rapidly, which makes the slip angle increase with a large slope; At the later stage of turning, the car gradually returns to the state of straight driving, the direction of its tires gradually tends to be consistent with the forward direction, and the slip angle rapidly decreases to zero.

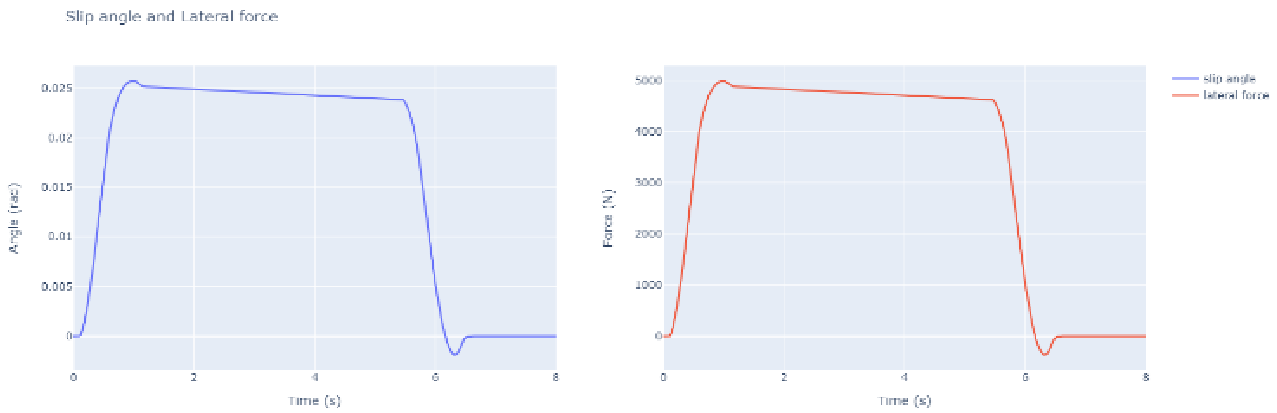

Figure 4. Slip angle and lateral force plot for M3 in U-turn

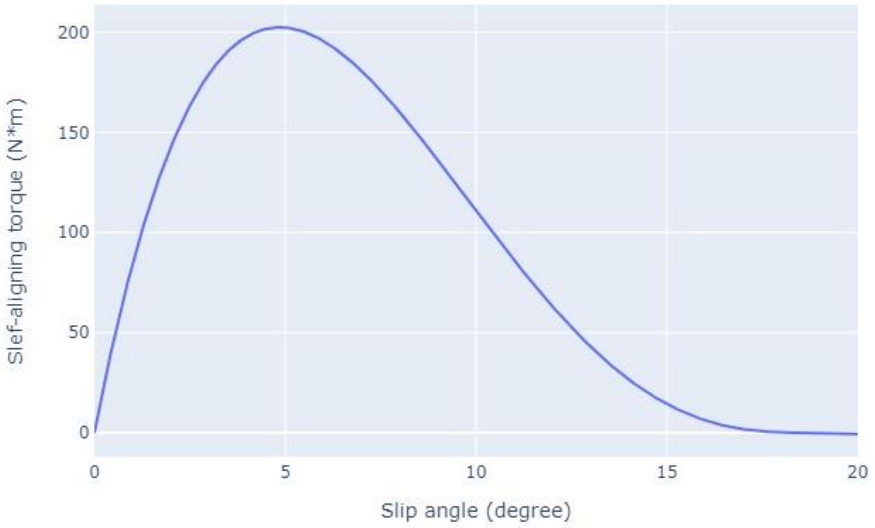

Figure 5. Self-aligning torque vs. slip angle

According to the calculation model of the self-aligning torque, the variation curve of the aligning torque with the slip angle during the driving of BMW can be obtained, as shown in Figure 5. As can be seen from the figure, when the value of slip angle is less than 5 degrees, the self-aligning torque is directly proportional to the slip angle. Since the maximum slip angle of BMW is less than 5 degrees in the process of U-turn, the change trend of its self-aligning torque should be the same as the slip angle in theory. As shown in Figure 6, the curve of the self-aligning torque with time is drawn. Similar to the predicted results, the self-aligning torque increases rapidly at the beginning, remains stable when it rises to 110N•m, and decreases rapidly to zero at the end of the turn. It can be seen that in the initial stage of U-turn, the vehicle changes from straight-line driving to counterclockwise turning. Under the action of increasing self-aligning torque, the trend of