Research on Combination Optimization Method of Equipment Maintenance Task Package based on Maintenance Cost Analysis

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Abstract. This paper proposes a maintenance task package determination method based on the analysis of generalized maintenance cost (including maintenance cost, loss cost and maintenance time), aiming at the preventive maintenance task of aircraft. In the constructed maintenance task packaging model, practical issues such as approach, workload, logical sequence of maintenance tasks and the will of operators are considered in the implementation process of preventive maintenance tasks, to clarify the priority of packaging strategy and avoid problems such as excessively large workload distribution of each task package and even the inability to be realized in the actual field due to the pursuit of low cost. Based on the actual requirements of maintenance engineering, make preventive maintenance task package to meet the requirements of operators.

Keywords: Maintenance Task Combination; Maintenance Task Packaging; Maintenance Interval; Maintenance Task Execution Area.

1. Introduction

Preventive maintenance tasks are determined by the aircraft operator according to the aircraft configuration, operating environment and maintenance experience, the implementation of maintenance program/regulations, airworthiness and operation regulations requirements and manufacturer recommendations. The maintenance task package is an organic combination of relevant preventive maintenance tasks to form a maintenance plan, so as to reduce the stoppage time of ordered inspection, improve maintenance efficiency and reduce maintenance costs. At present, maintenance cost accounts for 2/3 of aircraft purchase cost and 10-20% of direct operation cost [1]. With the continuous improvement of aircraft reliability and economy requirements, on the basis of ensuring the safety and reliability of aircraft operation, it has become one of the primary goals of the current development of airlines to optimize the maintenance plan to the maximum extent and improve the efficiency [2]. Therefore, it is necessary to study the method of determining maintenance task package based on cost analysis, so as to optimize and reduce maintenance cost and improve maintenance efficiency.

At present, in the process of optimizing maintenance costs, foreign scholars tend to put forward optimization models from the maintenance strategies of complex systems involved in their own industries to find the best maintenance time, so as to optimize analysis in terms of maximizing production and reducing maintenance costs. In 2015, Shafiee M proposed the optimal group optimization strategy based on service time and applied it to wind turbine bearings to ensure the minimum long-term maintenance cost of the system [3]. In 2018, Nzukam C proposed an opportunity maintenance plan, which uses Monte Carlo simulation method to consider the randomness of preventive maintenance summary opportunity duration, and seeks the best maintenance opportunity on the basis of ensuring equipment reliability and availability, so as to optimize maintenance costs [4].

Domestic scholars focus more on the introduction of the idea of opportunistic maintenance in the optimization of maintenance strategy. As early as 2007, in view of the high fixed maintenance cost of multi-component system, Cai Jing et al. proposed to adopt the opportunistic maintenance strategy, organically combine preventive maintenance and fault maintenance, so as to share the fixed maintenance cost of the system and realize the purpose of saving maintenance cost [5]. In 2015, Yang Qingxiu from Civil Aviation University of China compared and analyzed the advantages and disadvantages and applicability of the group optimization model organically combining direct and indirect maintenance groups and the group optimization model based on case-based reasoning and data mining, providing a theoretical basis for the group optimization of maintenance tasks of airlines [6]. In 2020, Wei Qidong also studied the combinatorial optimization problem of preventive maintenance tasks of civil aircraft systems, built a single objective and multi-objective optimization model of preventive opportunistic maintenance with the goal of minimizing the rate of unit time cost and maximizing the availability of congestion, and introduced the idea of opportunistic maintenance with the goal of minimizing the maintenance cost of civil aircraft components and the constraint condition of reliability. The maintenance tasks are combined and optimized according to their time and structure correlation [2]. In the study of combined maintenance strategy of wind farm in 2021, Zhao Hongshan determined the optimal preventive maintenance cycle of each component on the basis that the failure rate of each component obeyed Weibull distribution, and then determined the implementation time of follow-up preventive maintenance of each component. All maintenance tasks within a period of time in the future are grouped into multiple maintenance groups according to group schemes, and all maintenance tasks contained within a single maintenance group are arranged to be executed by multiple maintenance teams together by hierarchical optimization method, so that the maintenance team can have the shortest working time under the condition of the shortest downtime of wind turbines [7].

At present, domestic and foreign scholars aim at reducing maintenance cost and optimizing maintenance strategy, and most of them take minimizing system maintenance cost and maximizing availability as optimization objectives. For the whole plane, there are few studies considering the influence of objective and practical problems such as operator's will, logical order of maintenance tasks and workload on the maintenance cost and the availability of the whole plane in the actual maintenance process. In 2017, Tingting Lu proposed a method to formulate a combination assignment scheme of airline maintenance tasks oriented to the actual situation of airlines. According to the inherent attributes of maintenance tasks and the technical elements required by maintenance tasks, the maintenance tasks were grouped and bundled. The maintenance task package was taken as the basic element and combined with the maintenance ability of users and the use of the aircraft fleet. Establish a user-oriented airline maintenance task allocation model [8]. In the paper published by Zhang Peng et al in 2020, the manual hours and stopover time of maintenance task packages were taken as constraints, and the maximum cumulative execution time of all maintenance task packages at the post-flight stopover was taken as the objective function. The optimization model of civil aircraft maintenance task group packages was established and solved according to the construction algorithm of maintenance interval deviation stipulated in AC-121-53. Take B737NG as an example for simulation verification [9].

