Robustness analysis of highway network based on complex network

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Abstract. Taking Chongqing, Hunan, Shandong, Shaanxi and Sichuan as examples, this paper conducts a comparative study on the robustness of their highway networks, which is helpful for the subsequent construction of China's highway networks. The topology structure of highway networks is studied by complex network theory. The degree distribution, average degree, average clustering coefficient, average path length, network diameter and other parameters of the network were calculated, and the robustness of the highway network in five provinces and cities was compared from four aspects: connectivity, network efficiency, turn rate and robustness *r*. The results show that Chongqing and Shaanxi have a good performance of highway network robustness, Shandong and Sichuan have a more balanced performance, and Hunan has a weak performance. Enhance network robustness by placing route planning directions in place with fewer route options to provide drivers with more route options.

Keywords: Complex network, Highway network, Robustness, Topology.

1. Introduction

Since the 20th century, with the continuous improvement of China's scientific and technological level, China has attached more and more importance to the development of transportation, among which, highway is one of the important ways of travel. Highway network is a typical complex network, and the robustness of the network can measure the robustness of highways. Through comparative study on the robustness of the highway network, the highway construction in China can be further improved.

Jiang Bin analyzed and compared the distribution of highways in different cities and the small-world characteristics of urban highway networks [1]. Li Jin et al. studied the topology of subway networks in many Chinese cities, and conducted a comparative study on the robustness of subway networks in Beijing [2]. Gao Tianzhi et al. studied the network topology of subway networks in 10 Chinese cities and summarized the characteristics of China's subway networks [3]. Feng Chun et al. conducted a simulation study on the robustness of urban rail transit networks in 15 Chinese cities [4]. Gan Junjie et al. carried out complex network analysis and study on the robustness of Wuhan metro network [5]. Sun Junyan et al. used L-space to model the bus network and subway network of Xi 'an city and made a comparative analysis[6] .

Based on previous studies, this paper analyzes and compares the robustness of highway networks in Chongqing, Hunan, Shandong, Shaanxi and Sichuan provinces. In this paper, L-space network is constructed, and complex network theory is used to calculate the parameters of degree distribution, average degree, average clustering coefficient, average path length, network diameter, connectivity, network efficiency, turn rate and robustness *r* of highway network in five provinces and cities for comparison and research.

2. Network Construction

A highway network diagram can be converted to an undirected empowerment graph G=(V, E), which expressed in V highway nodes in a network, the node number is N=|V|; E said highway collection the line between the two nodes in the networks, number of edges for M=|E|. For the edge (v_i,v_j) has weight w_{ij}, construct the adjacency matrix $A=(a_{ij})_{n\times n}$, among which

$$a_{ij}=\begin{cases} w_{ij},\ (v_i,v_j)\in E \\ 0,\ other \end{cases} \tag{1}$$

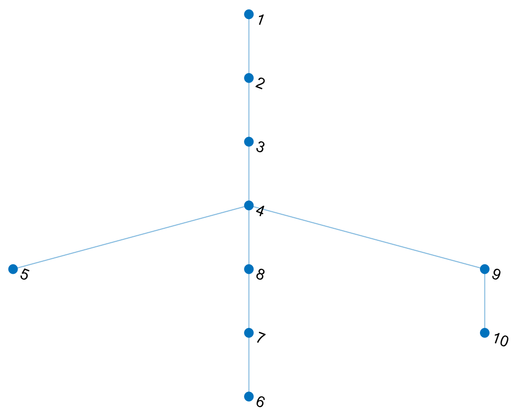
And the network graph used in this paper is undirected unweighted graph, then