restoring straight-line driving also increases gradually. After that, the self-aligning torque of the car remains unchanged, and the car has a tendency to return to straight-line driving in the process of turning. However, due to the certain included angle between the wheel angle and the angular position of the car, the car cannot return to straight-line driving. At the end of the turn, the rotation angle of the wheel is gradually reduced to zero. Under the action of the self-aligning torque, the vehicle returns to a straight line, and the self-aligning torque is also reduced to zero.

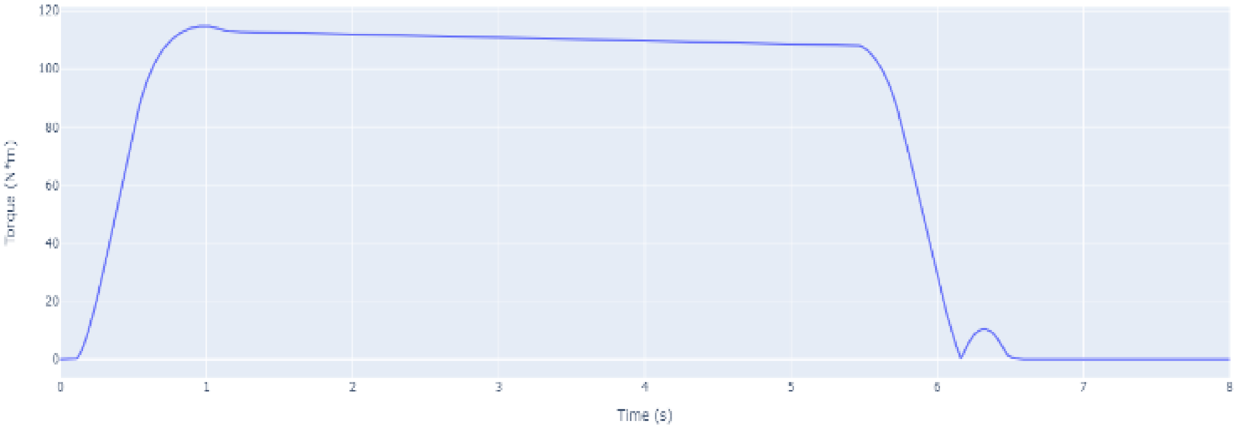

Figure 6. Self-aligning torques vs. time for M3 in U-turn

3.2. Analysis of self-aligning torque of vehicle during Lane-change

Taking BMW as an example, after solving the motion parameters in the process of lane change by using the existing dynamic model, the change of its motion trajectory and angular position with time is drawn, as shown in Figure 7.

It can be seen from the figure that the car runs in a straight line in the initial state, then turns counterclockwise and then turns clockwise, and finally returns to a straight line, which is in line with the definition of lane change movement. In the change curve of vehicle angular position, it can be seen that the angular position starts from zero, first rises to about 16 degrees, and then decreases to 0 degrees, which is in line with the motion trajectory of the vehicle.