Aiming at the preventive maintenance task of aircraft, this paper proposes a maintenance task package determination method based on the analysis of generalized maintenance cost (including maintenance cost, loss cost and maintenance time). In the constructed maintenance task packaging model, practical issues such as approach, workload, logical sequence of maintenance tasks and operator's will are considered in the implementation process of preventive maintenance tasks, so as to clarify the priority of packaging strategy and avoid problems such as excessively large workload distribution of each task package and even the inability to be realized in the actual field due to the pursuit of low cost. Based on the actual requirements of maintenance engineering, make preventive maintenance task package to meet the requirements of operators.

2. Annotation and Explanation

For ease of understanding, this section will explain the symbols used in the paper and their meanings, as shown in Table 1.

Table 1. Annotation and explanation

Annotation	Explanation
T_i	maintenance task *i*
I_i	Maintenance interval of maintenance task *i*
$f_i(t)$	The probability density function of the fault corresponding to the maintenance task *i*
λ_i	The probability distribution of the fault corresponding to the maintenance task *i*
τ_i	The execution time of the maintenance task *i*
c_i	Costs incurred in performing maintenance task *i*
$E[\tau_i]$	The expected loss of unit uptime corresponding to the maintenance task *i*
d_i	The repair cost generated after a fault occurs corresponding to a maintenance task *i*
δ_i	Operator's risk factor of failure mode *i*
L_i	The executable interval of the maintenance task *i*
L_i^-	The lower limit of the executable range
L_i^+	The upper limit of the executable range
$\overline{L}_i$	The executable interval except for maintenance task *i*
M_i	The workload of maintenance task *i*
P_j	Maintenance task package *j*
K_j	Interval of maintenance task packages *j*
W_j	The workload limit of the maintenance task package *j*
$\mathbf{Q}_{n\times n}$	Maintenance task correlation matrix

3. Analysis of the Feasible Interval between Maintenance Tasks According to the Costs

For aviation operators, in order to facilitate the arrangement of maintenance plans, multiple maintenance tasks are usually combined to form a task package. Therefore, it is necessary to adjust the execution time of maintenance tasks within a certain range, balancing the cost of preventive maintenance tasks and the risk cost of not taking maintenance tasks.

Since each maintenance task is to reduce the probability of a failure mode, we need to master the above trade-offs: the probability distribution of various types of failures $f_i(t)$, the cost of performing maintenance tasksc_i and the cost of repair and shutdown after the occurrence of failures d_i.

3.1 Cost Tradeoff Analysis of Maintenance Tasks

To simplify the process, we assume that the maintenance task will be able to repair the fault as fresh as possible. Therefore, the above tradeoff process can be expressed as the classic "age change strategy": when the operation time of the aircraft reaches a specified threshold valueτ_i $(\tau_i \leq I_i)$, the maintenance task *i* is performed on the aircraft; If the aircraft has a corresponding fault before τ_i, the corresponding repair work will be started. The essence of the problem is to determine the optimal τ_i to minimize the expected loss per unit of operating time ($E[\tau_i]$) of the aircraft in long service. Since it is assumed that maintenance tasks can repair faults as new, according to the update theory[10], the running time interval (denoted as a cycle) between any two adjacent maintenance tasks is independently and equally distributed. Therefore, the expected loss minimization problem of unit running time in long-term use can be simplified to the expected loss minimization problem of unit running time in one cycle:

$$\min_{0<\tau_i\leq I_i} E[\tau_i] = \lim_{t\to\infty} \frac{expected\ loss\ between[0,t]}{t} = \frac{expected\ loss\ in\ a\ cycle}{average\ period\ length} \tag{1}$$

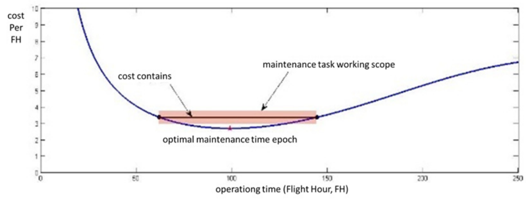
In Formula (1), the average expected loss in a period consists of two parts: The expected loss $c_i \cdot R_i(\tau_i)$arising from performing maintenance task i and the expected loss $d_i \cdot F_i(\tau_i)$ arising from failure, where $F_i(\tau_i) + R_i(\tau_i) = 1$. Similarly, average period length has two parts: The average operating time $\tau_i \cdot L\{X \geq \tau_i\}$of the aircraft under the condition of performing maintenance task i and the average operating time $X \cdot L\{X \geq \tau_i\}$of the aircraft under the condition of failure, where X is the operating time of plane. Therefore, Equation (1) can be written as:

$$\min_{0<\tau_i\leq I_i} E[\tau_i] = \frac{c_i \cdot R_i(\tau_i) + d_i \cdot F_i(\tau_i)}{\int_{\tau_i}^{\infty} \tau_i \cdot dF_i(t) + \int_0^{\tau_i} t \cdot dF_i(t)} = \frac{c_i \cdot R_i(\tau_i) + d_i \cdot F_i(\tau_i)}{\tau_i - \int_0^{\tau_i} F_i(t)dt} \tag{2}$$
$$= \frac{c_i \cdot R_i(\tau_i) + d_i \cdot F_i(\tau_i)}{\int_0^{\tau_i} R_i(t)dt}$$

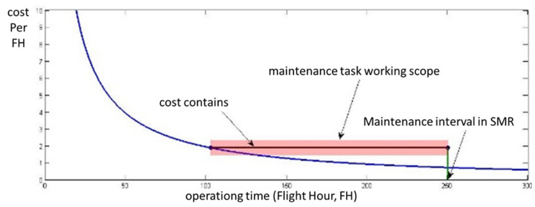
When the constraint $\tau_i^* \leq I_i$ is ignored, the sufficient and necessary conditions for the existence of a unique optimal solution τ_i^* to Equation (2) are: ①$d_i > c_i$, ②$r_i(t) = f_i(t)/R_i(t)$is strictly monotonically increasing function, ③$r_i(\infty) > d_i/\lambda_i(d_i - c_i)$. Then τ_i^* can be obtained from Equation (3)[10]:

$$r_i(\tau_i^*)R_i(t)dt - F_i(\tau_i^*) = \frac{c_i}{d_i - c_i} \tag{3}$$

And the minimum loss per unit time is $E[\tau_i] = (d_i - c_i) \cdot r_i(\tau_i^*)$, see figure 1-(a). If condition ③ is not satisfied, then $\tau_i^* = \infty$, and $E[\infty] = d_i/\lambda_i$, see figure 1-(b).



(a)



(b)

Figure 1. Schematic Diagram of $E[\tau_i]$

3.2 Determination of the Execution Interval of Maintenance Tasks

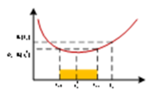
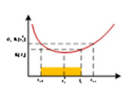
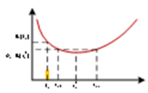
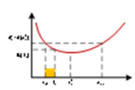
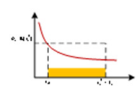
Ideally, the operator can perform maintenance tasks according to the minimum expected loss $\tau_i^*$of the cost balance. However, the credibility of the above trade-offs is affected by the accuracy of information such as the probability distribution of failures, the cost of maintenance tasks, and the cost of repair/downtime. In addition, scheduling maintenance work based on a single point of trade-off results in fragmented maintenance tasks, which is not conducive to flight scheduling by air operators. Therefore, what the operator wants is the executable range of each maintenance task, within which the maintenance cost is acceptable to the operator. The operator can then arrange a maintenance task package according to the execution interval of multiple maintenance tasks.

When the above steps are completed, the minimum expected loss per unit of running time of the maintenance task i can be obtained. Therefore, a feasible method to determine the executable interval of maintenance tasks is "according to the importance of the fault corresponding to the maintenance task i, the operator should give a risk coefficient δ_i(the value of which is greater than 1) about the expected loss $E[\tau_i]$ within the unit time based on its engineering experience and risk preference", that is, the actual acceptable value of the operator for the maintenance task floats to $\delta_i \cdot E[\tau_i]$. A smaller δ_i indicates that the operator is risk-averse to the fault corresponding to the task; otherwise, it indicates that the operator is willing to take relevant risks. The determination of risk coefficient should at least consider such factors as the impact of failure on operation reliability, maintenance task cost and other factors, and its determination method is not the focus of this paper.

By using the adjusted acceptable unit maintenance cost $\delta_i \cdot E[\tau_i^*]$ and combined with the constraint condition $\tau_i \le I_i$,the executable interval $L_i$of the maintenance task can be obtained (there are 5 cases, see Table 2).

For maintenance tasks that do not meet the first two judgment conditions, the maintenance cost trade-off method cannot be applied to determine its executable interval, and its execution interval should not exceed the specified value I_i (same as case 3). When generating maintenance task packages, it can be integrated into adjacent task packages according to relevant constraints. It is described in detail in Section 3.

