Power Diagrams and Rules of Cyclists in Road Cycling

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Abstract. Road cycling time trials have a long history in all sports, and the individual time trial is one of the most important events in road cycling time trials. Due to the long distance, complex sections, and uncertain natural environment of road cycling individual time trials, in addition to the performance of the bicycle itself, the rider's ability and physical ability, the athlete's planning of the ride during the distance is also an important factor affecting the performance of this sport. Therefore, it is important to create a model that can facilitate cyclists' planning decisions for cy- cling. In constructing the initial model, we compared the training and competition data of cyclists with other types of athletes by referring to various literature to find commonalities and individ- ualities between road cycling time trials and other sports, thereby refining the data, and finally determined a power curve model related to power functions using knowledge of ordinary differ- ential equations and fitting.The model can infer the instantaneous power from the rider's riding time, and when combined with the external force data such as wind and gravity, the rider's kinetic energy can be derived. The model also incorporates the fact that the total energy of the rider is limited, which guarantees the feasibility and reasonableness of the model

Keywords: Power curves; time trial specialist.

1. Introduction

In a road cycling race,each cyclist should ride a fixed track alone,and the winner is the rider who finishes in the shortest time.In general, the more power a rider generates, the less time the rider has to maintain that power before having to reduce it and recov- er.Additionally, as the race progresses, riders will become more and more fatigued due to power consumption.

This paper aims to provide guidance for different types of athletes and their Directeur Sportif to help them win the game.In order to allow both different athletes and their Di- recteur Sportif to understand the athletes' physical use during a race more intuitively, we use rider's power curve to indicate the power generated by the rider in a certain period of time.On that basis,we design a model which can be applied to various types of com- petitors to explore the relationship between the rider's position on the track and the force exerted by the rider.

Our specific tasks include the followings:

- Define power curves for different types of players and different genders.
- Applying our models to different individual time trial courses.
- Confirm the effect of weather factors on the power curve
- Confirm the extent to which rider deviation from the target affects the outcome
- Extending our model to team time trials
- Use our research to write a race's race guidance

2. Assumptions and Justifications

- We assump that left and right wind direction has no effect on the rider.the wind di- rection that mainly affects the rider's performance is downwind and upwind.(Wind direction left and right is relative to the rider during the race.)
- We assume that extreme weather will not have a significant impact on the competi- tors, as the race organizers will mostly change the course temporarily or interrupt the race in the event of extreme weather.

● We assume that the rider's power decreases evenly before the corner and increases evenly after the corner .The cornering time is short and the rider can control their power and speed subjectively, thus all the effects brought by wind and friction can be neglected.

● We assume that the male rider is 1.94m tall and weighs 79kg; the female rider is 168.7cm tall and weighs 57.9kg.

3. Model Preparation

3.1. Notations

The symbols are visualized as shown in Table 1.

Table 1. Symbols

Symbol	Discription	Unit
t	Time	-
PWR	Power to Weight Ratio	W/kg
a	Parameters in terms of strength, speed and explosive power	-
b	Parameters in terms of endurance level	-
α	Hill slope gradient	O

3.2. Glossary

Criterium a bicycle race that takes place on a closed course. The length can be spec- ified by a fixed number of laps or the most laps in a predetermined time period. **Directeur Sportif** a team's director who is responsible for managing the riders and staff, making race decisions, and deciding the team composition for a given race.

Individual Time Trial an event in which riders traverse a predetermined course one at a time. The riders are not allowed to work together or ride near one another. The time required to traverse the course is recorded for each rider. The lower the time the better the rider's final placement.

Power Curve a visual representation of the maximum power a rider can maintain for a particular length of time.

Climber a rider that specializes in races that have multiple long climbs.

Puncheur a rider that specializes in races that include many short, steep climbs or many sharp accelerations.

Rouleur

terrains.

Sprinter

a rider that is a generalist and can do well in races with a wide variety of

a rider that specializes in producing extremely high power for short periods

of time. These riders generally focus on winning at the end of a race or during the inter- mediate sprints (if a race has intermediate sprints).

