Modeling and Simulation for a Five DOF Robotic Arm Manipulator

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Abstract. Nowadays, service robots are used in a large number of applications in the service industry. With the booming service industry in third world countries, service robots are expected to be used in a wider range of applications, so it is of great importance to conduct research on service robots. This paper develops the model of a kinematic models for a 5 DOF robotic arm that used in service industry in SolidWorks. The robotic arm's mathematical model is predicated on Denavit-Hartemberg (DH) method, which determine the robot joints angle vector. The procedure aims at describing the forward and inverse kinematic analysis, and finally determine the end effector's position and orientation through the corresponding joints angle related to the coordinate system (CS), in order to control the robotic arm to reach the designated location in space. A simulation model of the robotic arm was built using the Matlab Robotics Toolbox to verify the correctness of forward and inverse kinematics, which also can simplify the calculation and analysis.

Keywords: Service robot, Forward and inverse kinematics, Robotic arm simulation.

1. Introduction

In the new century, with the rapid development of science and technology, all industries are gradually developing towards intelligence, integration and informatization [1]. At the same time, industries have increasingly high requirements for cost, accuracy and efficiency in production activities, so the phenomenon of robots replacing manual labor is becoming more and more common in various industries [2]. Because the use of robots instead of manual labor can not only avoid people caught in the duplication of labor, but also save the industry's labor costs and promote the development of the industry. In the service industry, the application of robots has also made great strides compared to the last century. Service robots generally need to have good interaction capabilities, and their cheap but necessary intelligence allows them to be used on a large scale in the service industry in scenarios such as material transportation, rescue and disaster relief, and hotel services [3]. Service robots with robotic arms, in addition to being able to have certain intelligent interaction capabilities, can also make specific movements according to demand or scenarios [4].

Robotic arm modelling including kinematic analysis. The kinematic model does not take into account the forces on the robot, but only the mutual argumentation of the joint parameters of its CS and the directions and the end-effector locations [5]. The DH method uses robot joint parameters to solve for the directions and locations of its end-effectors and is widely used in the kinematic analysis and robotic arms simulation [6]. This paper uses the designed service-oriented robotic arm as the research object, constructs its mathematical model using DH method, and performs forward and inverse kinematic analysis and calculation. Simulations were then performed using Matlab Robotics Toolbox to verify the positive and negative kinematic analysis calculations, in order to provide some theoretical basis for the completion of subsequent specific work such as item handling for the purpose of this model.

2. Kinematic Analysis of the Robotic Arm and Its Verification

2.1. Structural and Kinematic Analysis of the Robotic Arm

2.1.1. Structure of the model

In the choice of degrees of freedom, considering only need to complete simple movements, the service robot arm does not need too many degrees of freedom, but at the same time in a limited space need to be more flexible [7]. The robot arm with five degrees of freedom meets the above characteristics and has greater practicality in the service industry [8]. The built robotic arm has five degrees of freedom, and can basically perform the movements expected by the design. The design and assembly of each component is done in Solidworks (see Figure 1).

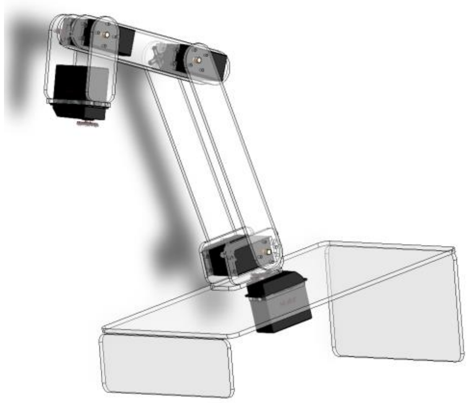

Figure 1. The structure of robotic arm

The dual motors on its base can complete the swinging and rotating action at the shoulder of the robotic arm, and complete the swinging with motors at each of the two succeeding joints. Finally, a motor is installed on the end-effector to complete the fine rotating action.

2.1.2. Forward kinematic model

There are two DH methods for mathematical modeling of the forward kinematics of robotic arms, one is the standard DH parametric modeling method (SDH) and the other is the modified DH parametric modeling method (MDH) [9, 10]. The main difference between the two is that the SDH method uses the latter joint CS of the connecting link as its fixation CS, while the MDH method uses the former joint CS of the connecting link as its fixation CS [11]. Based on the mechanism of the robot arm, its mathematical model is established by the standard DH method, and the established linkage coordinates system is shown in Figure 2.