Table 2. Rules for determining the conductible interval of a maintenance task

Judgement Condition	Condition 1	Condition 2	Condition 3	Condition 4	Condition 5
$d_i > c_i$	■	■	■	■	■
$r_i(t)$ is a monotonically increasing function	■	■	■	■	■
$r_i(\infty) > d_i / \lambda_i(d_i - c_i)$	■	■	■	■	
$\tau_i^* \le I_i$	■	■			
$\delta_i \cdot E[\tau_i^*] < E[I_i]$	■		■		
Interval of execution L_i	$[\tau_{i,l}, \tau_{i,u}]$	$[\tau_{i,l}, I_i]$	I_i	$[\tau_{i,l}, I_i]$	$[\tau_{i,l}, I_i]$
Sketch map					

4. Study on the Principle and Process of Maintenance Task Combination

4.1 Influence Analysis of Executable Interval on Maintenance Task Combination Principle

The traditional maintenance task combination is mainly based on the impact type of the fault mode corresponding to the maintenance task (impact safety/regulation, $T_i = 1$; Affect task/economy, $T_i = 0$), the interval $I_i$of each maintenance task, and the expected interval $K_j$of the maintenance task packagep_j. Where: For the fault mode affecting safety/regulations, the corresponding maintenance task can be executed in advance, and cannot be delayed. Generally, it will be added to the previous maintenance task package for early execution. For the fault mode that affects the task/economy, the corresponding maintenance task can be implemented or delayed. Generally, it will be added to one task package according to the relative position of its maintenance interval with that of other maintenance task packages.

$$T_i \in P_j \begin{cases} \min\limits_j \{I_i - K_j \ge 0\}, T_i = 1 \\ \min\limits_j \{|I_i - K_j|\}, T_i = 0 \end{cases} \quad (4)$$

Taking Figure 2 as an example, for the maintenance task $T_i (i \in \{1,2,\dots,7\})$, its maintenance interval is dispersed in the time axis, which is not conducive to the arrangement of maintenance plan. Therefore, several maintenance task packages P_j $(j \in \{1,2,3\})$ can be arranged to integrate the maintenance tasks. According to the traditional maintenance task combination principle, the maintenance task affecting safety/regulations $T_2/T_4/T_6$ can only be placed in the previous task package, while the maintenance task affecting the economy $T_1/T_3/T_5/T_7$ can be placed in the nearest task package between two adjacent task packages. Therefore, the maintenance task packages obtained after combination are as follows:

$$P_1 = \{T_1, T_2\}, P_2 = \{T_3, T_4, T_6\}, P_3 = \{T_5, T_7\}$$

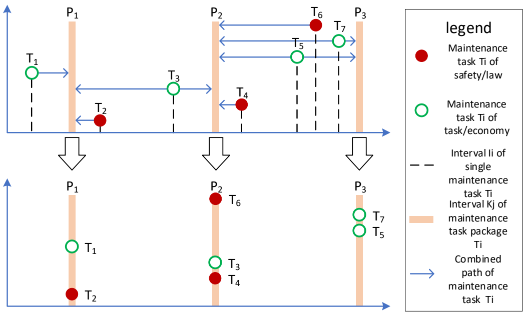


Figure 2. Schematic diagram of the traditional maintenance task packing rules

The packaging logic of this maintenance task combination is simple and clear, but the drawbacks of the packaging result are mainly reflected in the following aspects:

Excessive implementation of safety/regulatory impact maintenance tasks

Since such maintenance tasks are performed in advance in the traditional packaging strategy, the number of such maintenance tasks will be magnified from the perspective of the entire life cycle, resulting in an increase in the time the aircraft will be unavailable. In addition, in order to ensure the requirements of safety and relevant airworthiness laws and regulations in the process of aircraft design, over-margin design is usually adopted for the fault modes affecting safety and regulations. Coupled with the randomness of fault arrival, relevant preventive maintenance tasks mainly focus on inspection in the actual implementation and the possibility of really finding faults is extremely low. Therefore, frequent implementation of such maintenance tasks will significantly increase maintenance costs.

affect task/economic maintenance task force is insufficient

In the traditional packaging strategy, such maintenance tasks can be either advanced or delayed, so the maintenance cost of the life cycle will be increased for the maintenance tasks executed in advance. For the delayed maintenance tasks, the possibility of corresponding failure modes will increase and the loss due to task failure will increase.

maintenance tasks package workload uneven

Since the workload of each maintenance task is not considered in the traditional packaging strategy, the workload of the resulting maintenance task package may vary greatly, affecting the use of aircraft scheduling plan.

The above shortcomings are mainly due to the lack of analysis of the executable interval of each maintenance task from the perspective of cost in the process of maintenance task packaging, and the determination of maintenance task package is mainly based on the requirement of scheduling. However, with the emergence of the concept of situational maintenance, it is necessary to change the scheduled maintenance scheduling to situational dynamic maintenance scheduling.

Any failure mode, whether it affects safety/regulation or mission/economy, can be transformed into a cost perspective. For example, for a safety/regulatory failure, the cost of repair can be defined as a large value $(d_i = \alpha \to \infty)$; For faults that affect the task/economy, the loss after occurrence can be given an estimate based on engineering experience or historical statistics $(d_i = \beta)$. Accordingly, the maintenance cost required for the implementation of each maintenance task can also be given an estimate through engineering analysis or historical statistics ($c_i = \beta$). Therefore, the method presented in this paper can be used to determine the executable interval of each maintenance task, thus covering the consideration of the impact type of failure.

