The Progress of the Quantum Hall Effect

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Abstract. Contemporarily, various novel phenomena related to quantum Hall effect and many extended versions of Hall effects were discovered. This big family of Hall effect has given rise to the development in many fields including condensed matter physics, and relevant research (e.g., topological insulators, topological superconductors), remains a hot topic nowadays. This paper will focus on the application of quantum Hall effect in state-of-art researches, especially in superconductor-related field. Information retrieve and literature analysis are the main methods used in this review. First, a brief introduction of the definition for different kinds of Hall effects will be given. Afterwards, applications of quantum Hall effect will be discussed, including an unconventional CDW discovered in a kamoge superconductor, Josephson Junction presenting a quantum Hall effect and the quantum anomalous Hall effect-based Josephson Junction. Subsequently, the limitations in these researches and problems remaining to be solved by further research will be shown. These researches have great significance in achieving quantum computation. Overall, these results shed light on guiding further studies for Hall effect.

Keywords: quantum Hall effect, superconductivity, topological insulator.

1. Introduction

Hall effect was discovered by Hall in 1897. In 1980, Von Klitzing found that in a two-dimensional electron gas (2DEG), when the temperature is very low and the magnetic field is strong, the Hall resistance ρ_{xy} become quantized [1]. ρ_{xy}, which has a linear relationship with magnetic field B in OHE, now is step-like and has several plateaus. The Hall conductance can be mathematically described as follows:

$$\sigma_{xy} = n\frac{e^2}{h}, n\ is\ integer \tag{1}$$

Meanwhile, the longitudinal resistance ρ_{xx} becomes 0 in those plateaus. This can be explained by Landau level, if there are n Landau levels, σ_{xy} will be $n\frac{e^2}{h}$. The current is actually distributed on the edge of the material, a kind of chiral edge state (shown in Fig. 1) and there is no current inside the material between the edge.

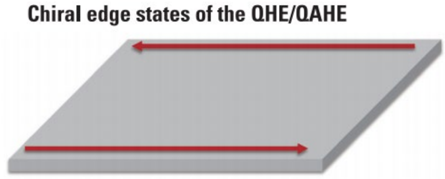

Figure 1. Chiral edge state.

In lower temperature, one can observe that these plateaus can also appear where $\sigma_{xy} = n\frac{e^2}{h}$. Here n is a fraction (e.g., 1/3, 2/5, 3/7, …,) where only 2/5 and 2/3 has filling factors that are even numbers, and others are all odd numbers [2]. Hall noticed that the Hall voltage in ferromagnetic conductors is much higher than non-magnetic conductors [3]. Because the ferromagnetic conductor can keep its spontaneous magnetization when the external magnetic field is removed, the anomalous Hall effect (AHE) can also be measured without the magnetic field. Since the QHE requires a strong magnetic field applied, which is a very tough demand to meet, and the AHE can be implemented without magnetic field, it's natural for scientists to think about how to realize quantized phenomenon by using AHE.

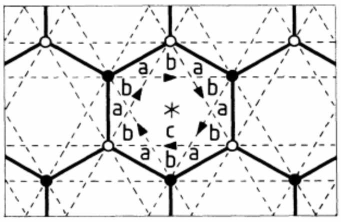

Figure 2. The honeycomb-net model [4].

Fortunately, Haldane gave a theoretical answer to the quantum anomalous Hall effect (QAHE) problem, which is a honeycomb-net graphene lattice as illustrated in Fig. 2 [4]. This model breaks the time-reversal symmetry by a periodic magnetic field applied. However, this paper didn’t offer any specific materials to realize this model. Scholars discovered the realization of QAHE through quantum spin Hall effect (QSHE) based on magnetic material which suppresses one spin channel in the QSH system, allowing one spin current to propagate. In a spin-orbit coupling system, the electrons will be biased to different directions in an external electric field if the electrons have different spin directions as schematically shown in Fig. 3. There is a spin flux in the direction perpendicular to the electric field without net charge current [5]. This Hall effect is named as spin Hall effect (SHE) can remain even when the external magnetic field is removed.

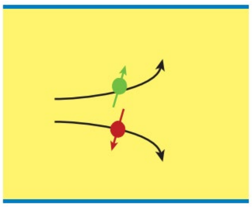

Figure 3. A sketch of spin Hall effect.