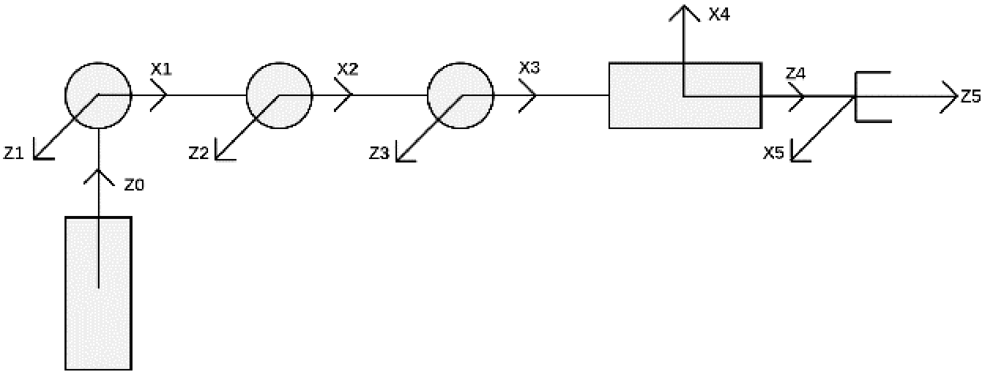

Figure 2. CS of the robotic arm

The DH method applies four main parameters, which includes link length (a_{i-1}), link twist (α_{i-1}), link offset (d_i) and joint angle(θ_i). Each linkage of the robot arm has been fixed to a CS, and the DH parameters of the robotic arm are listed in Table 1.

Table 1. DH parameters

i	Link twist α_i/(°)	Link length a_i/mm	Link offset d_i/mm	Joint angle θi/(°)
1	90°	0	0	θ_1
2	0	249(L_2)	0	θ_2
3	0	141(L_3)	0	θ_3
4	90°	0	0	θ_4
5	0	0	98(L_5)	θ_5

Then, we need to complete the transformation from the $i-1$ joint to the i joint, and so on, and combine all the transformations to complete the complete flush transformation equation of the robot arm, whose general equation is as follows:

$$T_i^{i-1}=Rot_{z,\theta i}Trans_{z,di}Trans_{x,\alpha i}Rot_{x,\alpha i} \\ =\begin{bmatrix} C_{\theta i} & -S_{\theta i}C_{\alpha i} & S_{\theta i}S_{\alpha i} & a_iC_{\theta i} \\ S_{\theta i} & C_{\theta i}C_{\alpha i} & -C_{\theta i}S_{\alpha i} & a_iS_{\theta i} \\ 0 & S_{\alpha i} & C_{\alpha i} & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

Where $S_{\theta i}$ represents Sin θ_i, and $C_{\theta i}$ represents Cos θ_i.

According to the recursive formula of T_i^{i-1}, the flush transformation matrix between each key solid CS can be obtained (T_1^0~T_5^4):

$$T_1^0=\begin{bmatrix} C_{\theta 1} & 0 & S_{\theta 1} & 0 \\ S_{\theta 1} & 0 & -C_{\theta 1} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$T_2^1=\begin{bmatrix} C_{\theta 2} & -S_{\theta 2} & 0 & L_2C_{\theta 2} \\ S_{\theta 2} & C_{\theta 2} & 0 & L_2S_{\theta 2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$T_2^1=\begin{bmatrix} C_{\theta 2} & -S_{\theta 2} & 0 & L_2C_{\theta 2} \\ S_{\theta 2} & C_{\theta 2} & 0 & L_2S_{\theta 2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$$T_3^2=\begin{bmatrix} C_{\theta 3} & -S_{\theta 3} & 0 & L_3C_{\theta 3} \\ S_{\theta 3} & C_{\theta 3} & 0 & L_3S_{\theta 3} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{5}$$

$$T_4^3=\begin{bmatrix} C_{\theta 4} & 0 & S_{\theta 4} & 0 \\ S_{\theta 4} & 0 & -C_{\theta 4} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{6}$$

$$T_5^4=\begin{bmatrix} C_{\theta 5} & -S_{\theta 5} & 0 & 0 \\ S_{\theta 5} & C_{\theta 5} & 0 & 0 \\ 0 & 0 & 1 & L_5 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

Now that the homogeneous transformation matrix for each linkage is listed, multiplying these individual transformation matrices gives the homogeneous transformation matrix representing the position and directions of the robotic arm's end-effector. This leads to the forward kinematic model of the robotic arm:

$$T_5^0=T_1^0T_2^1T_3^2T_4^3T_5^4=\begin{bmatrix} r_{11} & r_{12} & r_{13} & P_x \\ r_{21} & r_{22} & r_{23} & P_y \\ r_{31} & r_{32} & r_{33} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{8}$$

Where the first 3x3 r matrix and P(p_x, p_y and p_z) representing the rotation and displacement of the end-effector.