Time Trial Specialist a rider that specializes in the individual time trial events. **Time Trialist** a road bicycle racer who can maintain high speeds for long periods of time, to maximize performance during individual or team time trials.

PWR Power to Weight Ratio,which used to identify the more realistic riding ability of different cyclists.It is calculated as: power output / own weight, with higher values

representing greater ability. For riders whose goal is to climb, such as participating in hill climb races or wanting to improve in the sprint, the primary consideration is PWR.

4. Model and Solutions

4.1. Model Building

To define the power profile of the riders, first we need to build a model. 1. The model we build is composed of three parts.

● Now is the first part.Here we use power data from a rider of unknown gender and unknown type riding for about an hour(The data is visualized as shown in Figure1)[4].

From Figure 1,we decided to use an exponential fit to fit these data.By fitting, we derived the ideal power curve as in Figure 2.

At the same time,we derived the data fitting equation.

$P = a \times t^b$

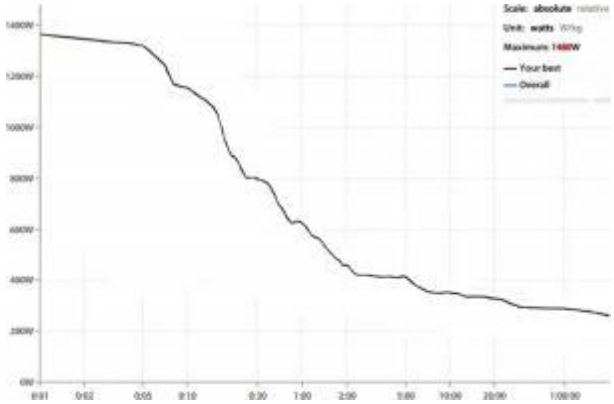

Figure 1. Unknown riders.

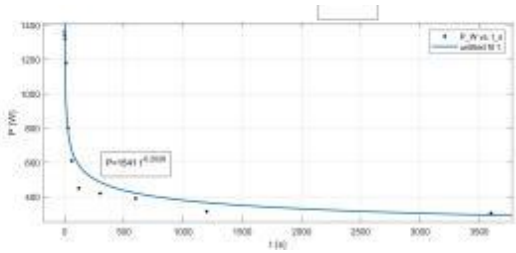

Figure 2. Power curve of unknown riders.

However, it is difficult to conclusively confirm the data fitting equation we need with just one person's data. So again, we fit the data by processing data from five different types of riders: time trial specialist, climber, sprinter, rouleur, and puncheur. First ,we take the time trial specialist as an example.Assuming that the power trend of a time trial specialist is approximately the same as that of a Time Trialist within 20 mins of cycling.Here we present a table of their power and power ratio data as in Table 2.

Table 2. Power and power ratio data

Times	5s	1min	5min	20min
PWR(W/kg)	18.37	9.37	6.05	5.33
P(W)	1451.23	736.28	477.95	421.07

Fitting these data and the data of the other three types, we produce the corre- sponding ideal power curve shown in Figure 3[5].

From this we can conclude that although the data vary for the four different types of riders, they can all be fitted with the equation$P = a \times t^b$, which leads to a power curve.

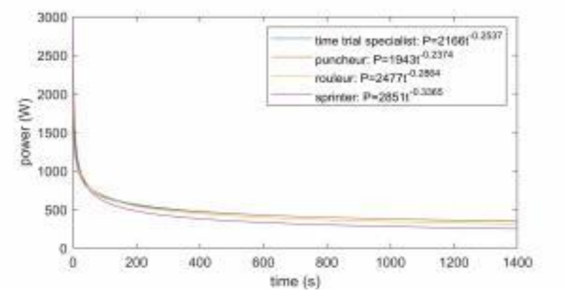

Figure 3. Four types of Power Curves.