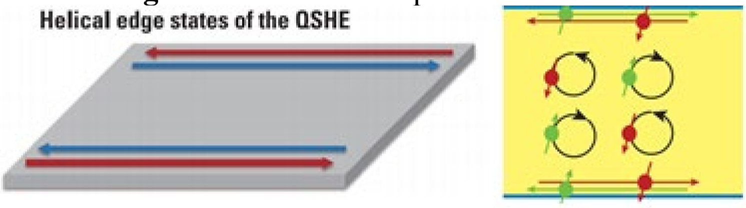

Figure 4. A sketch of quantum spin Hall effect.

QSHE is related with a pair of spin-polarized edge states caused by spin-orbit coupling counter-propagating at each edge of the material, and the inner area of the material carries no current as illustrated Fig. 4. Additionally, this effect is protected by time reversal symmetry, which does not need the external magnetic field and shows a quantized Hall conductance.

The current researches mainly focus on topological insulators (TIs) and topological superconductivity deriving from topological insulators. 2D topological possesses a topological invariant $\mathbb{Z}_2$ and shows the character of QSHE [6]. Topological superconductor is able to be constructed by doping metallic elements in to TIs, or, using the proximity effect, producing a heterostructure of an s-wave superconductor and a TI to get the topological superconductor. In topological superconductor, Majorana Fermions that are the antiparticles of themselves are predicted to exist, which may realize the qubits in quantum computation [7]. However, creating coexisting superconductivity and quantum Hall states is still challenging, bringing difficulty to the finding of exotic topological excitations [8], and our paper will discuss some resent work related to that. Some researches about superconductor with quantum Hall states and the potential to produce quantum Hall states are introduced and works regarding anomalous Josephson Junction will be discussed, which are all relevant to quantum Hall effect and are significant for the quantum computation. Afterwards, the limitations of these researches and further study to be done will be discussed.

2. Unconventional charge density wave in kagome superconductor

To achieve the QAHE in topological materials, a charge density-wave-like order was proposed but is hard to realize experimentally. However, researchers discovered and unconventional chiral charge order in a kagome material $\mathrm{KV_3Sb_5}$ through high-resolution STM. After theoretical analysis, they found that unconventional chiral charge density order in a frustrated kamome material can lead to a large QAHE and can be the precursor of unconventional superconductivity. The atomic structure of $\mathrm{KV_3Sb_5}$ is shown in Fig. 5. Through experiment, it's discovered that $\mathrm{KV_3Sb_5}$ has a superconducting transition temperature of $T_C = 0.93K$[9], which features a $\mathbb{Z}_2$ topological invariant [9].

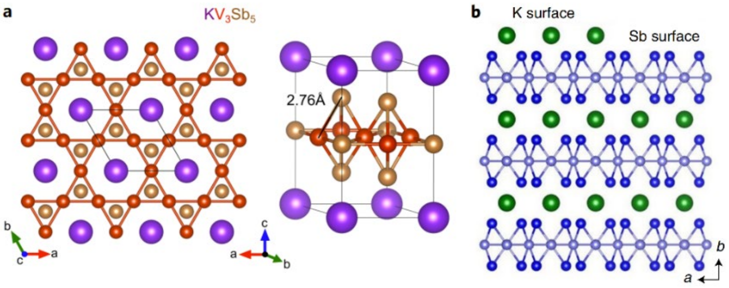

Figure 5. The structure of KV_3Sb_5 [9, 10].

Compared with other kagome lattice systems, $\mathrm{KV_3Sb_5}$ is different in that there is an additional 2×2 superlattice modulations for K surface and Sb surface. This 2×2 superlattice modulations disappear at 80k and peaks at 4.2K. They also tested on that material and recorded dI/dV data. From that data they found there is a gap-like feature on the Sb surface, which will disappear at 80K (given in Fig. 6(a)), i.e., it's natural to think there is some relations between the gap and the 2×2 modulation. Besides, in one-principle calculation, the local density of states also shows an energy gap (seen from Fig. 6(b)) around the Femi level, which fits that gap observed experimentally. That gap does not react to an external magnetic field applied in the c-axis direction as depicted in Fig. 6(c). Furthermore, the dI/dV imaging shows a type of charge intensity reversal in Fig. 6(d) and 6(e) and the modulation vector is non-dispersive demonstrating a static electronic order Fig. 6(f) and 6(g). All of these findings are consistent to a CDW (charge density wave) gap in the classical Peierls mechanism.