$$a_{ij}=\begin{cases} 1,\ (v_i,v_j)\in E \\ 0,\ other \end{cases} \tag{2}$$

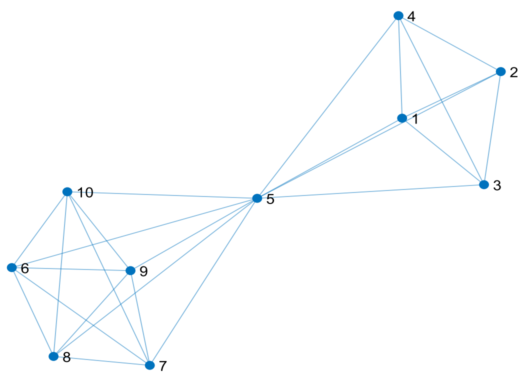
(1) L-space network model. In this model, it is assumed that node i and node j are two adjacent nodes in the network, so there is an edge between node i and node j. Then in this network model, the node degree k in the network is the number of nodes connected to this node in the network. Figure 1(a) shows the L-space network topology model, using 1,2,...,10 is the number of nodes on the two lines of the highway network. The nodes on line 1 are numbered 1-5, and the nodes on line 2 are numbered 6-10, in which node 4 is the intersection of the two lines.

(2) P-space network model. In this model, it is still assumed that node i and node j are two nodes in the network, but node i and node j need to be in the same connecting edge, then there can be a connecting edge between node i and node j, and node degree k is the number of nodes that directly reach the node without going through other nodes in the network. Figure1(b) shows the topology structure model of P-space network, using 1,2,...,10 is the number of the two lines in the highway network, the node number on line 1 is 1-5, and the node number on line 2 is 6-10, where in node 5 is the connection point of the two lines.

In this paper, L-space network topology model is adopted. Five adjacency matrices are established by taking highways in Chongqing, Hunan, Shandong, Shaanxi and Sichuan provinces as examples. The network topology model is a good simulation of the actual highway network architecture.



(a)



(b)

Figure 1. (a)L-space network space topology diagram (b)P-space network space topology diagram

3. Network Parameters

3.1. Node degree and degree distribution

Node degree k_i is the number of other nodes directly connected with node i. Node degree can be obtained by the adjacency matrix of network G constructed in the second part of this paper

$$k_i=\sum_{i=1,\,j=1}^{N} a_{ij} \tag{3}$$

The average of all node degrees in the network is the average degree of the network

$$<k>=\frac{1}{N}\sum_{i=1}^{N}k_i \tag{4}$$

In formula (4), N is the number of nodes in the network topology diagram.

The average degree of the network is related to the standardized average degree of the network as follows:

$$<k>_b = \frac{<k>}{N-1} = \frac{1}{N(N-1)}\sum_{i=1}^{N} k_i \tag{5}$$

In the whole network, arrange the degrees of nodes in the network in the order from small to large, and get the proportion $P(k)$ of nodes with degree k in the whole network nodes. Meanwhile, the proportion $P(k)$ can also be regarded as the probability that the degree of a randomly selected node in the network is k. We call the probability distribution of node degrees degree distribution, and can make the corresponding degree distribution diagram.

3.2. Average clustering coefficient

The average clustering coefficient is an important index in complex network analysis, which is used to measure the closeness of the connection between a node and its neighbors in a network diagram. The clustering coefficient C_i is with node v_i and the ratio of the actual number of connected edges between all nodes connected, the clustering coefficient

$$C_i = \frac{M_i}{(k_i(k_i-1))/2} = \frac{2M_i}{k_i(k_i-1)} \tag{6}$$

Average clustering coefficient

$$<C> = \frac{C_i}{N} = \frac{2M_i}{k_i(k_i-1)N} \tag{7}$$

In equations (6) and (7), M is the actual number of connected edges between nodes and all nodes in the network topology diagram.

3.3. Average path length

In the network topology diagram, the average path length L is the average distance between any node [7]

$$L = \frac{2}{N(N-1)}\sum_{1\le i,j\le N}^{N} w_{ij} \tag{8}$$

The average path length is the average number of nodes in the shortest connection chain between two nodes in the network, which can reflect the degree of separation between nodes in the network.