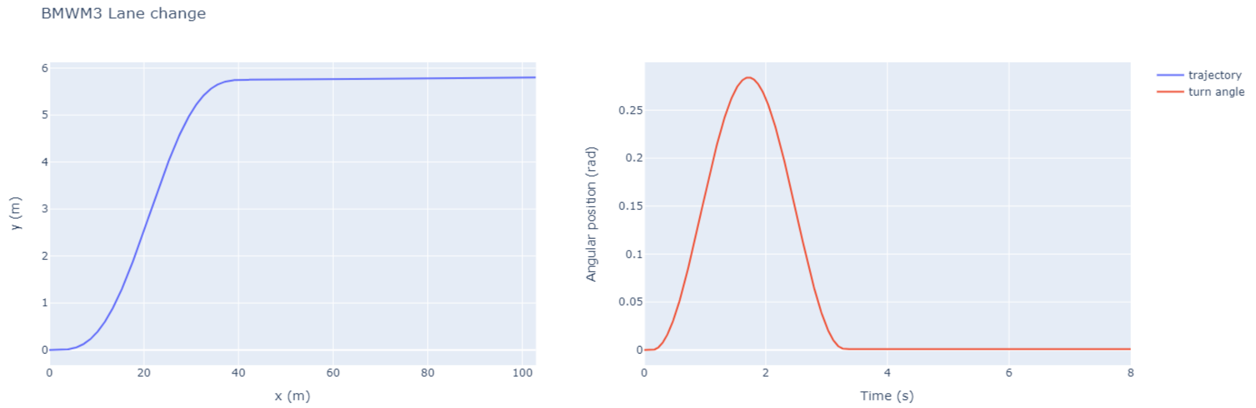

Figure 7. Trajectory and turn angle plot for M3 in lane change

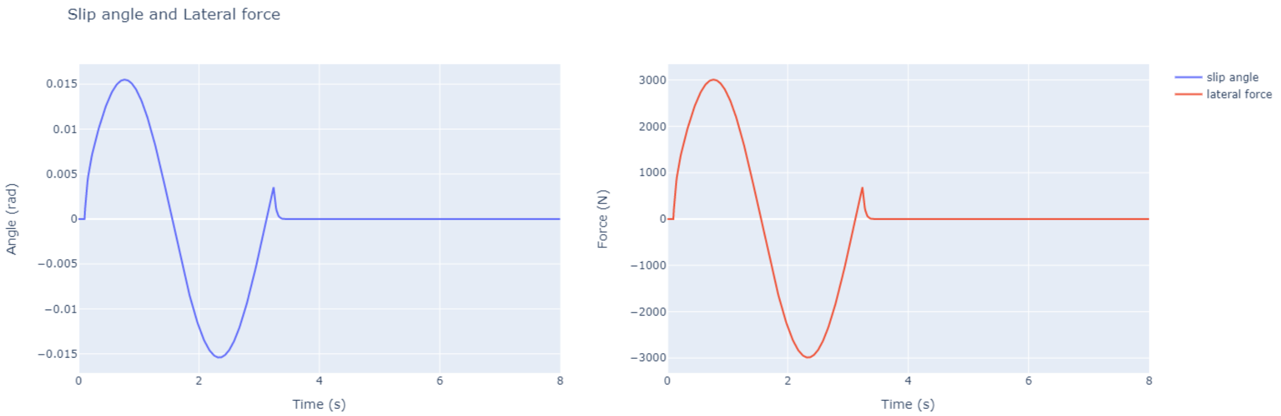

Figure 8. Slip angle and lateral force plot for M3 in lane-change

Draw the variation curve of slip angle and lateral force with time using known parameters, as shown in Figure 8. It can be seen that the change of slip angle with time first increases to 0.9 degrees, then decreases to 0 degrees, then reversely increases to 0.9 degrees, and finally returns to 0 degrees. In the first stage, the car turns counterclockwise. At this time, the change speed of the car's angular position lags behind the change speed of the car's tire angle, and the slip angle increases. In the next stage, in order to complete the lane change action, the car needs to turn clockwise. At this time, the corner direction of the tire needs to be opposite to that in the first stage. After turning the steering wheel in the opposite direction, the steering of the car tire first reaches the same angle as the corner position, and then increases in the opposite direction. Accordingly, the slip angle first decreases to 0 degrees and then increases in the opposite direction. In the last stage, in order to resume straight driving, the tire angle of the car needs to be kept in the same state as the angle position, and at the same time, the slip angle gradually decreases to 0 degrees. In the linear model, the lateral force is directly proportional to the slip angle, so its variation trend is exactly the same as the slip angle. Firstly, it rises from zero to 3000N, then decreases to zero, then increases to 3000N in reverse, and finally decreases to zero.

Finally, through the calculation model, the variation curve of self-aligning torque with time under the condition of lane change is obtained, as shown in Figure 9. Since the variation range of slip angle is within 5 degrees, the self-aligning torque increases with the increase of slip angle. Theoretically, the variation trend of self-aligning torque should be the same as that of slip angle. The situation in the figure is the same as the predicted result, first increasing from zero to 76 and then decreasing to 0. This process will be repeated in the next stage of motion. In the process of vehicle lane changing, the self-aligning torque of the vehicle has experienced four stages: increase, decrease, increase and decrease, which always makes the vehicle have the trend of returning to straight driving. When the second stage and the third stage alternate, the car returns to a straight line from a short time.

