

Theoretical Progress and the state-of-art models for Plasma Quantization

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Abstract. Plasma, known as the fourth state of matter, attracts the curiosity of scholars. Specifically, this paper will discuss the quantization models and theoretical progress for plasma, starting with the brief conclusion of fruition of former researchers. In addition, the novel perspective for the process will be proposed in order to expansion of investigating plasma relevant effects. According to the analysis of the quantization process of plasma, (i.e., the Villasov equation describing the basic motion), the basic theory of quantum plasma is introduced. Moreover, the state-of-art models and results on the quantization of plasma field and interstellar plasma will also be introduced. These results shed light on guiding further exploration on nonlinear plasma physics.

Keywords: quantum plasma, Villasov equation, alactic plasma.

1. Introduction

For hundreds of years, human beings had thought that all of the matters in this universe must belong to one of the three fundamental states of solid, liquid and gas. However, during the development of modern technology scholars noticed that we all ignored an being that takes the supreme status in our daily life, which is the sun. The state that the sun belongs to had perplexed a lot of people. With further studies, one knew that it is constructed by hydrogen and helium atoms but only known this cannot satisfy scholars' curiosity until the plasma, the fourth states of matter, was found.

Plasma firstly discovered in 1879 by Sir William Crookes by his self-invented vessel which later been called "Crookes Tube". The Crookes Tube was an apparatus which used to demonstrate the negative beam was built by electrons. In the Crookes Tube, there is a positive electrode and vacuum [1]. Crookes did this experiment by rising the temperature of gases too high that even overcame the electrostatic movement. Being noticed as the neutral outcome of positively charged ions and negative electrons, plasma was stated to have the quasi-neutrality that implies the electron density in the plasma is nearly equals to the ion density and the common density of the substance [2]. The major difference between plasma and gas is that when the plasma in an electromagnetic field often be affected by the field while such phenomenon won't happen on gaseous states substances [3].

The following contents is the integration of the previous knowledge and some expansion during on our studying process. The rest part of the paper is organized as follows. The second section illustrates the basic descriptions of plasma the fundamental definitions and the quantum effects in plasma. The third section includes the theory of the quantum plasma and the results of simulation and analytical progress. The fourth section shows the limitations of current studies with prospects of the future. The fifth section will be the conclusion and summarization.

2. Basic Description

2.1 Maxwell equations

Maxwell equations developed by James C. Maxwell describe the relationship between the electric field and the magnetic field. While Maxwell was coming-up with the equations, he predicted the

existence of electromagnetic wave. Maxwell assembled the equations by summarizing previous scholars' equations with his own understanding. Maxwell First equation is Gauss's Law measures the proportionality between the electric flux, which is the number of electric field line across a close surface and its charge distribution as

$$\nabla \cdot E = \frac{\rho}{\epsilon} \quad (1)$$

This equation states the electric field E moves out ward or inward form the electric charge, where the ϵ standards for permittivity. To have a more direct model, one can suppose that there is a three-dimensional plan surface, and the electric field lines across the line for one time will be the available flux. Noticing that the lines that entering the surface at one point and exist should not be counted in number of fluxes. In a sphere with certain surface area the electric flux across, it can also be measured by the electric field line starting from the positive charge from the middle and will extend outward through the surface in all direction. Maxwell Second Equation, the Magnetism Monopoles are usually seen as the Gauss's Law for magnetic flux rather than electric flux. The magnetic field line exists from the North pole of the magnet will back to the South Pole, which causes the overall magnetic flux to be 0, stated as:

$$\nabla \cdot B = 0 \quad (2)$$

where B states the magnetic field at this place. The rudimentary theory of the Magnetism Monopoles is quite similar with what we talked above at the Gauss's Law part. To build a model, we will firstly examine a magnet. As it known to all, magnets are always existing at a dipole state and the magnetic field line will always follow a loop that starts from the North pole, crossing through the bar to the South pole and round back to the North. Due to this characteristic, if we consider there is a gaussian surface in the middle of the magnet, we will notice that the number of magnetic field lines enter the surface will equals to the magnetic field lines. Therefore, the net magnetic flux will equal to 0. Maxwell's third law is the Faraday's Law, which states that the proportionality of the electromagnetic field line to the change of magnetic flux through the circuit. It can be shown as