After the first part, the second part follows. The title mentions that there is ● a limit to the amount of energy a rider can consume during a race. Therefore,we assume that the maximum amount of energy a rider can expend during a race is

We can easily conclude that

$$\int Pdt \leq W$$

● Then comes the third part.Since in the race, in addition to the rider's own power, there are downwind propulsion, upwind resistance, friction with the ground and other influencing factors(we call these forces collectively F) can affect the power of the rider,then we can obtain an equation related to the energy conversion of the rider:

$$\frac{1}{2}mv^2 = \int pdt + \int fvdt$$

In summary, the model we developed is as follows

$$\begin{cases} P = a \times t^b \\ \int Pdt \leq W \\ \frac{1}{2}mv^2 = \int Pdt + \int fvdt \end{cases}$$

PS.Something about road curves.[3]

Based on the inertia of a turn, it is known that a person's body is tilted inward when riding a bicycle, assuming a tilt of θ °· Because athletes ride faster in road cycling time trials on non-curve sections, they need to decelerate when entering a curve from a straightaway and accelerate when entering a straightaway from a curve.

From the force analysis method in the literature, the optimal speed when passing through the curve is derived to avoid deviation from the race course due to too fast or too slow speed.

As the athlete into the curve and into the straight road when the speed corresponds to the power increase or decrease is subject to the subjective will, so it may be bold to assume that the power into the curve and into the straight road are to u_1 W / s to do uniformly decreasing and uniformly increasing power movement. Assume that its speed before entering the curve is v_0 and power is P_0,where v_0 can be derived from $\frac{1}{2}mv^2 = \int pdt + \int fvdt$ in the model.Since the change in speed due to the work done in the bend comes mainly from the rider's own work state, and the time after the bend is short, it can be considered that most of the work is provided by the rider himself, so the influence of friction, wind and other factors on the work done by the rider can be ignored, so at this time it can be considered $\int Pdt = \Delta \frac{1}{2}mv^2$ .We treat the cornering process as if it were a circular motion, and we can see that

$$\Delta\frac{1}{2}mv^2=\frac{1}{2}m(gR\tan\theta-v_0^2)=P\Delta t-\frac{1}{2}u_1\Delta t^2$$

So we can find out the required time Δt and then the required distance to complete the simulation of the cornering situation. As the power changes symmetrically when acceler- ating and decelerating, the acceleration after the corner can be symmetrically analogous. In the simulation of the total distance, the time for each corner is too short compared to the total time, so it can be ignored.

4.2. Definition of The Power Profile

When building the model, we based our discussion on the power data for male riders. So let's look at the data for female riders next.[1]

Our data is visualized as shown in Figure 4

According to the data provided in the original URL of Figure 4, it can be seen that the power table of this female rider while riding to the hills is as follows in Table 3.

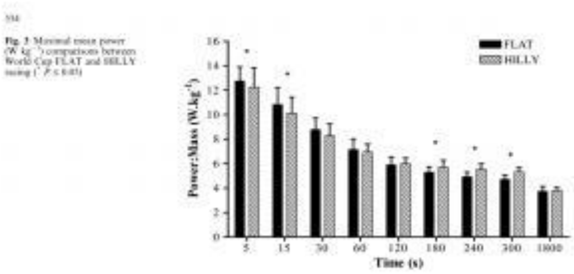

Figure 4. Data of female riders.

Table 3. The power table of a female rider while riding to the hills

Times(s)	5	15	30	60	120	180	240	300	1800
P(W)	706.38	579	480.57	405.3	341.61	301.08	283.71	272.13	237.39

Similarly, the power table for a female rider on the flat is as follows in Table 4.

Now, we got a comparison chart as shown in Figure 5

Table 4. The power table for a female rider on the flat

Times(s)	5	15	30	60	120	180	240	300	1800
P(W)	741.12	636.9	515.31	416.88	341.61	306.87	283.71	272.13	225.81

Here we can infer that the wind factor has little influence on the rider in the hills. If

the wind is blowing forward from behind the rider, i.e. a downwind situation, which can be compared to a downwind situation on flat ground because the hillside is less obstruc- tive to the downwind at this point and while in the case of headwind, the hillside has a more pronounced hindering effect on the headwind, so the power is reduced more slowly and the parameter b is relatively small under hilly conditions.