After obtaining the executable interval of each maintenance task, the maintenance task can be packaged according to the overlap of the maintenance interval. Because the executable interval $L_i$of one maintenance task may intersect with the executable interval $\overline{L_l}$of multiple maintenance tasks, the combination of maintenance tasks can be based on the correlation between maintenance tasks (e.g., same maintenance area, related maintenance steps) and the maintenance time limit of each task package.

Taking Figure 3 as an example, for the maintenance task $T_i(i \in \{1,2,3\})$, the executable intervals $L_i$of each task are obtained respectively. According to the intersection of the intervals, it can be known that the combination can be carried out either by moving T_2forward to T_1or extending back to T_3, because the economic affordability of $T_2$is consistent throughout the wholeL_2. Here we make two sets of assumptions: ①Maintenance tasks $T_2$and $T_3$are more relevant than $T_2$and T_1, ②The interval within L_1 L_2which maintenance can be scheduled is less than $M_1 + M_2$. Under these two assumptions, the same combination result of maintenance tasks can be obtained, namely $P_1 = \{T_1\}$and$P_2 = \{T_2, T_3\}$.

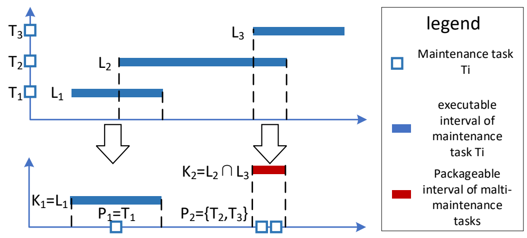


Figure 3. Schematic diagram of the maintenance task combination rules based to the conductible intervals

The biggest difference between the maintenance task combination method based on cost analysis and the traditional maintenance task combination method is: The traditional combination method is to determine the task package interval framework first, which has obvious planning characteristics, while the combination method based on cost analysis is to generate the task package interval finally, which has obvious dynamic characteristics according to the situation. In addition, the task package interval obtained by the traditional combination method is fixed, while the combination method based on cost analysis finally obtains the executable interval I $L_i$of the maintenance task package, which is convenient for users to arrange the maintenance task package according to the use plan of the aircraft and has stronger flexibility.

4.2 Study on Maintenance Task Combination Process based on Cost Analysis

The process of maintenance task combination packaging method proposed in this paper mainly includes three steps: ①Data/information acquisition②cost analysis③task combination.

(1) Data/information acquisition

In this step, you need to obtain the following data: cost data (including the cost $c_i$generated by performing maintenance task i and the repair cost $d_i$generated after the corresponding fault mode occurs), and fault rule information (including: The probability distribution density function $f_i(t)$and its mean valueλ_i of the fault mode corresponding to the maintenance task i), the fault risk preference information (risk coefficientδ_i), the maintenance task information (including: maintenance intervalI_i, maintenance workload M_i and the correlation matrix $Q_{n\times n}$ of the maintenance task), and the workload upper limit $W_j$of the maintenance task package.

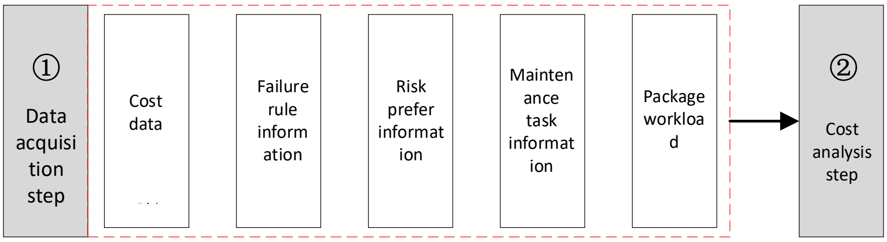


Figure 4. Requirement of data acquisition

(2) cost analysis

This step will use the data of each maintenance task and its corresponding fault to determine the executable interval L_i of the maintenance task according to the judgment conditions listed in Table 2 and formula (2-3).

Firstly, the maintenance cost data and fault rule information are used to determine whether there is an executable interval for each maintenance task. If the judgment condition is not met, the execution time of the maintenance task cannot be later than the specified maintenance interval I_i. After the completion of other maintenance task combinations, the maintenance task is tried to be placed into the adjacent task package.

Secondly, $r_i(\infty) > d_i/\lambda_i(d_i - c_i)$ will be used to determine whether there is a unique optimal solution to formula (2). If the maintenance task does not exist, then $\tau_i^* = I_i$, and then from $E[\tau_{i,l}] = \delta_i \cdot E[I_i]$ formula (2) is used to obtain the lower limit of the executable interval $\tau_{i,l}$ of the maintenance task, and thus its executable interval is $L_i = [\tau_{i,l}, I_i]$.