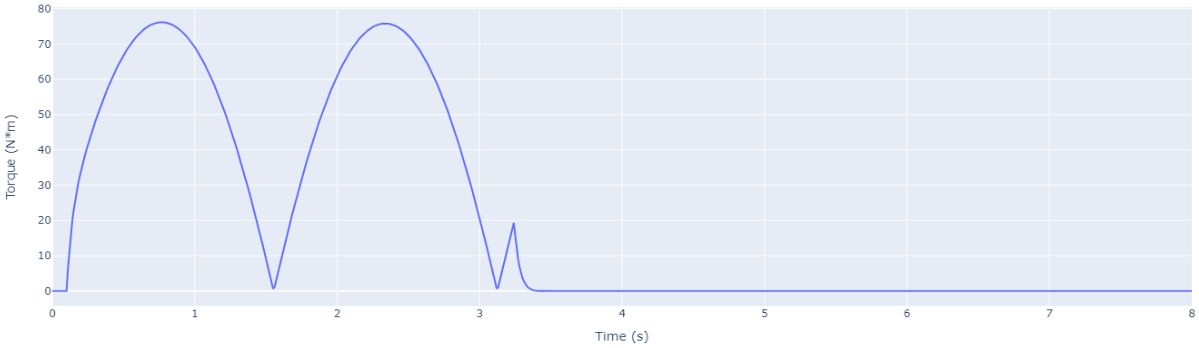

Figure 9. Self-aligning torques vs. time for M3 in lane change

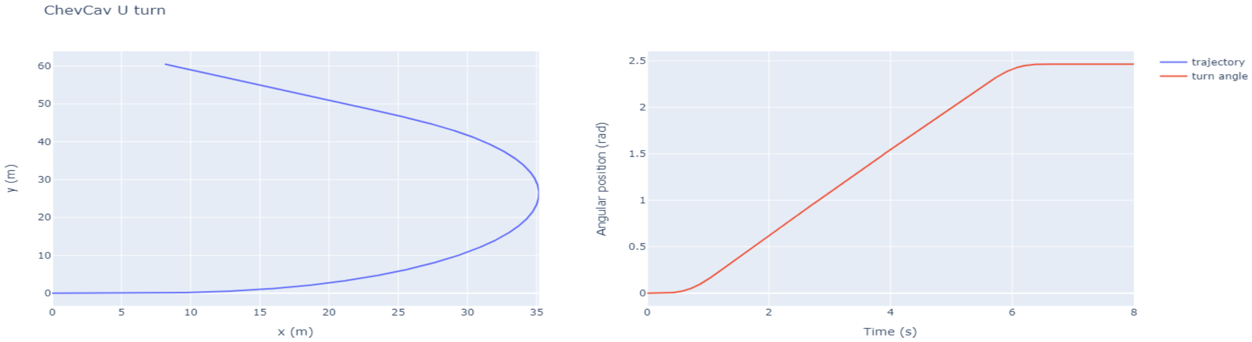

Figure 10. Trajectory and turn angle plot for Chevrolet in U-turn

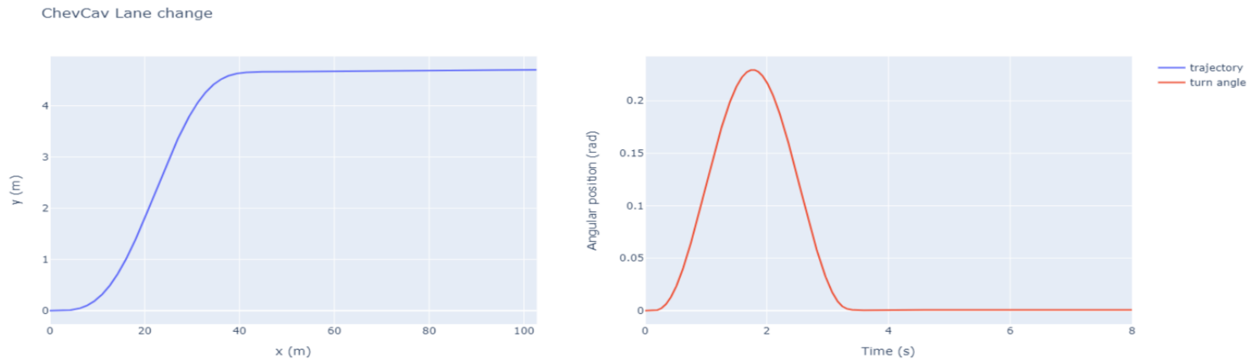

Figure 11. Trajectory and turn angle plot for Chevrolet in lane change

3.3. Comparison of changes of self-aligning torque between BMW and Chevrolet

The dynamic analysis of Chevrolet in U-turn and lane change is carried out, and the dynamic parameters in the process of motion are obtained. Draw the change curve of Chevrolet's motion track and angular position under the two modes, as shown in the Figure 10 and Figure 11. As can be seen, the motion trajectory and angular position changes of Chevrolet are different from those of BMW, but they are basically consistent. Figure 12 compares the self-aligning torque of Chevrolet and BMW under the condition of U-turn and lane change. In the case of U-turn, the variation trend of self-aligning torque of BMW and Chevrolet is the same, but the specific values are different. The maximum self-aligning torque of BMW during turning is 110N•m, while that of Chevrolet is 122N•m. This is because the turning stiffness and vertical force of the vehicle have a direct or indirect impact on the self-aligning torque, and these two parameters are related to the manufacturing process of the vehicle. Similarly, in the case of lane change, the parameter change of Chevrolet is consistent with that of BMW, and the value of maximum