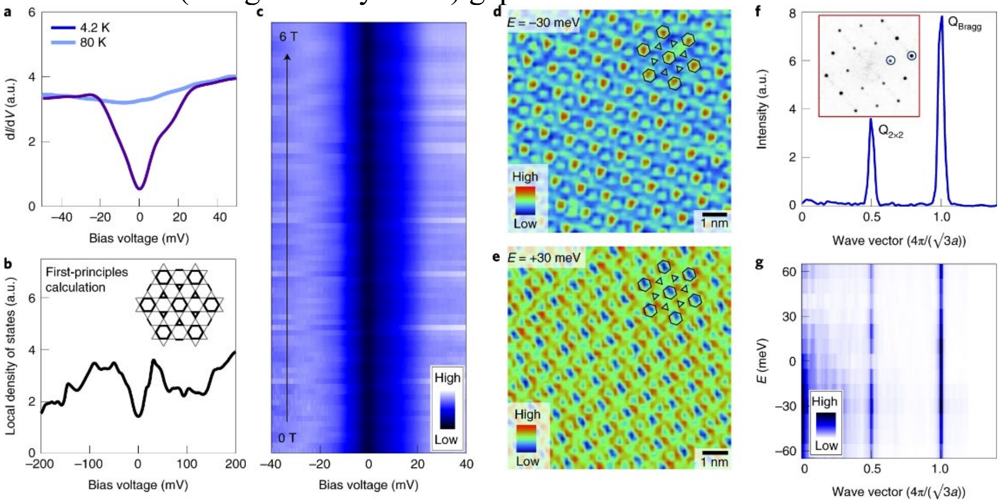

Figure 6. (a) the gap-like feature of dI/dV on Sb surface detected in experiment. (b): the first-principle calculation. (c): the response of the gap-like feature of dI/dV to the magnetic field perturbation. (d), (e): Automatically resolved imaging of the same Sb surface with different energy [9].

The charge modulation vectors has shown the intensity anisotropy, which can be the result of the chiral CDW order. The chirality is defined as the direction from the lowest to the highest vector peaks, which is either clockwise or anticlockwise. In this case, it's found that the magnetic fields with different directions applied in c-axis can change the chirality in the same atomic area.

3. Coexisting quantum Hall effect and superconductivity in Josephson junction

Topological matters with Quantum Hall state have the potential for stable quantum information storage which is a major application. Contemporarily, many researches have been focused on hybrid superconductor or semiconductors because of the recent discovery of new bound states with non-trivial braiding properties. Josephson Junction devices with a III-V two-dimensional electron gas(2DEG) interfaced with Niobium (Nb) superconducting contacts are made. These devices show the potential to study the coexistence of superconductivity (SC) and QH. Scientists grew the 2DEG heterostructure by Molecular Beam Epitaxy (MBE), and consists of an $\mathrm{In}_{0.75}\ \mathrm{a}_{0.25}\mathrm{As}$, $In_{0.75}Ga_{0.25}As$, InAs heterostructure, as shown in Fig. 7. This heterostructure has the sheet density of $n2_D = 6.24 \times 10^{-11} cm^2$, the mobility of $\mu_e = 1.6 \times 10^2 cm^2/(Vs)$, the mean free path $l_{mfp} = 2.16\mu m$ and the effective electron mass $m^* = (0.030 \pm 0.001)m_e$ [11].

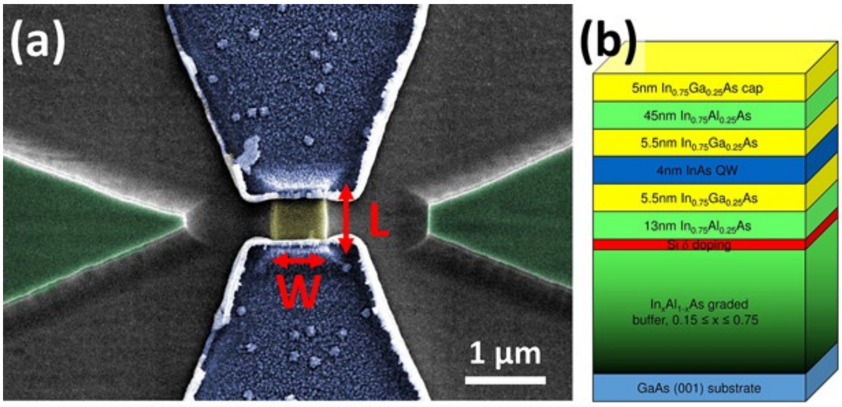

Figure 7. (a) The green parts of the side gates and the blue parts are the niobium (the superconductor). (b) The heterostructure [11].