3.4. Network Diameter

In practical applications, the network is not connected everywhere. The network diameter D is usually the distance between the two farthest nodes in the network topology diagram, that is, the maximum distance between any two nodes in the network topology diagram is the network diameter

$$D = \max_{i,j} d_{ij} \tag{9}$$

3.5. Network Efficiency

Assuming that the shorter the distance between two nodes, the higher the efficiency of information transfer between the two nodes, the efficiency of information transfer between the two nodes is proportional to the reciprocal of the distance between the two nodes, then the average path length and the network efficiency are reciprocal to each other, so the network efficiency GE is defined

$$GE = \frac{2}{N(N-1)} \sum_{1 \le i, j \le N}^{N} \frac{1}{w_{ij}} \tag{10}$$

3.6. Robustness evaluation index

3.6.1 Connectivity

As one of the robustness evaluation indexes, connectivity γ is oriented towards the whole network, rather than the area of one or several nodes [8 - 9]. Its formula is

$$\gamma = \frac{|M|}{3|N|-6} \tag{11}$$

3.6.2 Turn rate

Throughout the network system, the robustness indicator reflects the ability of the network to take alternative paths. In this paper, the winding number μ in Berge's definition is used to represent the ability of the network to have alternative paths [8]

$$\mu=|M|-|N|+1 \tag{12}$$

The winding number will increase with the continuous expansion of the whole network system, but it still does not show that the more winding number the stronger the network robustness, because when the network is expanded, the number of nodes will increase, the number of edges will also change, so the ratio of winding number to the number of nodes should be used to represent the winding rate, with the winding rate μ^T to reflect the robustness of the entire network

$$\mu^T = \frac{\mu}{|N|} \tag{13}$$

3.6.3 Robustness

Robustness r is another measure of network robustness. It translates robustness into the ratio of the number of paths that can be switched to the total number of nodes in the network

$$r = \frac{\ln(M-N+2)}{N} \tag{14}$$

The robustness of the corresponding network will be enhanced with the increase of the path to the corresponding exchangeable destination, so as to compare the robustness of different networks.

4. Analysis of complex characteristics of highway network

4.1. Basic network indicators of each province

According to the adjacency matrix, this paper uses MATLAB software to calculate the highway networks of Chongqing, Hunan, Shandong, Shaanxi and Sichuan (Table 1).

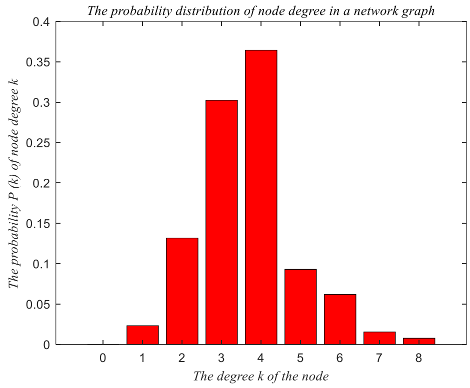
Table 1. Basic indexes of highway networks in five provinces and cities in China

Serial number	City	N	M	$\langle k \rangle$	$\langle k \rangle_b$	k_{max}	$\langle C \rangle$
1	Chongqing	129	236	3.6589	0.0286	8	0.1243
2	Hunan	125	204	3.2560	0.0263	7	0.0417
3	Shangdong	120	218	3.6333	0.0304	14	0.0708
4	Shaanxi	98	162	3.2959	0.0340	10	0.0738
5	Sichuan	119	204	3.8571	0.0327	12	0.1073

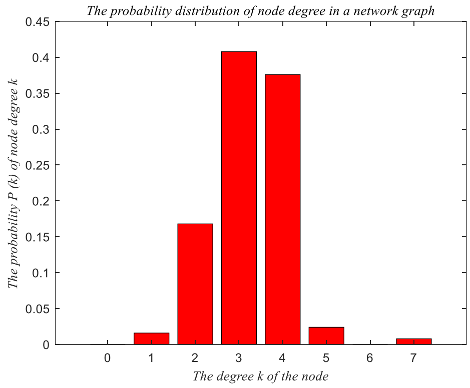
It is calculated that the average degree of highway network in Sichuan Province is 3.8571. Compared with the other four provinces and cities, the average degree of highway network in Sichuan