The influence of gravity and friction is greater on hills than on flat land, so these factors suppress the power output, resulting in a smaller maximum power output on hills than on flat land, with a relatively large parameter a for the flat condition.

At this point, we can perform a male and female time trial specialist comparison.

In the previous article, we saw that the male time trial specialist power curve is in Figure 6

Combining the power data of the female rider in the flat and hilly conditions in the previous article, and the resulting image of female rider from fitting the data is in Figure 7 Compare the two curves in Figure 8

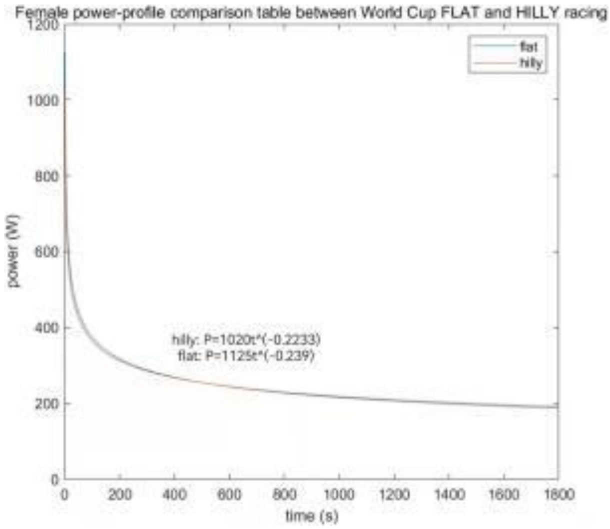

Figure 5. Hilly vs flat.

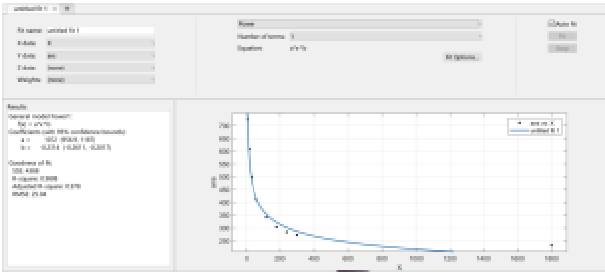

Figure 6. Hilly and flat combination.

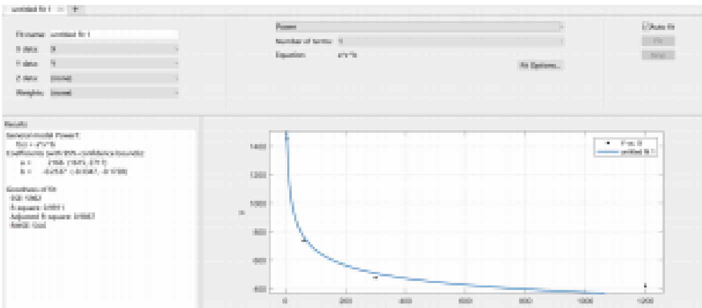

Figure 7. Time trial specialist.

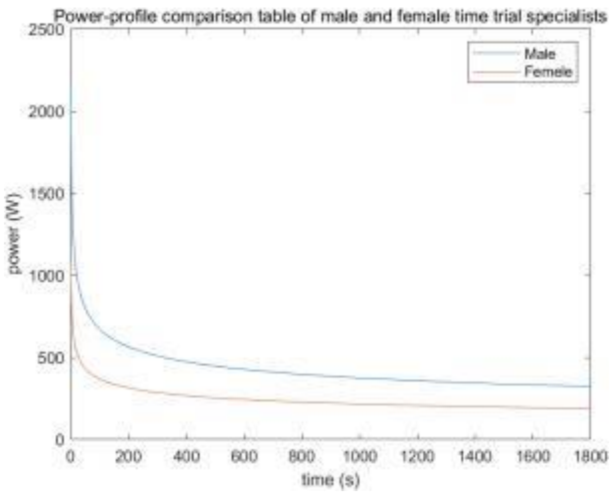

Figure 8. Male vs female.