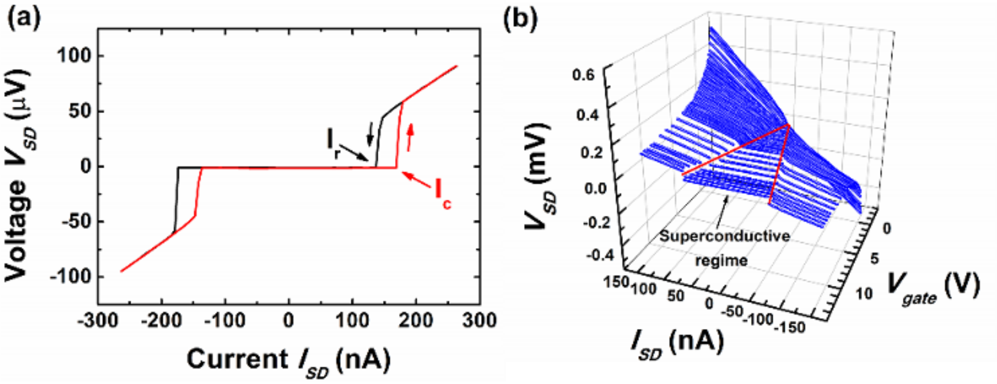

Figure. 8. (a): the I-V curve of the Josephson Junction, the arrows show the direction of the sweep. (b): the image of the I-V curves changing with gate voltages. The two red lines here represents the transition from superconductive regime and the dissipative regime [11].

As for the superconductivity property of this Josephson Junction, the critical temperature under no magnetic field is $T = (8.14 \pm 0.01)K$, and the critical magnetic field under the temperature of $T = 320mK$ is $B = (2.77 \pm 0.02)T$. The I-V character of the Josephson Junction is shown in Fig. 8(a), I_{SD} being the DC current applied and VSD being the source-drain voltage. The central plateaus in this figure demonstrates the superconductive regime, the Josephson state. Additionally, the normal resistance $R_n = (0.75 \pm 0.01)k\Omega$ and the excess current $I_{exc} = (0.57 \pm 0.01)\mu A$. The side gates control the supercurrent magnitude and the R_n, by increasing and decreasing the V_{gate} as given in Fig. 8 (b) [11].

The Josephson junction displays the quantum Hall effect. When V_{gate} is fixed at 3V, the Hall conductance shows four QH regimes and it decreases as the magnetic field applied increases. As the gate voltage increases more Landau levels are filled, increasing the electron density in the 2DEG. From Hall resistance, the density of electron can be calculated [11].

$$n_s = \frac{R_H e}{b} \tag{2}$$

The relation between density of electron and gate Voltage V_{gate} is displayed in Table 1. From those data, it's concluded that side gates can control the density of electron in 2DEG in this InAs quantum well.

Table 1. V_{gate} and n_s at $T = 350mK$.

$V_{gate}(V)$	$n_s(10^{11}cm^{-2})$
+4	19.4 ± 0.08
0	1.57 ± 0.07
-1	1.45 ± 0.07
-1.7	1.05 ± 0.06

4. QAH-based anomalous Josephson junction

Josephson junction is an indispensable part in superconducting electronics field, which is able to bring supercurrent when the superconducting electrodes have the same superconducting phase. However, it's still hard to observe the anomalous current in realistic. Since the quantum anomalous Hall insulators (QAHI) can be transformed into a superconductor, scholars have constructed a domain wall structure based on QAHI [12]. Domain wall is the region where transition of the magnetization direction happens because of the difference of magnetization directions in the neighbouring regions.

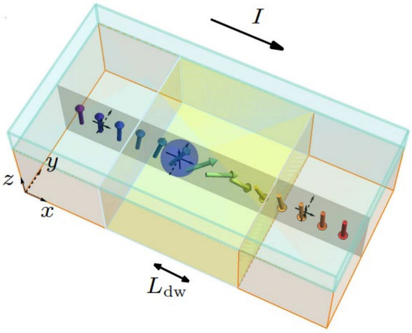

Figure 8. The bottom is the QAHI-based Josephson junction with a domain wall structure and the top is the s-wave superconductor [12].

