Application of Motion Interface Tracking CVOFLS Method to Zalesak Disk Problem

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Abstract. The interface curvature calculation is not accurate in VOF method and the interface mass is not conserved in Level Set method, A new interface tracking method CVOFLS is proposed (Coupled Volume of Fluid and Level Set method). This method combines the advantages of VOF and Level Set, The VOF and Level Set functions are simultaneously solved according to the fluid velocity, The interface obtained by the VOF function is used to correct the fluid quality, The Level Set function is used to calculate the interface norma, The Level Set function reinitialization process is omitted, Thus, the deficiencies of the two methods are overcome effectively. An example of interface tracking numerical simulation shows that, this method can guarantee high precision of free interface tracking and good mass conservation, and it can improve the calculation efficiency.

Keywords: Interface Tracking; CVOFLS; Interface Accuracy; Conservation of Mass; Computational Efficiency.

1. Introduction

Many problems in life are closely related to moving interfaces, such as paraffin melting, crystal growth, solidification and melting interfaces, as well as explosion and combustion gas interfaces [1], Therefore, the study of free interface numerical simulation has application value. At present, the free interface tracking methods can roughly divided into Lagrangian method, Eulerian method and Lagrangian-Eulerian hybrid method according to different reference systems. Among them, Eulerian method is the most common method used to describe fluid by fixing the reference system to the space domain. Among Eulerian methods, VOF methods[2] and Level Set methods[3] are widely used because of their advantages of simple implementation, small memory consumption, and easy handling of interface movement problems with complex topology changes. The VOF method was first proposed by Hirt[4] in 1981, He has successfully performed numerical simulations of dam failure and free surfaces of surges, as well as rayleigh-Taylor instability. Although this method has good mass conservation, it is difficult to calculate the normal and curvature of the interface accurately. Subsequently, Osher 1988 proposed the Level Set method, which can accurately calculate the normal direction and curvature of the interface by constructing an isosurface function to trace the interface implicitly, but the conservation is poor. Some scholars proposed CLSVOF(Coupled Level Set and Volume of Fluid) which combined the advantages of the two methods to solve the problems of the above two methods.

In 2010, Sun Dongliang Tao Wenquan etc[6] presented another form of CLSVOF method, which used geometric principles to calculate the Level Set function near the interface and omitted the transport process of the Level Set function. Although this method simplifies the original CLSVOF method to some extent, its geometry principle and interface reconstruction process are complicated. In 2011, Li Qiang [7] improved CLSVOF method and proposed a CLSVOF method with a quality correction formula. Although this method achieves good accuracy, the calculation efficiency is not high because of the large number of iterations in the re-initialization process of the Level Set function. Wang [8] in 2012, a new VOF method was proposed, which also uses the idea of coupling VOF with Level Set method. Based on the reconstructed interface of VOF method, the distance function with the accuracy of second order was constructed, thus improving the calculation accuracy. However, this method requires two VOF function interface reconstruction and Level Set function re-initialization, so the implementation process is complicated.

Although CLSVOF method combines the advantages of VOF and Level Set method to achieve high precision effect. However, there are still some shortcomings such as large amount of calculation, difficult implementation of the program and not accurate enough calculation. The VOF method also has the advantages of good conservation and high efficiency, but the poor interface normal and curvature become its defects. If the relatively accurate interface normal and curvature calculated by Level Set method are coupled to the VOF method, its defects can be overcome. Based on this, a new method of Coupled Volume of Fluid and Level Set(CVOFLS) is proposed in this paper. In this method, the relatively accurate interface method is obtained by Level Set function to correct the interface normal of VOF function, which saves the re-initialization process of Level Set function. Therefore, more accurate VOF function and higher computational efficiency can be obtained. The simulation of classical interface motion problem verifies the validity and accuracy of the proposed method.

2. Transport Equation

In this paper, VOF function is used to track the moving interface, and Level Set function is used to correct the interface normal direction in VOF function to ensure the accuracy of solution. The following are the transport equations satisfied by the VOF function and the Level Set function in a moving medium with a velocity field:

$$\frac{\partial F}{\partial t} + \mathbf{u} \cdot \nabla F = 0 \tag{1}$$

$$\frac{\partial \phi}{\partial t} + \mathbf{u} \cdot \nabla \phi = 0 \tag{2}$$

In Formula (1), VOF function F represents the volume fraction of fluid in each volume element F value is between 0 and 1 , F is 1 in the volume element filled with fluid, F is 0 in the volume element without fluid.

In Formula(2), Level Set function ϕ is defined to solve the symbolic distance function between the point in the region and the fluid interface, Where the value of the point in the fluid ϕ is positive, in the absence of fluid the value of the point ϕ is negative, the value of the point at the fluid interface is zero.

Reinitialization is required $\phi(\mathbf{x},t)$ during the calculation of the Level Set function, In order to ensure that the Level Set function has a very good smoothness in the whole solution region and maintains a good numerical stability in the solution process, make it become a symbolic distance function from point $\mathbf{x}$ to interface $\Gamma(t)$ again. By solving the initial value problem

$$\begin{cases} \dfrac{\partial \phi}{\partial \tau} = sign(\phi_0)(1 - |\nabla \phi|) \\ \phi(\mathbf{x},0) = \phi_0(\mathbf{x}) \end{cases} \tag{3}$$

the symbolic distance function is reinitialized. Where τ is pseudo time in order to make the result smooth, reinitialization process $sign(\phi_0)$ the following approximation is used $sign(\phi_0) = \dfrac{\phi_0}{\sqrt{\phi_0^2 + \left[(\min(\Delta x, \Delta y))\right]^2}}$, Δx , Δy is the spatial step of the grid in the corresponding coordinate axis direction.