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (3)$$

Different from the original Faraday's Law, it is said that changing magnetic field will lead to an electromotive force. Maxwell gave us a more precise and general version. From this equation, one can clearly notice two important factors that will heavily affect the magnitude of the electromagnetic induction. Firstly, the more rapidly changed the magnetic field, the greater the induced emf. This means the smaller the ∂t is, the larger the emf can be. Secondly, the larger the area of the wire, the larger the emf can be. Finally, the Ampere Law illustrate a relationship between the magnetic field and currents.

$$\nabla \times H = J + \frac{\partial D}{\partial t} \quad (4)$$

Based on Ampere Law, one can calculate the magnetic field that is produced by electric current moving in a wire in any given shape. The fundamental ideal of the Ampere Law is to cut the total length of the wire into infinite dL and put the calculated H into Stroke's Theorem to obtain the current density J. Based on the former research, Maxwell took the change of Electric Flux Density into consideration. By adding the Displacement Current Density, one finally derives the whole equation.

2.2 Debye Shielding

Debye Shielding is the basic measure in plasma physics to evaluate the extent of which charges of the plasma shield out the effect of electric field. The length of Debye Shielding effect is called the Debye length, expressed by the following equation.

$$\lambda_D = \left(\frac{\epsilon_0 k T_e}{n e^2} \right)^{\frac{1}{2}} \quad (5)$$

The equation was firstly founded by Debye and showed the proportion relationship between the square root of Temperature and Debye Length [4]. In order to exhibit the collective behavior of plasma, the Debye Length of it must much less than its identical length so that outer potential won't

influence most of the molecules in the plasma [1]. In another angle, Debye Length can also measure the potential length that the electric potentials can fluctuate in a plasma substance [4].

2.3 Plasma Oscillation

Plasma Oscillation or the Langmuir Wave is used to measure the movement of ions and electrons in plasma. As the plasma is often considered as neutral, the net force of the plasma is also thought to be 0. The theory of Langmuir Wave states that as the negative electron in the plasma moves the positive charge will form an electrostatic attraction on it to main the electron at its static position. Owing to the strong interaction force between electrons, most of them are vibrating at one particular frequency that depends on the characteristics of the plasma [5]. Plasma has the factor of oscillation frequency directly proportional to the square root of its density [6]

$$\omega_{pe} = \left(\frac{N_e e^2}{\pi m_e} \right)^{\frac{1}{2}} \quad (6)$$

2.4 Quantum plasma

The theory of quantum plasma can be traced back to Moyer and Wigner first attempt and others to find a suitable form to describe multibody quantum systems, and from this basis, a rich field emerged [7]. Plasma physics involves many phenomena in statistical mechanics related to kinetic processes. Therefore, the structure and properties of the basic kinetic equations that control the dynamic behavior of plasma must first be studied.

In quantum mechanics, the state of a system is defined as the integral of the square of the module of a vector in the state space S Hilbert space, and the operator corresponding to the state can be measured. For the mixed state, we built the density operator:

$$\hat{\rho} = \sum_n c_n |\psi_n\rangle\langle\psi_n| \quad (7)$$

where c_n is the probability that the system is in state, the density operator can be seen as the average of many systems, ψ represents a multiparty quantum wave function, resulting in a multiparty density matrix. The expected value of any observable value is given by the following equation:

$$\langle \hat{A} \rangle = \int da \langle a | \hat{\rho} \hat{A} | a \rangle \quad (8)$$

where A is the operator that corresponds to making the measurement of quantity A and $|a\rangle$ is a complete spectrum of eigenstates. The temporal evolution of the density operator is given by von Neumann's equation:

$$i\hbar \frac{\partial}{\partial t} \hat{\rho} = [\hat{H}, \hat{\rho}] \quad (9)$$

which is derived from Schrödinger's equations, \hat{H} is the Hamiltonian operator. These are the basis for plasma quantization. In the plasma problem, a quantum system with both positional and spin-free degrees of freedom is described by the Pauli Hamiltonian quantity:

$$\hat{H} = \left[\frac{(\mathbf{P} - q\mathbf{A})^2}{2m} + q\phi \right] \hat{1} - \mu\boldsymbol{\sigma} \cdot \mathbf{B} \quad (10)$$

where A and ϕ are vector potentials and scalar potentials of the fields, μ is the magnetic moment of the electron, B is the magnetic induction intensity, the term in parentheses is only the Schrödinger Hamiltonian quantity, and the second term corresponds to the dipole energy produced by the spin. Combining the Wigner-Weyl transform and the spin transform, we get the description in extended phase space, which includes two spin coordinates in addition to the usual seven spatial, momentum, and temporal coordinates. By bringing this Hamiltonian quantity into von Neumann's equation and performing the above transformations, the evolution of the system in phase space can be obtained [8]. Under the limit of the scale length of the Longer than de Broglie waves, it can be simplified to:

$$\frac{\partial}{\partial t} f + V \cdot \nabla_x f + \frac{q}{m} (E + v \cdot B) \cdot \nabla_v f + \frac{\mu}{m} \nabla_x [B \cdot (s + \nabla_s)] \cdot \nabla_v f + \frac{2\mu}{\hbar} (s \times B) \cdot \nabla_s f = 0 \quad (11)$$

The two terms proportional to μ here come from the spin effect, the first of which is the correction of the spin gradient due to the quantum properties of the spin due to the even force of the spin in the magnetic field. The second represents the spin precession effect. Thus, Maxwell's equations now can be described in following form:

$$\begin{cases} n = \int d\Omega f(x, v, s, t) \\ J = q \int d\Omega v f(x, v, s, t) + 3\mu \nabla \times \int d\Omega s f(x, v, s, t) \end{cases} \quad (12)$$

among them are the integrating elements on the spin and velocity space. The second term on the right side of the second equation is the magnetization current generated by the spin.

3. Theory of quantum plasma

3.1 Field quantization

First, introducing the process of quantization of electromagnetic fields in plasma, we assume an infinite, uniform, isotropic plasma. According to the analytical model of Endonca et al. [9], who proposed and made an energy operator:

$$W = \sum_{\lambda=1}^3 \int w(k, \lambda) \frac{d\vec{k}^3}{(2\pi)^3} \quad (13)$$

where, the energy density operator is defined as $w(k, \lambda)$:

$$w(k, \lambda) = \hbar\omega_k(\lambda) \left[a^*(k, \lambda)a(k, \lambda) + \frac{1}{2} \right] \quad (14)$$

The sum $a^*(k, \lambda)a(k, \lambda)$ indicates that the wave vectors = 1, 2, 3 are annihilation and creation operators of the k available photon state, respectively, and the corresponding photon frequencies are determined by the dispersion relationship:

$$w_k(\lambda) = \sqrt{k^2 c_\lambda^2 + \omega_p^2} \quad (15)$$

Where $c_\lambda = c$ for $\lambda=1$ and 2, and $c_\lambda = s_e$ for $\lambda = 3$. Obviously, the first two modes or photon states correspond to the two transverse photons, and the third one to the longitudinal photon, or plasmon. It is also clear from this dispersion relationship that the electromagnetic field in a plasma is a mass vector field in which the equivalent mass vector can be defined in the condition of $\lambda c_\lambda = c\lambda = 3c_\lambda = S_e$

$$m_\lambda = \frac{\hbar}{c_\lambda^2} \omega_p \quad (16)$$

This definition extends the concept of photon equivalent mass to include the equivalent mass of longitudinal photons. One sees that the effective mass of the plasma exciton is much greater than the effective mass of the transverse photon, because we usually have $c^2 \gg S_e^2$. Using annihilation and creation operators for Heisenberg's equations, as well as the transformation relationship between the operators, it is not difficult to obtain:

$$\frac{d}{dt} a^*(k, \lambda) = i\omega_k(\lambda) a^*(k, \lambda) \quad (17)$$

The equations about $a(k, \lambda)$ it are similar to the above equation, which show that the field behaves similar to a quantum oscillator, whose frequency ω_k is determined by Equation (15). In nonlinear systems, it can also be known by the quantization equations (13) and (14) that we can replace the classical quantity sum with an operator $\vec{A}_k A_k^*$

