Based on the Difference Equation of Forest Ecological and Economic Optimization Management Solution

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Abstract. Forests lay emphasis on meeting ecological balance, carbon sequestration, social material needs, and promoting economic development. The paper will explore an integrated forest management and utilization model to maximize the benefits in all aspects of forest. To find the factors that affecting forest carbon sequestration, the grey correlation method was used to analyze the age structure and annual precipitation. Then the key index, age structure, which is affecting forest carbon sequestration was obtained. For qualitative and quantitative research on forest carbon sequestration of the trend of change over time, Saihanba was selected as the research object, then ArcGIS visualization analysis was conducted. Furthermore, polynomial function was utilized in Matlab data fitting toolbox to fit the data, then a logistics regression model in line with the situation of forest carbon sequestration was obtained. According to this model, we developed the integrated forest management model based on difference equation. This model can facilitate the finding of an optimal cutting intensity coefficient K to obtain the optimal forest management scheme. It also paves the road in getting the relationship between forest carbon sequestration with time.

Keywords: Grey Correlation Method; Logistics Regression Model; Difference Equation Model; Carbon Sequestration.

1. Introduction

Since the industrial revolution, the ever increasing consumption of fossil fuels such as crude oil by human activities and the massive destruction of forest have led to an surge of the greenhouse gases emission such as CO_2 [1] .So far the CO_2 content in the atmosphere has reached 360 ppm, which is 1.25 times higher than the level in the 1900s. The sea level rise, the deterioration of world agriculture and ecology, the elevation of natural disasters, they all indicating the development of the global warming. Given this, it is urgent to take active actions to deal with climate change [2]. Reducing carbon emissions and increasing carbon sequestration is considered one of the most important ways tomitigate climate change [3]. The Kyoto Protocol, which adopted in 1998, signed by the developed countries in order to cut emissions of CO_2 and other greenhouse gases by at least 5% below the levels of 1990 during the 2008-2012 commitment period. The Paris Agreement adopted in 2015 further clarified that the rise in average global temperature in the 21st century should be controlled within 2℃, and the global temperature rise should be controlled within 1.5℃ above the pre-industrial level[4].

2. Model Preparation

2.1 Factors Affecting Forest Carbon Sequestration

Carbon sequestration refers to measures that increase the amount of carbon in carbon pools other than the atmosphere, including both physical and biological sequestration. Physical sequestration involves long-term storage of carbon dioxide in exploited oil and gas wells, coal seams, and deep oceans. Plants use photosynthesis to convert carbon dioxide from the atmosphere into carbohydrates, which are fixed in the form of organic carbon in the body or soil. Biological carbon sequestration is the use of plant photosynthesis to improve the ability of ecosystems to absorb and store carbon so as to reduce the concentration of carbon dioxide in the atmosphere and slow down global warming [5]. The main factors affecting forest carbon sequestration are:

a. Age structure

The carbon sequestration of forests is closely related to the age composition of forests. According to their ages, general forests can be divided into young forest, middle forest, near mature forest, mature forest and overmature forest. Among them, the carbon sequestration rate is the largest in the middle forest ecosystem, while the carbon absorption and release of mature forest/overmature forest are basically balanced because their biomass basically stops growing. The age structure of forests depends not only on the development and evolution of forests but also on the influence of external disturbance. The higher the frequency of disturbance, the larger the composition of young forest, the less its carbon sequestration.

b.Annual precipitation

Precipitation can promote plant growth thus increase plant and biomass productivity. Therefore, precipitation can promote carbon sequestration in forest ecosystems. In the area where the combination of hydrothermal factors favor plant growth, the plant biomass and the carbon density of vegetation is high. In arid regions, precipitation is the main limiting factor of NPP (plant net primary productivity). NPP decreases with the reducing of the ratio of precipitation to potential evaporation, and the corresponding carbon sequestration capacity of plants decreases with the declining of productivity.

c.Geographic location

Topography affects the distribution and growth of forest preparation types to a certain extent by influencing temperature, precipitation, light, and other factors, which further affecting the input of forest ecosystem. In addition, different slopes and elevations receive different degrees of human disturbance. With the increase of slope or altitude, the chance and degree of human disturbance become smaller and the carbon sequestration capacity remains high or become higher.