3. The Numerical Method

Although CLSVOF method [8] has high accuracy, it has too many iterations in the re-initialization process of Level Set function, resulting in low computational efficiency. The CVOFLS method in this paper is an improvement on the CLSVOF method. Its basic idea is to discretize THE VOF and

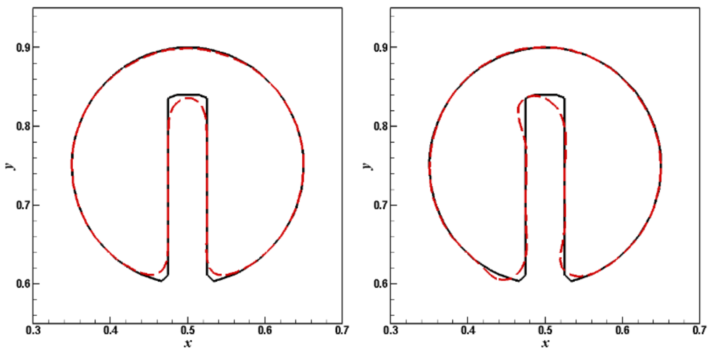
Level Set equations according to the fluid velocity, take the SOLUTION of THE VOF equation as the main body, and use the Level Set function to correct the interface normal (Let's call it n), In this process, the Level Set function reinitialization in CLSVOF method is omitted. Since the interface normal obtained by Level Set function is more accurate than that obtained by VOF function, a more accurate VOF function can be obtained by replacing the interface normal obtained by Level Set function with the interface normal obtained by SOLVING VOF function. Where, the expression of interface unit normal $\mathbf{n}$ obtained by Level Set method is $\mathbf{n} = \nabla\phi / |\nabla\phi|$, $\nabla\phi$ discretization with central difference scheme.

4. The Numerical Example

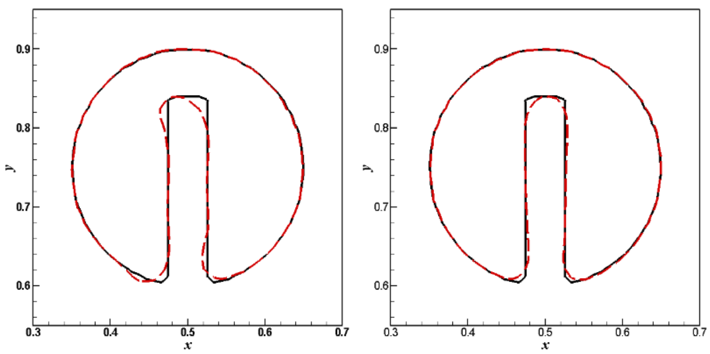
In order to verify the effectiveness of the CVOFLS method proposed in this paper, the following numerical experiment is conducted with the moving interface capture under the rotating flow field as an example.

Zalesakdisk problem is often used to verify the free interface tracing capability. The notch disk has a radius of 0.15, The center of the circle is at (0.50,0.75), The notch width is 0.05. The calculation region of the whole flow field is $[0,1]\times[0,1]$,the time step is 0.0025s. The velocity field is given as follows $u = (\pi/3.14)(0.50 - y)$, $v = (\pi/3.14)(x - 0.50)$

The rotation center of the velocity field is located at (0.50,0.50).



(a) Level Set methods (b) VOF methods



(c) CLSVOF methods [7] (d) CVOFLS methods

Figure 1. Results of Zalesak problem simulated by different methods(solid line: initial shape, dotted line: shape after one rotation)

Figure 1 is a discrete numerical simulation of Zalesak disk problem using grid. In the figure, the simulation results of the Level Set method, VOF method, CLSVOF method [8] and CVOFLS method in this paper for the rotation of the notch disk under the above velocity field are presented respectively. As can be seen from the figure, the simulation results of CLSVOF and CVOFLS method in this paper are better, and the relative mass errors of the results obtained are both less than 0.002%, while the results of CVOFLS method in this paper are slightly better. However, in terms of computational efficiency, Table 1 shows the comparison of simulation time between the two methods under different grids. As can be seen from Table 1, the computational efficiency of CVOFLS method in this paper is significantly higher than that of CLSVOF method in the corresponding grid. Therefore, the CVOFLS method proposed in this paper has the advantages of high precision, high efficiency and good mass conservation.

Table 1. The simulation time of Zalesak disk problem by CLSVOF method [8] and CVOFLS method under different computing grids

Computational grid	CLSVOF method (s)	CVOFLS method (s)
100×100	612.78	65.03
150×150	1336.09	156.98
200×200	2372.28	287.00

5. Conclusion

Because the method of coupling VOF and Level Set has the advantages of both, this paper proposes a new coupling method based on the idea of coupling VOF and Level Set method, namely CVOFLS method. Numerical results show that the simulation results of Zalesak disk by CVOFLS method verify the accuracy of the method, and the simulation interface of the method has high tracking accuracy and good mass conservation. In addition, CVOFLS method is easy to implement and only uses the Level Set function equation to solve the normal direction of the interface without the re-initialization process, so the calculation efficiency is greatly improved.

Acknowledgments

Ministry of Education of China 2018 Yue Qian Technology Industry University Cooperation and Collaborative Education Program (No. 201802153104), Xi'an Eurasia University Key course construction fund project (No. 2018KC032).

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