self-aligning torque is also greater than that of BMW, which is 108N•m, while that of BMW is 76N•m. Therefore, under the same driving conditions, Chevrolet has a stronger tendency to return to straight-line driving, that is, it has better self-aligning performance.

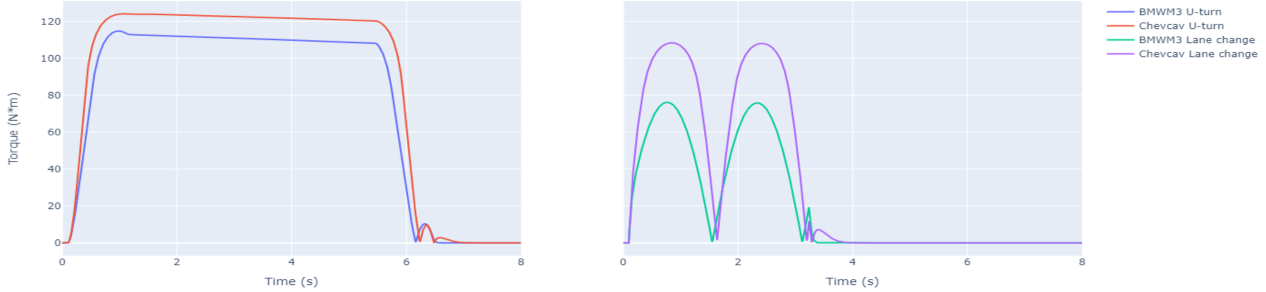

Figure 12. Self-aligning torques vs. time for both cars and in both turn types

4. Conclusion

In this paper, the mathematical model of vehicle dynamics analysis is established, and the variation of vehicle dynamics parameters with time under the condition of U-turn and lane change is obtained by solving the differential equation. Using the calculated dynamic parameters, the change of vehicle self-aligning torque is calculated, and the influence of vehicle self-aligning torque on vehicle dynamic performance is analyzed. Through mathematical modeling and dynamic analysis, the following conclusions are obtained:

(1) In the case of U-turn, the self-aligning torque of the vehicle first increases, then remains stable, and finally decreases to zero.

(2) In the case of U-turn, the self-aligning torque of the vehicle first increases to the peak value, then decreases to zero, then increases to the peak value, and finally returns to zero, and the values of the two peaks are the same.

(3) BMW and Chevrolet have the same change trend of self-aligning torque under U-turn and lane change, but the peak value of Chevrolet's self-aligning torque is greater.

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