$$\vec{A}_k \rightarrow \sum_{\lambda=1, 2} \sqrt{\frac{\hbar}{2\epsilon_0 \omega_k}} a_k(\lambda) \vec{e}_k(\lambda) \quad (18)$$

and

$$\vec{A}_k^* \rightarrow \sum_{\lambda=1, 2} \sqrt{\frac{\hbar}{2\epsilon_0 \omega_k}} a_k^*(\lambda) \vec{e}_k^*(\lambda) \quad (19)$$

where, it is the unit polarization vector, which means that the classical quantities of nonlinear systems can be replaced with the following equation $\vec{e}_k(\lambda) |A_k|^2$:

$$|A_k|^2 \rightarrow \sum_{\lambda=1, 2} \sqrt{\frac{\hbar}{2\epsilon_0\omega_k}} a_k^* a_k \quad (20)$$

Thus, one can derive a new definition of the photon charge operator:

$$Q_k(\lambda) = -\frac{e\hbar}{4m\omega_k} \frac{\omega_p^2}{v_k^2} a_k^*(\lambda) a_k(\lambda) \quad (21)$$

It is also possible to calculate the average of certain quantum states of this operator, to determine the total average of the photon charges, which can be written as follows:

$$Q = \sum_{\lambda=1,2} \int q_k n_k(\lambda) \frac{d\vec{k}}{(2\pi)^3} \quad (22)$$

Where n_k is the average of the photon occupancy number, or the usual number operator, and the number q_k is the photon charge, or the equivalent charge of a single photon $n_k(\lambda) = \langle a_k^*(\lambda) a_k(\lambda) \rangle$:

$$q_k = -\frac{e\hbar}{4m\omega_k} \frac{\omega_p^2}{v_k^2} \quad (23)$$

This is the quantization of the plasma field.

3.2 Plasma in universe

The cosmic plasma was developed by Alfven H, who built a model of the plasma universe, which is known as an alternative to the traditional "visual universe". Figure 1 presents a sketch of plasma universe [10]. Laboratory (size 10^{-1} m) and plasma experiments in the ionosphere, as well as in situ measurements in the magnetosphere (10^8 m) and found that the properties of the plasma were very different from those commonly thought [11]. We have reason to believe that many of the results of laboratory and magnetosphere studies can be further extrapolated to the plasmas of interstellar ($\sim 10^{17}$ m) and intergalactic regions ($< 10^{26}$ m).

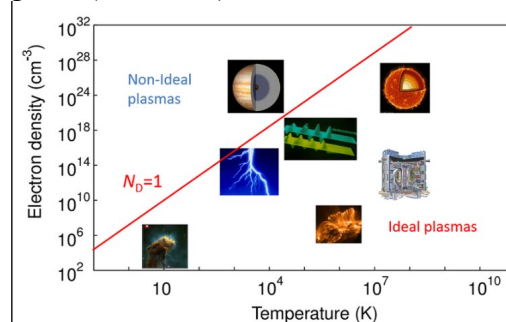


Figure 1. The sketch of plasma types [10].

The study also goes on to assume that the fundamental properties of plasma are the same everywhere, thus describing this inference as an "expansion of knowledge" that begins with a study in the laboratory. When the knowledge generated by laboratory and magnetosphere studies is combined with observations of plasma phenomena in galaxies and interstellar spaces, its expansion intensifies with the advent of the space age, and astrophysics ushers in a new era based on plasma cosmic models.

The properties of the plasma universe inferred from laboratory and magnetosphere phenomena in the study are summarized in the literature [12] and are roughly as follows: First of all, this universe has microporous structures, possibly antimatter, and is not made up of the traditional Big Bang. The plasma universe is penetrated by a network of currents, transmitting energy over long distances, creating a double electrolayer that accelerates particles to very high energies. Plasma is usually a dust critical velocity phenomenon and provides a new approach for the origin of the sun.