Seen from the sketch illustrated in Fig. 8, the magnetization in the domain rotates gradually from +z to -z. Through the Hamiltonian of the QAHI with a domain wall H_{QAHI}, the QAHI-based Josephson junction with a domain wall can be described.

$$H_{QAHI} = \Sigma_p \Psi_p^\dagger H_{QAHI}(\mathbf{p}) \Psi_p \tag{3}$$

$$H_{QAHI}(p) = \mathbf{v_F} p_y \boldsymbol{\tau_z} \boldsymbol{\sigma_x} - \mathbf{v_F} p_x \boldsymbol{\tau_z} \boldsymbol{\sigma_y} + m(p)\tau_x\sigma_0 + \mathbf{M} \cdot \boldsymbol{\sigma} \tag{4}$$

Here, $\mathbf{p} = (p_x, p_y) = \left(-i\frac{\partial}{\partial x}, -i\frac{\partial}{\partial y}\right)$ and $\Psi_p = [\psi_{p,t\uparrow}, \psi_{p,t\downarrow}, \psi_{p,b\uparrow}, \psi_{p,b\downarrow}]^T$. $\mathbf{v_F}$ is the Fermi velocity. The $\psi_{p,t\uparrow/\downarrow}$ and $\psi_{p,b\uparrow/\downarrow}$ are annihilation operators for the upper and lower layers and the up and down spin directions. $\boldsymbol{\sigma}_{x,y,z}$ and $\boldsymbol{\tau}_{x,y,z}$ are Pauli Matrices in the base of spins and layers. $m(p) = m_0 - m_1(p_x^2 + p_y^2)$ describes the upper and lower layers coupling. $\mathbf{M}$ is the magnetization which can be written in a vector form: $(Msin\theta sin\varphi_{az}, Msin\theta cos\varphi_{az}, Mcos\theta)$,$\boldsymbol{\sigma}_0$ is the indentity matrix, and $\boldsymbol{\sigma}$ is $(\boldsymbol{\sigma}_x, \boldsymbol{\sigma}_y, \boldsymbol{\sigma}_z)$. From the rotation of magnetization $\mathbf{M}$, the relationship of polar angleθ and x can be figured out, which can be written as following

$$cos\theta(x) = -tanh\frac{x}{L_{dw}} \tag{5}$$

After discretizing the Hamiltonian in the lattice square, using Bogoliubov de Gennes (BdG) method and considering the topological property and the existence of superconductor, the anomalous current in the QAHI-based Josephson junction with the zero bias can be calculated. From the

Hamiltonian in different regions and the current conservation, the current formula at the interface can be derived. When there is no bias, the value of I is shown as below.

$$\begin{aligned} I &= e < \frac{d\widehat{N}_L(t)}{dt} >= -\frac{i}{e} < \widehat{N}_L)(t), H_{BdG}(t) > \\ &= -\frac{e}{\hbar}\sum_{i_y} \mathrm{Tr}\left(T_x \mathrm{i}\left\langle \Psi^\dagger_{(-1/2,i_y)}(t) \otimes \Psi_{(1/2,i_y)}(t)\right\rangle + \text{ H.c. }\right) \end{aligned} \tag{6}$$

Substituting the nonequilibrium Keldysh Green's functions and the BdG representation, the calculation can be done and the supercurrent I is:

$$I = -\frac{e}{h}\int \mathrm{d}\varepsilon \mathrm{Tr}[T_{\mathrm{LR}} G^{<}_{\mathrm{RL}}(\varepsilon) - G^{<}_{\mathrm{LR}}(\varepsilon) T_{\mathrm{RL}}] \tag{7}$$

Scholars analyzed the symmetry characters of this structure, they find that the condition for the existence of anomalous current is

$$U H_{BdG}(\varphi) U^\dagger = H_{BdG}(-\varphi) \tag{8}$$

and find that if the y-direction component M_y of the magnetization in domain wall is zero.