$r_{11}= C_{\theta 5} [C_{\theta 4} (C_{\theta 1}C_{\theta 2}C_{\theta 3} - C_{\theta 1}S_{\theta 2}S_{\theta 3}) + S_{\theta 4} (-C_{\theta 1}C_{\theta 2}S_{\theta 3} - C_{\theta 1}C_{\theta 3}S_{\theta 2})]+ S_{\theta 1}S_{\theta 5}$
$r_{12}= C_{\theta 5}S_{\theta 1} - S_{\theta 5} [C_{\theta 4} (C_{\theta 1}C_{\theta 2}C_{\theta 3} - C_{\theta 1}S_{\theta 2}S_{\theta 3}) + S_{\theta 4}(-C_{\theta 1}C_{\theta 2}S_{\theta 3} - C_{\theta 1}C_{\theta 3}S_{\theta 2})]$
$r_{13}=-C_{\theta 4}(-C_{\theta 1}C_{\theta 2}S_{\theta 3} - C_{\theta 1}C_{\theta 3}S_{\theta 2}) + S_{\theta 4}(C_{\theta 1}C_{\theta 2}C_{\theta 3} - C_{\theta 1}S_{\theta 2}S_{\theta 3})$
$r_{21}=-C_{\theta 1}S_{\theta 5} + C_{\theta 5}[C_{\theta 4}(C_{\theta 2}C_{\theta 3}S_{\theta 1} - S_{\theta 1}S_{\theta 2}S_{\theta 3}) + S_{\theta 4}(-C_{\theta 2}S_{\theta 1}S_{\theta 3} - C_{\theta 3}S_{\theta 1}S_{\theta 2})]$
$r_{22}=-C_{\theta 1}C_{\theta 5} - S_{\theta 5}[C_{\theta 4}(C_{\theta 2}C_{\theta 3}S_{\theta 1} - S_{\theta 1}S_{\theta 2}S_{\theta 3}) + S_{\theta 4}(-C_{\theta 2}S_{\theta 1}S_{\theta 3} - C_{\theta 3}S_{\theta 1}S_{\theta 2})]$
$r_{23}=-C_{\theta 4}(-C_{\theta 2}S_{\theta 1}S_{\theta 3} - C_{\theta 3}S_{\theta 1}S_{\theta 2}) + S_{\theta 4}(C_{\theta 2}C_{\theta 3}S_{\theta 1} - S_{\theta 1}S_{\theta 2}S_{\theta 3})$
$r_{31}= C_{\theta 5}[C_{\theta 4}(C_{\theta 2}S_{\theta 3} + C_{\theta 3}S_{\theta 2}) + S_{\theta 4}(C_{\theta 2}C_{\theta 3} - S_{\theta 2}S_{\theta 3})]$
$r_{32}=-S_{\theta 5}[C_{\theta 4}(C_{\theta 2}S_{\theta 3} + C_{\theta 3}S_{\theta 2}) + S_{\theta 4}(C_{\theta 2}C_{\theta 3} - S_{\theta 2}S_{\theta 3})]$
$r_{33}=-C_{\theta 4}(C_{\theta 2}C_{\theta 3} - S_{\theta 2}S_{\theta 3}) + S_{\theta 4}(C_{\theta 2}S_{\theta 3} + C_{\theta 3}S_{\theta 2})$
$P_x=L_3C_{\theta 1}C_{\theta 2}C_{\theta 3} + L_2C_{\theta 1}C_{\theta 2} - L_3C_{\theta 1}S_{\theta 2}S_{\theta 3} - L_5C_{\theta 4}(-C_{\theta 1}C_{\theta 2}S_{\theta 3}-C_{\theta 1}C_{\theta 3}S_{\theta 2})$
$+ L_5S_{\theta 4}(C_{\theta 1}C_{\theta 2}C_{\theta 3} - C_{\theta 1}S_{\theta 2}S_{\theta 3})$
$P_y=L_3C_{\theta 2}C_{\theta 3}S_{\theta 1} + L_2C_{\theta 2}S_{\theta 1} - L_5C_{\theta 4}(-C_{\theta 2}S_{\theta 1}S_{\theta 3} - C_{\theta 3}S_{\theta 1}S_{\theta 2}) - L_3S_{\theta 1}S_{\theta 2}S_{\theta 3}$
$+ L_5S_{\theta 4}(C_{\theta 2}C_{\theta 3}S_{\theta 1} - S_{\theta 1}S_{\theta 2}S_{\theta 3})$
$P_z=L_3C_{\theta 2}S_{\theta 3} + L_3C_{\theta 3}S_{\theta 2} - L_5C_{\theta 4}(C_{\theta 2}C_{\theta 3} - S_{\theta 2}S_{\theta 3}) + L_2S_{\theta 2} + L_5S_{\theta 4}(C_{\theta 2}S_{\theta 3} + C_{\theta 3}S_{\theta 2})$