When there is a unique optimal solution (i.e., the most economical execution time of the maintenance task), the optimal solution τ_i^* is obtained by (3), and the expected loss cost corresponding to the optimal solution and the specified maintenance interval needs to be judged. If the judgment condition $E[\tau_i^*] < E[I_i]$ is not satisfied, the lower limit $\tau_{i,l}$ of the executable interval $L_i = [\tau_{i,l}]$ of the maintenance task is obtained by the inverse solution of formula (2).

Finally, it is necessary to determine the relationship between the most economical execution time τ_i^* of the maintenance task and the specified maintenance interval I_i. When $\tau_i^* \le I_i$, it means that both upper and lower limits exist in the executable interval of the maintenance task. Then from $E[\tau_{i,l}] = E[\tau_{i,u}] = \delta_i \cdot E[\tau_i^*]$, $\tau_{i,u}$ and $\tau_{i,u}$ can be obtained by formula (2), the executable interval $L_i = [\tau_{i,l}, \tau_{i,u}]$. Otherwise, it indicates that the optimal execution time of the maintenance task is the specified maintenance interval. After the completion of other maintenance task combinations, the maintenance task is attempted to be placed into the adjacent task package.

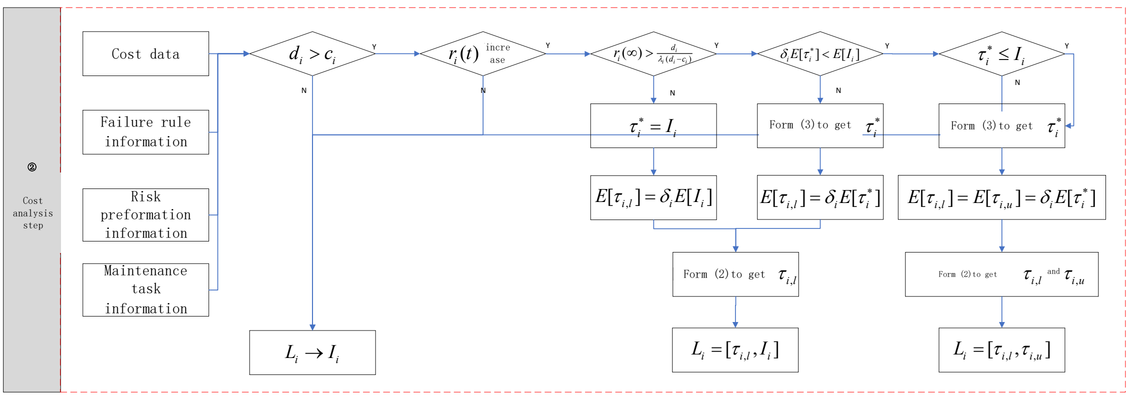


Figure 5. Process for performing cost analysis

(3) task combination

After obtaining the executable interval for all the maintenance tasks, this step determines which maintenance tasks to combine into one task package in terms of both their relevance and the workload constraints of the maintenance task package.

Firstly, the correlation of maintenance tasks ($u, v \in \{1,2,\dots,n\}$) is represented by a matrix $Q_{n\times n} = [q_{uv}]_{n\times n}$. If repair tasks T_u and T_v related, $q_{uv} = q_{vu} = 1$, otherwise $q_{uv} = q_{vu} = 0$ ($q_{uu} = 0$). Therefore, when the executable interval of one maintenance task overlapped with the executable interval of multiple maintenance tasks, the maintenance tasks related to it were combined first. The maintenance task correlation matrix should be determined before the task combination step begins by combining with engineering experience. In the process of maintenance combination, once a certain maintenance task has been combined with other maintenance tasks into a task package, the corresponding row/column elements of the included maintenance task in the correlation matrix should be adjusted to$q_{ujk} = q_{jkv} = 0$, so as to indicate that when the next maintenance task package is generated, Tasks that have been fully packaged will not be reconsidered.

Secondly, the maintenance task package workload constraints are predetermined by the operator based on the expected usage plan. In the process of composite packaging, the workload of maintenance tasks combined into the same task package is linearly accumulated $\sum M_i$and ensured to meet the constraint $\sum M_i \le W_j$. When a certain maintenance task is added to the task package and the resulting situation $\sum M_i > W_j$ occurs, it is necessary to find a combination solution that maximizes $\sum M_i$ and meets the constraints from the current task package, and remove the multi-domain maintenance tasks from the maintenance task package. This problem can be equivalent to a single constrained integer programming problem, which will not be described in detail here.