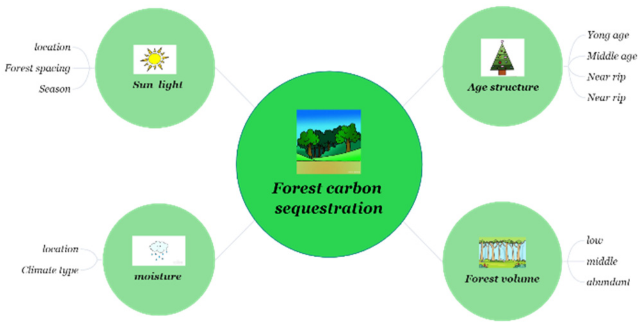

Figure 1. Factors affecting forest carbon sequestration

2.2 Saihanba Data

Saihanba is located in Chengde City, Hebei Province. The forest farm covers a total area of 94000 hm2. The forest coverage rate reaches 78.1%, and the total forest stock is 8106,388 m^3 .The precipitation is mainly rainfall, supplemented by snowfall. The annual average precipitation is 452.2 mm, and the maximum annual precipitation is 636.0 mm.

By querying the land use data in 2021, ArcGIS software was used to search the regional map of Saihanba, and the boundary map of the study area was obtained. The forest cover in different areas was extracted by the mask method, and the quantitative relationship between the vegetation cover index and various vegetation areas was calculated according to the formula.

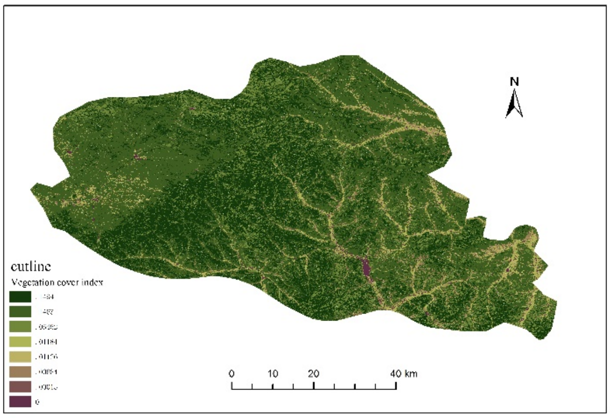

Figure 2. Vegetation Coverage Index of Saihanba

3. Model Establishment and Solution

3.1 Fitting the Relationship between Carbon Sequestration and Time Change

In the research of forest carbon sequestration quantity relationship changes over time, Saihanba was selected as our object. Solid carbon was chosen as the dependent variable while time was selected as the independent variable to construct a scatter diagram. MATLAB software was used to run the data fitting. To acquire the amount of forest carbon sequestration under normal change over time. The forests were divided into five groups: under 30 years, 30-60 years,60-90 years as nearly mature, 90-120 years as mature forest, and over 120 years for overmature forest [6].

Table 1. Division table of each age forest

Type	Young Age	Middle Age	Near Mature	Mature Forest	Over Mature
YEAR	≤30	30-60	60-90	90-120	≥120

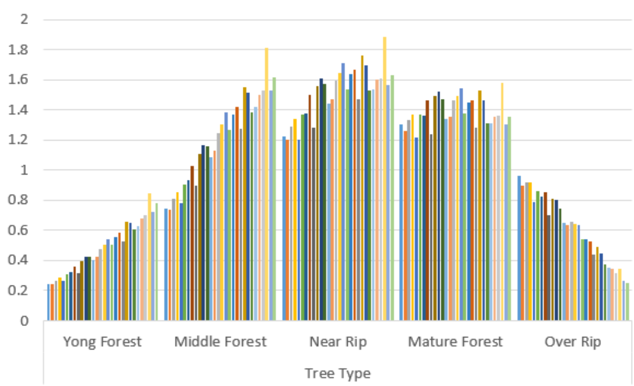

Figure 3. Bar Chart of Carbon Sequestration of Each Age Forest

As seen in the figure, the amount of carbon sequestration in young trees keeps increasing over time, and the increasing rate are also enlarging, the increasing rate of middle-age trees first increases and then decreases, the increasing rate of near-mature forest decreases, the amount of carbon sequestration in mature forest remains almost unchanged, and the amount of carbon sequestration in overmature forest keeps decreasing.