Province is the highest. According to the node degree distribution in Figure 2 (e), it can be seen that there are about 2-4 nodes around each node degree that can be reached directly, among which the highest probability of the node whose degree value is 4 is 36.97%. It indicates that the highway in Sichuan province relies more on small and medium-sized nodes (degree value is 4) so that drivers can choose routes by themselves. In addition, there is a second peak value in the degree distribution diagram of this node, the node whose degree value is 8, which indicates that Sichuan Province may place the highway route planning on large nodes (degree value is 8) in the future to provide more route choices for drivers and improve the highway network.

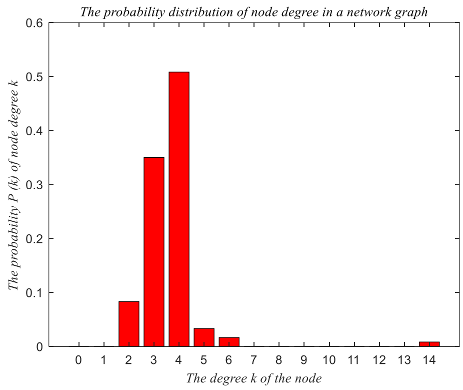
As can be seen from the node degree distribution diagram of 5 provinces (Figure 2), almost all provinces and cities on average have about 3-4 nodes around each node that can be reached directly, and more rely on small and medium-sized nodes (degree value 4) to provide drivers with more route choices, and the future highway route planning may continue to favor these nodes.



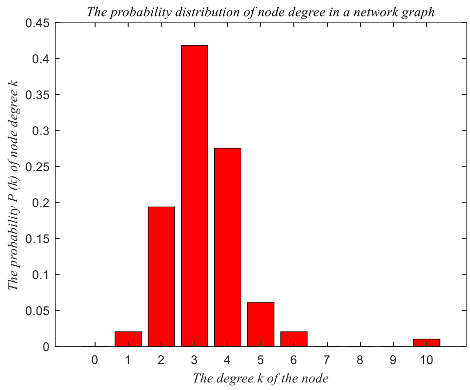
(a) Node degree distribution of Chongqing



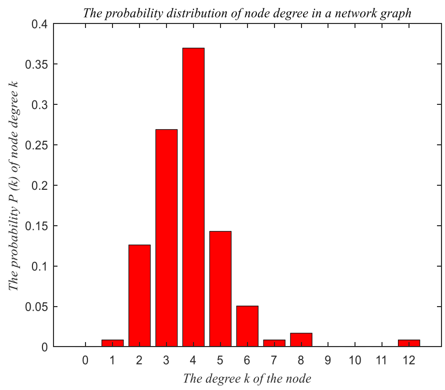
(b) Node degree distribution of Hunan



(c) Node degree distribution of Shandong



(d) Node degree distribution of Shaanxi



(e) Node degree distribution in Sichuan

Figure 2. Degree distribution of highway network in 5 provinces and cities

According to the calculation of the average clustering coefficient of highway networks in the five provinces and cities and the relevant definition of the clustering coefficient, it can be seen that the highway networks in the five provinces and cities have clustering characteristics. The large clustering coefficient of highway networks in Chongqing and Sichuan are 0.1243 and 0.1073, respectively, indicating that the highway networks in these two provinces and cities are relatively compact. The average clustering coefficient of highway network in Hunan Province is the minimum 0.0417, indicating that the tightness of the province is sparse.

4.2. Analysis of network robustness

According to each basic index value in Table 1, the characteristic index of the highway network is calculated (see Table 2).