According to Alfven, theories about the evolution of galactic clouds and the formation of stars and solar system nebulae must also be revised, pointing to a new paradigm in which interstellar clouds should be seen as an extrapolation of magnetosphere studies. Some suggestions were given for this, a) the current in "vacuum" interstellar space assists in the gravitational collection of matter through the hoop shrinkage effect, thus forming interstellar clouds. b) These have developed under the combined action of mechanical and electromagnetic forces. The volume occupied by the current may be only a small part of the total volume, so the plasma region is not noticeable in the average of the

measurements with insufficient resolution. However, filamentous flow networks may play a decisive role in the evolution of clouds. Its evolution independent of its surroundings is only correctly viewed in the absence of an electric current connecting the interstellar cloud to its surroundings. c) As noted above, the general view that electromagnetic forces are opposed to cloud contraction is not necessarily correct. The shrinkage effect may help with compression, in fact, causing clouds with masses several orders of magnitude less than Jeans' masses to collapse. d) It seems possible to form a "stellesimal" from a cloud of dust.

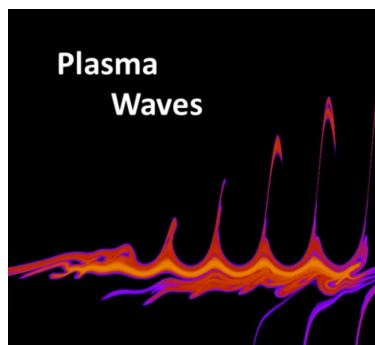


Figure 2. Plasma waves.

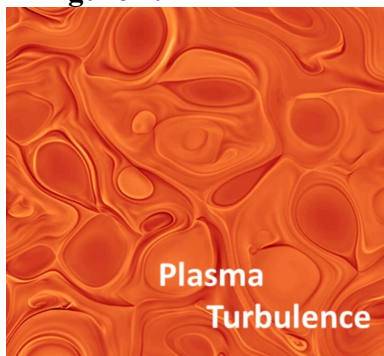


Figure 3. Plasma turbulence.

Finally, Alfven also discusses the application of models in cosmology. The Sun is thought to have formed from a dusty interstellar cloud through the above process. It has a certain mass, spin and magnetization strength. The remnants of the clouds form small clouds facing the sun, and according to plasma cosmology, they are housed in regions that reach critical velocities. Angular momentum is transmitted from the sun. These processes are controlled by plasma and mechanical effects. According to the "paradox principle", the formation process of moons and planets is basically the same [13]. The next process is plasma deionization and the formation of starlets. This plasma-astrological transition (PPT) is associated with the contraction of a factor that should be approximated, but some minor effects are expected to reduce the value by a few percent. The stars gather into planets. Around some planets, the same process is repeated in miniatures, which leads to the formation of satellite systems. Their cosmology is similar to that of planetary systems. $\Gamma = 2:3$ The formation of Earth galaxies was studied with examples, and the results obtained were also applicable to the formation of planets. Focused on the more special C ring, scholars also found some shadows that are predicted to be the origin of the universe. In addition, in the recent research, the Vlasov program developed by A. Grassi of the University of Pisa is used to simulate the electronic phase space during the rupture of one-dimensional Langmuir wave in relativistic plasma [15]. Fig. 2 and 3 illustrates the simulation of two-dimensional magnetized plasma turbulence is simulated by using the Marconi supercomputer of cineca.

4. Limitation & future prospects

Research in this field is still in its infancy, and the biggest problem is that the quantization work in the field of plasma is still incomplete. In addition, the revision of the interstellar cloud theory

requires a lot of work and is difficult to predict the outcome in detail. Future models of interstellar cloud formation and evolution remain on a few conjectures that need to be refined and summarized to produce truly usable models. In future work, one can continue to improve quantization from this aspect. If one can really break through the problems between these fields, we are expected to develop a new unified theory.

5. Conclusion

In summary, this paper investigates plasma based on quantum theory. According to the analysis, this study introduces some foundations of plasma mass ionization and its extension in some fields. Specifically, the very important Vlasov equation describing plasma dynamics is demonstrated and derived. The state-of-art researches on field quantization and intergalactic plasma are introduced. In the future, there may be greater breakthroughs in this theory. In general, these results provide guidance for the quantization of the whole plasma field.

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