From the graph we can find that parameter a is higher for male riders than for fe- male riders. This data indicates that male time trial specialists are stronger than females in terms of strength, speed and explosive power. However, parameter b is lower for males than females, indicating that male time trial specialists are somewhat lower in endurance

level than females. This is in line with common scientific knowledge of male and female physiology.

Having discussed the power profiles of different gender riders, let's next discuss the power profiles of different types of riders.We selected time trial specialist and sprinter for comparison.

According to the previous article, we can know the approximate power curve of time trial specialist and sprinter. The two are put together to produce the following compari- son chart as figure9[5]

You can see that parameter a of Time trial specialist is smaller than that of sprinter, but parameter b is larger than that of sprinter.Sprinter athletes are characterized by the ability to fight wind resistance on flat roads with more speed, a stronger physique, and more muscle. Generally, road bike races start with a flat stretch of road, which is prob- ably why the parameters of the sprinter are greater. The more power the rider has at the start, the greater the burst of speed.

Sprinter's strong muscles, while allowing them to be faster on the sprint course, can be- come a burden on hilly terrain. Although the discussion of flat and hilly conditions in the Women's World Cup found that hilly terrain reduces the parameter b, which means less wind and slower power, the effects of friction and gravity are related to the athlete's own weight, and the difference between downwind and upwind thrusts on any athlete is small.It is very common to see hilly terrain on road bike racing courses, which is why the sprinter decreases power faster and with smaller parameter b by the end of the race.

5. Sensitivity Analysis

5.1. Rider's deviations from The Target Power Distribution

The total course is divided into multiple sections, each section has a certain slope, and different sections of the course are considered. Considering the athlete's outburst,

the athlete's outburst curve can be inserted into the athlete's basic distance-power curve Since the work done by the athlete is similar for each outburst,he outburst curve can bedesigned as a cycle-like situation considering the inuence of the athletes previous workdone on the outburst. Since the previous ride and the burst situation will affect the athletes subsequntbusts, the starting position and the highest position of the burst curve

are influenced by the work done by the athlete (translated into distance). The maximum work A that the athlete can do on this segment of the curve is determined by inserting the burst curve into the athlete's basic distance-power curve so that he or she performs one or more bursts over the course of the segment. The athlete can not burst in the first half of the time, but in the second half of the time every T0 seconds can be burst, because the time difference between the two bursts is long, the former burst on the latter burst of influence is small, so the two bursts before and after the necessary connection, and to ensure that the total work done after multiple bursts can still remain below A. This determines the best outbreak solution. Delay the time of one of the outbursts, let the delay time be At , after which the change of the outburst time will lead to a change of the overall duration,

resulting in a change of the total time, i.e., AT . For the case of work done beyond the original time, the average power can be used instead because the power and the amount of power change is small when the athlete is close to completing the stage. In this way, more time needed is calculated. After that, the sensitivity analysis of this model can be completed with $\frac{\Delta t}{\Delta T}$. The symbols are visualized as shown in Table 5.

Table 5. Symbols in this Section

Symbol	Discription	Unit
$T0$	Cycle time for the emergence of the burst curve	s
At	Outbreak curve delay time	s
t 1	Original outbreak time	s
t 2	Postponement of the post-burst	s
AT	Amount of change in total time due to delay in outbreak curve	s
$T1$	Total time of the original plan	s
$T2$	Total extended time due to delayed outbreak curve	s