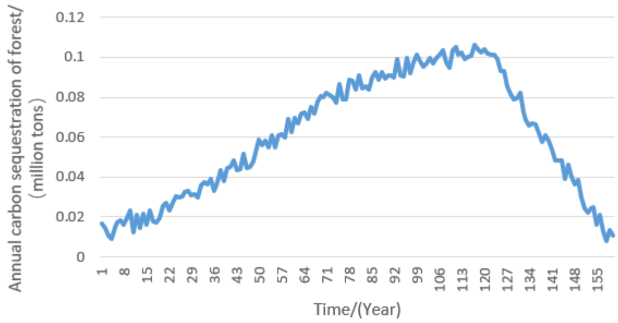

Figure 4. Changes in forest carbon sequestration

Figure 4 shows the change in carbon sequestration amount of a forest will go through four stages, in details, the initial stage of low carbon sequestration rate or regeneration stage after interference, the stage of maximum logistics growth of carbon sequestration rate, the mature stage of decreasing carbon sequestration rate, and the stage of forest death when carbon is decomposed into soil in the absence of human intervention,. Thus, the carbon dynamics of forests are largely determined by their age-level changes.

The MATLAB data fitting toolbox was used to fit the data and get the relationship between forest carbon sequestration in different times. In the initial and middle stages of the forest, that is, at the young age, middle age, near maturity, and maturity stages, the change of forest carbon sequestration roughly presents the change of logistics distribution.

According to the fitting result, we can get the function relation:

$$y_1(x) = \frac{1.1568}{0.9854 + e^{-0.9552x}} \tag{1}$$

At the later stage of the forest, the overripe stage, the forest carbon sequestration showed a downward trend because the biomass did not change and the respiration increased gradually. The fitting result was:

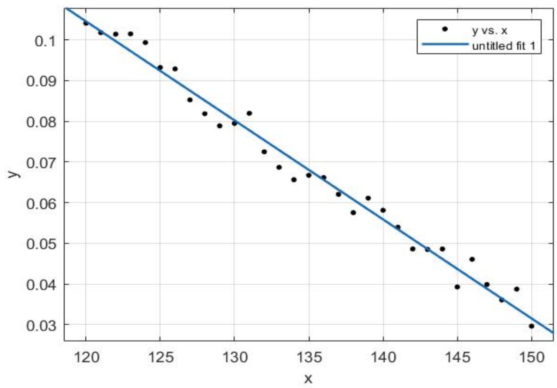

Figure 5. Logistic data fitting diagram

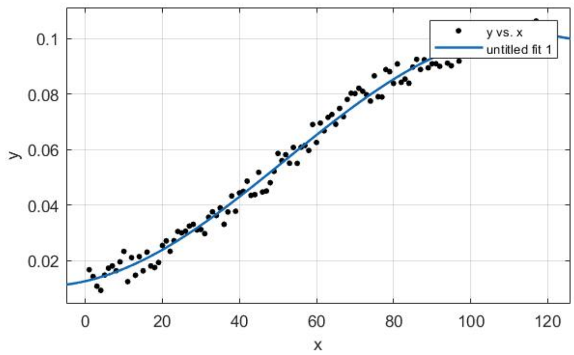

Figure 6. Linear regression data graph

According to the fitting result, we get the function relation:

$$y_2 = -0.4852\text{x} + 2.2359 \tag{2}$$

3.2 The Establishment and Solution of Difference Equation Model

The carbon sequestration effect of forests refers to the process of converting carbon dioxide and water into biomass and releasing oxygen in the process of photosynthesis. The specific reaction process is shown in the figure.

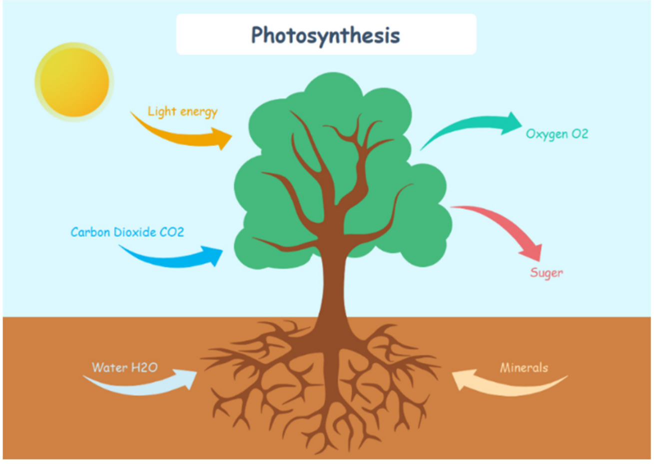

Figure 7. Diagram of photosynthesis

Forest realizes carbon sequestration through photosynthesis, converts CO_2 into organic matter in nature, and releases a large amount of oxygen and energy [7]. The reaction equation of photosynthesis is

$$6\mathrm{CO}_2 + 6\mathrm{H}_2\mathrm{O} \overset{\text{photosynthesis}}{\longrightarrow} \mathrm{C}_6\mathrm{H}_{12}\mathrm{O}_6 + 6\mathrm{O}_2 + ATP$$

The equation shows that the forest product of photosynthesis is mainly oxygen and organic materials. The volume of the organics mainly exist in the form of trees and forest products, thus we can calculat the volume of trees and the ownership to indirectly indicate the forest carbon sequestration capacity. In sustainable development of the forest, not under the premise of expansion of the scale, most deforestation can achieve maximum amount of carbon sequestration, to achieve optimal forest management.