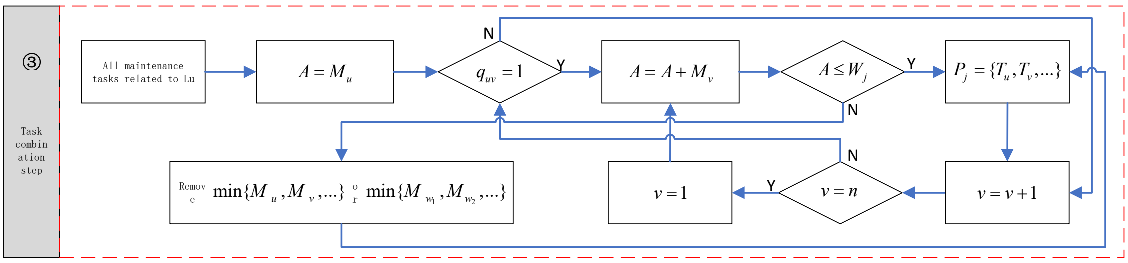


Figure 6. Process for performing maintenance task packing

The flow of maintenance task combination is shown in Figure 6. First, on the basis of obtaining the executable intervals $L_i$of each maintenance task, all intersecting executable intervals L_i should be filtered for the maintenance task T_u(assuming the minimum number is u), and the initial workload of the maintenance task package should be $A = M_u$. Secondly, the task T_v related to maintenance tasks T_u (that is $q_{uv} = 1$) should be combined and packaged first, and the workload of the maintenance task package should be accumulated. At the same time, the workload of the task package should be judged whether the workload of the task package exceeds the limit value W_j. If the constraint conditions are met, the maintenance task package is expanded to$P_j = \{T_u, T_v\}$. The same logic is used to determine whether the next repair task can be added to the task pack $P_j = \{T_u, T_v, \dots\}$. Otherwise, you need to remove the least repairable (i.e.min $\{M_u, M_v, \dots\}$) tasks from the current task package.

After all relevant maintenance tasks with $T_u$are added into the maintenance task package, and the total workload of the task package still does not exceed the limit value, we can try to successively add the task with the minimum workload $M_w$of maintenance task T_w into the task package $P_j = \{T_u, T_v, \dots, T_w, \dots\}$among the remaining maintenance tasks ($q_{uv} = 0$). If a step is added to cause the workload of the task package to exceed the limit value, then the non-relevant maintenance tasks with the minimum maintenance workload (i.e.min $\{M_u, M_v, \dots\}$) need to be removed from the current task package.

Combining and packaging maintenance tasks according to the above process can integrate as many maintenance tasks as possible into the same task package on the premise of meeting workload constraints, and make the total workload of the formed maintenance task package as close to the limit value as possible.

5. Example

In this section, a numerical example will be used to explain the implementation process of the maintenance task packaging proposed in this paper, in order to verify the feasibility of the method. To demonstrate the difference from the traditional maintenance task combination approach, this example assumes that seven maintenance tasks need to be packaged.

According to the maintenance task combination process given above, the first step needs to obtain relevant data. Table 3 lists the data of maintenance tasks and their corresponding failure modes. Where,$f_1(t)$: $W(4{,}100;t)$ it means that the fault distribution corresponding to the maintenance task T_1 follows Weibull distribution with shape parameter 4 and scale parameter 100 (other similar). In Figure 2, $T_2/T_4/T_6$ corresponds to the failure mode affecting safety/regulations. Therefore, in this example, the maintenance costs and losses of these three tasks are quite different, which reflects the feature of huge losses once the failure affecting safety/regulations occurs. In fact, in actual statistics, this assumption must be true, because for this kind of failure mode, the economic loss caused by it is much greater than the cost of preventive maintenance. In addition, it is also necessary to obtain the correlation matrix between each maintenance task as shown below. Finally, this example assumes that the total amount of work generated for each repair task package does not exceed $W_j = 8$.

$$\mathbf{Q}_{7\times 7} = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Table 3. Data/information of maintenance task T_i and its corresponding failure

T_i	c_i	d_i	$f_i(t)$	λ_i	δ_i	I_i	M_i
1	80	140	W(4,100;t)	0.0091	1.05	75	2
2	300	5000	W(10,300;t)	0.0032	1.01	115	4
3	50	200	W(2,200;t)	0.0044	1.1	170	3
4	600	8000	W(8,200;t)	0.0047	1.03	210	4
5	75	150	W(3,100;t)	0.0089	1.08	265	3
6	1200	10000	W(6,150;t)	0.0062	1.02	275	4
7	150	400	W(5,300;t)	0.0031	1.15	290	2

Based on the above data, the feasible interval of each maintenance task can be obtained by using the cost analysis process given in Figure 5, as shown in Table 4. For T_1andT_3, because the condition $\delta_i E[\tau_i^*] < E[I_i]$is not met, the upper limit of the executable interval is the specified maintenance interval $L_i^+ = I_1 = 75$and$L_i^+ = I_1 = 170$; For, because the condition $\tau_2^*$is not met, there is no executable interval, and maintenance is performed according to the specified maintenance interval $I_2 = 115$; For T_i(i=1,2,3,4), since the above two conditions are met at the same time, the upper and lower bounds of the executable interval can be obtained by the inverse solution of formula (2)

Table 4. Determining the conductible interval of a maintenance task based on cost analysis