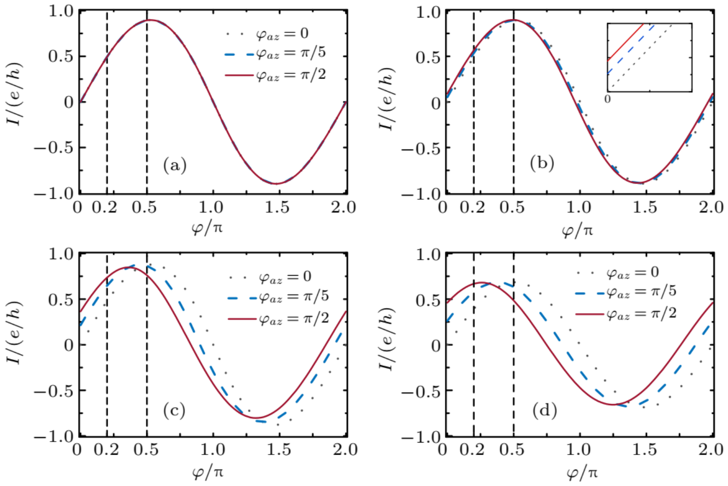

Figure 9. The current-phase relation with changing parameters of the amplitude of magnetization M and the azimuth angle φ_{az} representing the configuration of the domain wall [12].

Through experiments, more findings are available. As for phase shift, the relation between phase shift and azimuth angle or the amplitude of the supercurrent is measured through experiments. This relationship is shown in Fig. 9. It's found that phase shift has a sinusoidal relation to the azimuth angle and the max phase shift appears to be where the azimuth angle is $\frac{\pi}{2}$. However, the width of the growth of junction hardly affects the phase shift but enlarges the supercurrent. Additionally, for superconducting phases with Chern number being 0,1,2, corresponding to amplitude of magnetization M =0.05,0.2 and 0.35, the anomalous Josephson current all appears, supporting that the anomalous Josephson current is independent of the superconducting phases. Furthermore, the current gets greater as M grows larger [12].

Besides magnetization, azimuth angle also influences the supercurrent. With the increasing azimuth angle φ_{az} changes from 0 to $\frac{\pi}{2}$, the anomalous current (i.e., the current I at the point where the phase difference $\varphi = 0$) increases. When $\varphi_{az} = 0$, the current I remains to be 0. In summary, the phase shift and current amplitude of anomalous current changes periodically with azimuth angle. In a Bloch type-domain wall, the increasing magnetization or the thicker domain wall will lead to a larger maximum phase shift which makes continuous adjustments of the phase shift possible, and with a wider junction, the supercurrent will become bigger.

5. limitation

The topological superconductivity and the realization of Majorana zero mode have a great significance for quantum computation. However, despite the efforts scientists have made, an easy way of realizing them still demands further researches in this field and in technologies relevant to them. As for the unconventional CDW in kagome superconductor, it can be studied in atomic layers to find other properties for some topological phenomena, e.g., QAHE and chiral Majorana modes.

Based on the Josephson Junction with the property of integer quantum Hall effect, further researches in the fraction quantum Hall effect in proximity to SC contacts with improved material of Josephson Junction is waiting to be carried out to study exotic states like parafermions.

6. Conclusions

In summary, this paper has discussed the state-of-art approaches regarding quantum Hall effect and superconductivity. First, kagome lattice KV_3Sb_5 was found to have an unconventional CDW which leads to an anomalous Hall effect and can be a precursor of superconductivity and may also have topological quantum phenomenon including QAHE and chiral Majorana modes which requires more research. The nature of kagome lattice KV_3Sb_5 which has a unique 2×2 modulation is introduced. The experimental details fit Peierls mechanism for CDW order, but the chirality can be switched by magnetic field, indicating the breaking of time-reversal symmetry, which is unconventional. Such unconventional CDW can hold anomalous quantum Hall effect and can be a precursor of superconductivity. Second, the Josephson Junction with coexisting integer quantum Hall effect and superconductivity is constructed, which can be helpful for investigating hybrid systems that can lead to Majorana bound states. Finally, QAHI-based Josephson junction with a domain wall leads to anomalous current which can be modulated by magnetization, domain wall thickness and the width of the junction. It will have the potential for making phase battery and helping to realize superconducting quantum computation.

In the future, researches of the kagome lattices system in atomic layers, relevant study in coercive field of chirality switching effect, and the study of fractional QH effect in Josephson Junction with a III-V 2DEG can be potential points of breakthroughs. As for the QAHI-based Josephson junction, changing the configuration of the domain wall or the magnetization strength may bring about other interesting results in the future. Overall, these results offer a guideline for other researchers hoping to find solution to Majorana zero modes and topological superconductor.

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