Since we are only discussing the sensitivity issues related to the time anomaly at the point of eruption, then for the convenience of the discussion here, ignoring the effect of external forces on the power curve and discussing only the work done by the athlete, then the equation related to kinetic energy becomes

$$\frac{1}{2}mv^2 = \int Pdt$$

In the original model $P = a \times t^b$, because at this time b is negative, so can not be integrated from 0, then the starting point of integration is changed to 1. Suppose that after adding one or more outbreak curve to the original model $P = a \times t^b$, the total work obtained from the integration from 1 to T_1 seconds is A, that is, the ideal model of riding to T_1 seconds is the above, assuming that at the moment $t1$ originally had an outbreak, for some reason, the outbreak time without affecting the other moments of the outbreak, postponed to t_2 , noted $\Delta t = t_2 - t_1$. Suppose that after postponing the outbreak curve from 1 to T_1, the new total work obtained by integration A'¡A, then as described above, after T_1 with a uniform power straight line extension to T_2 seconds, so that the total work obtained by integrating the changed curve from 1 to T_2 is equal to A, to the original ideal model of the vision. Denote $\Delta T = T_2 - T_1$. then this is the requested sensitivity.

Now, we model the power curve $P = 2166 \times t^{-0.2537}$ for T_0 =1000s, T_1 =4000s, male time trial specialist as an example and discuss one of the sensitivities. To satisfy the requirement of the question that additional power recovery needs to be reduced after the burst, it may be assumed that the burst curve satisfies the trigonometric form sin t. Assuming that the athlete was originally scheduled to burst at the moments of travel to 2000s and 3000s, the power curve is shown in Figure 9.

For $P = 2166 \times t^{-0.2537}$,when t=4000s,P=264. 1288 , so it needs to increase $\frac{3026.816}{264.1288} \approx$ 11.46 to reach the original total work, then the sensitivity is about $\frac{500}{11.46} \approx 43.63$

From the graph15,it can be shown that the sensitivity is around the range of 43.63. A comparative graph of the curves after the outbreak anomaly is shown in Figure 10.

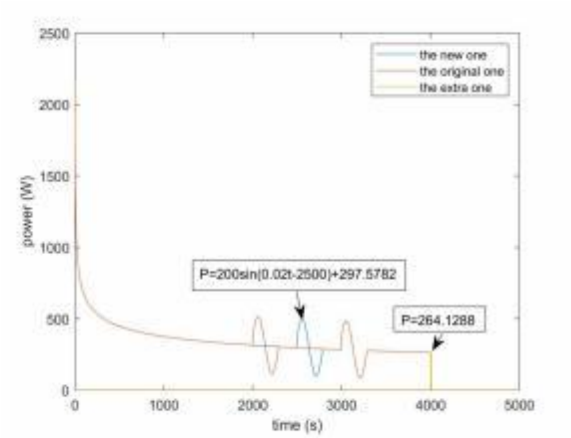

Figure 9. Original plan power curve.

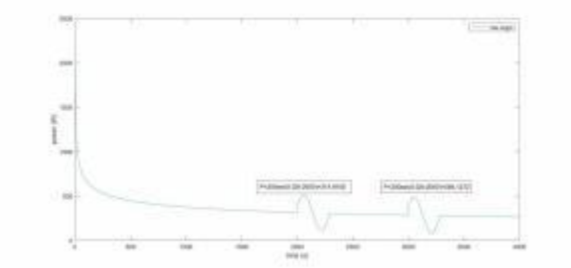

Figure 10. Curve after outbreak anomaly

6. Conclusions

Through our exploration, we finally came up with the following model:

$$\begin{cases} \mathrm{P} = a \times t^b \\ \int Pdt \le \mathrm{A} \\ \frac{1}{2}mv^2 = \int Pdt + \int fvdt \end{cases}$$

In a road cycling time trial, the power curve of the athlete satisfies the power function form and there is a limit to the total energy that can be utilized by the athlete. After combining the work done by the external forces, the kinetic energy of the athlete during the race can be found.

7. Model Evaluations and Further Discussion

7.1. Strengths

Our model is based on real data and takes into account the effects of wind speed and friction on the condition of the athletes and the competition to a large extent, resulting in a model that is more friendly to directors and riders without a mathe- matical background.

7.2. Weaknesses

For the modeling of curves, we lack more practical data and lack consideration of wind and friction on curves

In putting the model in three real terrain simulation only considered the changes in terrain undulations, not much exploration of curves and outbursts.

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