T_i	$r_i(t\to\infty)$	$d_i/\lambda_i(d_i-c_i)$	τ_i^*	$\delta_i\cdot E[\tau_i^*]$	$E[I_i]$	L_i
1	∞	256.5	82	1.4115	1.3577	[66,75]
2	∞	332.5	183	1.8363	2.6114	115
3	∞	303.1	118	0.9767	0.8879	[73,170]
4	∞	230.1	115	6.1425	14.9089	[103,124]
5	∞	224.7	81	1.5891	1.6798	[58,121]
6	∞	183.3	82	17.7715	71.8608	[75,90]
7	∞	516.1	205	1.0586	1.1350	[149,271]

After obtaining the executable interval of each maintenance task, the process given in Figure 6 will be used to determine the task combination scheme by comprehensively considering the correlation of maintenance tasks and the workload constraints of the maintenance task package. First, for T_1, from the first row of the correlation matrix, it can be seen that the tasks related to it include $T_2/T_5/T_7$, but since L_1only intersects to L_5, and $M_1+M_5=5$, the two tasks can be packaged as $P_1=\{T_1,T_5\}$. In addition, the executable interval $L_1\cap L_5=[58,75]$of the combined maintenance task package can also overlap with the executable interval of T_3/T_6. However, since it is assumed in this example that the total amount of work of each task package is$W_j=8$, it can only combine with T_3 , and the total amount of work of the combined task package meets the constraint condition$\sum M_i\le W_j$, namely, the obtained first combined task package is $P_1=\{T_1,T_5,T_3\}$, The task package can be selected to arrange related maintenance tasks within the time range $L_1\cap L_5\cap L_3=[73,75]$.

After the completion of the first task combination, the correlation matrix needs to be updated to replace all the row/column elements corresponding to $T_1/T_3/T_5$ with 0 respectively. Then, T_2 can be packaged. As can be seen from the second row of the correlation matrix, there is only correlation between T_2 and T_2, but because the executable intervals of the two maintenance tasks do not intersect, they cannot be combined temporarily.

$$\mathbf{Q}_{7\times7}=\begin{bmatrix}0&0&0&0&0&0&0\\0&0&0&0&0&0&1\\0&0&0&0&0&0&0\\0&0&0&0&0&1&0\\0&0&0&0&0&0&0\\0&0&0&1&0&0&0\\0&1&0&0&0&0&0\end{bmatrix}$$

Then, T_4 can be packaged. According to the fourth row of the correlation matrix, it can be seen that there is only correlation between T_4 and T_6. However, since the executable interval of the two maintenance tasks does not cross, the task package combination cannot be carried out temporarily. The situation is similar for T_6 andT_7 and we will not repeat it here.

After the maintenance tasks based on correlation are packaged, the maintenance tasks that have not been packaged need to be packaged again according to the intersection of their executable intervals. For example, for T_2, the executable interval is intersected with T_4, and the total workload of maintenance task package after combination is $M_2+M_4=8$, which meets the constraint conditions. Therefore, a second maintenance task package can be formed as $P_2=\{T_2,T_4\}$, which can arrange related maintenance tasks at time $L_2\cap L_4=115$.

Finally, for T_6 and T_7, due to the large gap between the executable intervals of maintenance tasks, there is no need to carry out combination and packaging, and relevant maintenance work can be arranged in the respective executable intervals. The resulting maintenance task package is shown in Table 5.

Table 5. Result of the maintenance task packing for the numerical example

Task package	Execute time	Package time
$P_1 = \{T_1, T_5, T_3\}$	$K_1 \in [73, 75]$	8
$P_2 = \{T_2, T_4\}$	$K_2 = 115$	8
$P_3 = \{T_6\}$	$K_3 \in [75, 90]$	4
$P_4 = \{T_7\}$	$K_4 \in [149, 271]$	2

6. Conclusion

In the traditional maintenance task packaging process, qualitative analysis is carried out only from the impact type of the fault mode corresponding to the maintenance task (affecting safety/regulations, affecting tasks/economy). As a result, it is difficult to judge whether the maintenance task should be advanced/delayed, the coverage scope of the main task package cannot be determined, and the combination packaging of maintenance tasks relies too much on engineering experience. This paper presents a maintenance task combination packaging method based on cost analysis.

First of all, this method conforms to the basic idea of maintenance packaging/optimization, that is, to reduce the possibility of fault release, reduce the cost of fault repair and improve the affordability of products by adjusting the scheduling plan of maintenance tasks reasonably. The method presented in this paper firstly uses maintenance cost data, fault rule information and operators' risk preference for different faults to determine the executable interval of each maintenance task from the perspective of minimizing maintenance cost, which changes the traditional analysis of maintenance task which only considers the single point of optimization. Secondly, the process and method of maintenance task combination considering the overlapping of the executable intervals of multiple maintenance tasks are given, and the correlation between maintenance tasks (maintenance tasks within the same area) and workload constraints of maintenance task packages are considered in the combination packaging process simultaneously, avoiding the disadvantage of large workload differences among multiple maintenance task packages. Finally, a numerical example is used to verify the operability of the proposed cost analysis method and maintenance task combination process. The method proposed in this paper will be analyzed by practical data of relevant engineering applications in subsequent studies.

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