Starting from the problem of tending and thinning of the pure plantation, we consider the hypothesis of optimal forest management under the condition of single site:

Although the stand growth rate of each tree species is different, the tree species can be divided into 5 age groups, which are young forest, middle forest, near mature forest, mature forest and overmature forest. They are denoted as

i_1, i_2, i_3, i_4, i_5 , a_1, a_2, a_3, a_4, a_5 are the average carbon sequestration per tree in each age group, denoted as W_1, W_2, W_3, W_4, W_5 respectively.

Since the plantations were considered, the effects of natural conditions are very small, and the possibility of their natural wither could be ignored, and only the artificial harvesting rate was denoted as k , the amount of artificial tending regeneration was denoted as β , the amount of regeneration survival was denoted as β', and the survival rate was denoted as

$$r = \frac{\beta'}{\beta} \tag{3}$$

When the artificial logging rate k is equal to the survival rate r. It means that the forest can meet the requirements of sustainable carbon sequestration. Therefore, the annual harvesting of trees should be accompanied by timely tending of new trees.

Although the increase in the total amount of trees is discrete, for the forest, we can artificially change the total amount of trees with time is continuous. Scientific tending thinning can only cut the mature forest and overmature forest, and the ratio of cutting intensity between mature forest and overmature forest is denoted as μ, and the ratio of cutting intensity are μk and k . Since the number of trees is continuously changing, the number of 4 and 5 age trees is only related to the cutting rate. It can be seen that the changes in trees in all ages should meet the following requirements:

$$\frac{dx_i(t)}{dt} = -\rho x_j(t),\ i = 1,2,3,4,5 \tag{4}$$

Where ρ represents the logging rates of different age grades. The logging rates of grades 1, 2 and 3 are zero, and those of grades 4 and 5 are μk and k, respectively. From the above formula, it can be obtained:

$$x_i(t) = x_i(0)e^{-\rho r} \tag{5}$$

The cutting amount of mature forest trees per unit time is proportional to the total number of mature forest trees, and their proportional coefficient, namely, the cutting intensity coefficient k, remains constant, and only 4 and 5 trees are cut in the cutting period. Therefore, the cutting intensity coefficient k determines the number change rule of 4 and 5 aged trees in the cutting period as follows:

$$\frac{dx_4(t)}{dt} = -\mu kx(t) \tag{6}$$

Sustainable logging requires the total number of trees in all age groups are the same at the beginning of each year. Under this equilibrium condition, the cutting intensity directly affects the annual harvest. To get the highest annual harvest, we can obtain the following equation by considering the preceding equation:

$$W = W_4 \int_0^T \mu k x_4(t)dt + W_5 \int_0^T kx_5(t)dt \tag{7}$$

As shown in the above, the problem is converted to a one-parameter extremum problem. Corresponding to different tree species, we only need to substitute the corresponding data to get the corresponding optimal cutting intensity K value. According to the cutting intensity K, we can choose the appropriate logging amount, to obtain the maximum carbon sequestration, namely the optimal forest management mode, so as to make a certain contribution to the sustainable development and utilization of forest resources.

4. Model Evaluation and Further Discussion

The visualization work is done successfully by us, such as the framework of the research method in the introduction, the bar chart of carbon sequestration made in the integrated forest management model, the optimal felling cycle per year in the comprehensive forest utilization model and so on. Boring data may reflect the tendency, but not as intuitive as so many images.

The analysis of evaluating the impact of Saihanba restoration on the environment can be more accurate if we have more comprehensive data.

We have analyzed and obtained the logistic regression model of Saihanba in the early and middle stages, and the functional relationship is as follows: $y_1(x) = \frac{1.1568}{0.9854+e^{-0.9552x}}$, and the linear regression model is presented in the late stage, and the functional expression is as follows: $y_2 = -0.4852\mathrm{x} + 2.2359$. According to the integrated forest management model, we can obtain the relationship between the maximum carbon sequestration and the optimal cutting intensity coefficient:

$$W = W_4 \int_0^T \mu k x_4(t)dt + W_5 \int_0^T kx_5(t)dt \quad (8).$$

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