Table 2. Characteristic indexes of highway network in 5 provinces and cities in China

Serial Number	City	L	D	$1\text{-}L_b$	E	γ	μ^T	r
1	Chongqing	7.6664	20	0.6167	0.1966	0.6194	0.8327	0.0158
2	Hunan	6.7715	16	0.5768	0.2019	0.5528	0.6400	0.0152
3	Shandong	6.7575	18	0.6246	0.2083	0.6158	0.8250	0.0167
4	Shaanxi	7.4327	23	0.6768	0.2072	0.5625	0.6633	0.0186
5	Sichuan	6.0474	17	0.6443	0.2275	0.5812	0.7227	0.0163

According to $<k>_b$ in Table 1 after sorting them, it can be concluded that Shaanxi has the least number of side connections but the strongest robustness, followed by Sichuan, Shandong and Chongqing, while Hunan has the most number of side connections but the weakest robustness. However, the degree value more reflects the connectivity of the network, so the less the number of connecting edges, the stronger the performance of the robustness, and it does not show that the network size is related to the robustness of the network.

4.2.1 Analysis of average path length and network efficiency

The average path length L of the highway network can reflect the average number of nodes that need to be passed to complete a journey from the starting point to the destination, and represent the average degree of separation between nodes. The shorter the average path length is, the more compact the network is connected, and the stronger the robustness is. The network diameter represents the network scale, which is the maximum distance. L_b can be obtained by using the average path length and network diameter, and then use $1\text{-}L_b$ measure the average path length of the network, the $1\text{-}L_b$ larger and the network larger, the more robust it is [10]. According to the data in Table 2, the $1\text{-}L_b$ of Shaanxi's network can be obtained if the maximum value is 0.6768, the highway network of Shaanxi is relatively robust, followed by Sichuan, Shandong, Chongqing and Hunan.

From the perspective of network efficiency, the average path length and network efficiency are reciprocal to each other. The shorter the average path length, the higher the efficiency, and the stronger the robustness. It can be seen from the data in Table 2 that the highway network in Sichuan Province has the shortest average path and the highest network efficiency, so the highway network in this province has the strongest robustness.

4.2.2 Connectivity analysis

The two indexes of connectivity γ and clustering coefficient $<C>$ adopted in this paper are used to analyze the robustness of highway network. According to the analysis of related indexes in Table 1 and Table 2, it is found that: The higher the clustering coefficient, the better the tightness, the higher the connectivity, the better the robustness, the connectivity of the five provinces is about 0.55-0.62 range, among which Chongqing highway network connectivity is the highest, the clustering coefficient is also the highest, so the robustness of Chongqing is strong, followed by Shandong, Sichuan and Shaanxi.On the contrary, Hunan has the lowest connectivity and the sparsest tightness, so its robustness is weak.

4.2.3 Analysis of turn rate

As the number of network nodes increases and the scale expands, the number of laps will increase, so the number of laps rate should be used to reflect the robustness of the highway network. According to the data obtained from Formula (13) and Table 2, it can be seen that the lap rate is proportional to the number of laps. Chongqing has the highest turn-around rate and the most turn-around rate, so its network has the strongest robustness, followed by Shandong, Sichuan, Shaanxi and Hunan.

4.2.4 Analysis of Robustness

According to Formula (14) and the data calculated in Table 2, it can be seen that node N is inversely proportional to robustness r, that is, the more nodes, the lower the robustness. The robustness r of Shaanxi highway network is the highest 0.0186 among the five provinces and cities, and the number of nodes is also the lowest. Therefore, Shaanxi highway network has strong robustness, followed by Shandong, Sichuan, Chongqing and Hunan.

5. Conclusion

According to the complexity analysis of the highway network from multiple perspectives in Part 4, the robustness of the highway network varies, but on the whole, Chongqing and Shaanxi have a better robustness, Shandong and Sichuan have a more balanced performance, and Hunan has a weak performance.

In the construction process of highway network in the future, route planning direction can be placed in places with fewer choices to provide drivers with more route choices, so as to enhance the robustness of the network.

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