2.1.3. Inverse kinematic model

Robot inverse kinematics, an important part of robot kinematics is solving for the angle of each joint (θ_1, θ_2, θ_3, θ_4, θ_5) through the end-effector pose matrix:

$$T_5^0=T_1^0(\theta_1)T_2^1(\theta_2)T_3^2(\theta_3)T_4^3(\theta_4)T_5^4(\theta_5) \tag{9}$$

Both ends of equation (9) are simultaneously pre-multiplied by the$[T_1^0(\theta_1)]$-1:

$$[T_1^0(\theta_1)]\text{-}1T_5^0=T_2^1(\theta_2)T_3^2(\theta_3)T_4^3(\theta_4)T_5^4(\theta_5) \tag{10}$$

According to the elements (3, 4) on both sides of the equation (10):

$$\mathrm{PxS1\text{-}PyC1}=0 \tag{11}$$

Where C_1 represents Cos (θ_1), S_1 represents Sin (θ_1) and S_{12} represents Sin (θ_1+θ_2).
Thus, the value of θ_1 can be found as:

$$\theta_1=\mathrm{Atan2}(\mathrm{Py},\mathrm{Px}) \tag{12}$$

According to the elements (1, 4) and (2, 4) on both sides of the equation (10):

$$\begin{cases} P_xC_1 + P_yS_1 = L_2C_2 + L_3C_{23} + L_5C_{234} \\ \qquad P_z = L_2S_2 + L_3S_{23} - L_5C_{234} \end{cases} \tag{13}$$

And according to the elements (1, 3) and (2, 3) on both sides of the equation (10):

$$\begin{cases} r_{23}S_1 + r_{13}C_1 = S_{234} \\ r_{33} = -C_{234} \end{cases} \tag{14}$$

Substituting equation (14) into equation (13) eliminates the term S_{234} and C_{234}:

$$\begin{cases} P_xC_1 + P_yS_1 = L_2C_2 + L_3C_{23} + L_5(r_{23}S_1 + r_{13}S_1) \\ P_z = L_2S_2 + L_3S_{23} + L_5r_{33} \end{cases} \tag{15}$$

Square both ends of equation (16):

$$\begin{cases} (K_1 - L_2C_2)^2 = (L_3C_{23})^2 \\ (K_2 - L_2S_2)^2 = (L_3S_{23})^2 \end{cases} \tag{16}$$

Where:

$$\begin{cases} K_1 = P_xC_1 + P_yS_1 - L_5(r_{33}S_1 + r_{13}C_1) \\ K_2 = P_z - L_5r_{33} \end{cases} \tag{17}$$

Adding the two equations of equation 16:

$$(K_1 - L_2C_2)^2 + (K_2 - L_2S_2)^2 = L_3^2 \tag{18}$$

Thus, the value of θ_2 can be found as:

$$\theta_2 = Atan\left(\frac{S_2}{C_2}\right) \tag{19}$$

Substituting equation (14) into equation (13), and organize the terms containing S23 and C23 to one side of the equation:

$$\begin{cases} P_xC_1 + P_yS_1 - L_2C_2 - L_5(r_{23}S_1 + r_{13}C_1) = L_3C_{23} \\ P_z - L_2S_2 - L_5r_{33} = L_3S_{23} \end{cases} \tag{20}$$

Solving θ_{23} by equation (20):

$$\theta_{23} = Atan(P_z - L_2S_2 - L_5r_{33})/(P_xC_1 + P_yS_1 - L_2C_2 - L_5r_{23}S_1 - \mathrm{L}_5r_{13}C_1) \tag{21}$$

$$\theta_3 = \theta_{23} - \theta_2 \tag{22}$$

Now that the joint angles of the first three joints are known, pre-multiply the inverse matrix of the kinematics of the first three joints:

$$[T_3^2(\theta_3)]\text{-}1[T_2^1(\theta_2)]\text{-}1[T_1^0(\theta_1)]\text{-}1T_5^0{=}T_4^3(\theta_4)T_5^4(\theta_5) \tag{23}$$

According to the elements (1, 4) and (2, 4) on both sides of the equation (23):

$$\begin{cases} P_zS_{23} - L_3 - L_2C_3 + P_xC_1C_{23} + P_yS_1C_{23} = L_5S_4 \\ P_zC_{23} - L_2S_3 + P_xC_1S_{23} + P_yS_1S_{23} = -L_5C_4 \end{cases} \tag{24}$$

Thus, the value of θ_4 can be found as:

$$\theta_4 = Atan\left(\frac{-P_zS_{23}+L_3+L_2C_3-P_xC_1C_{23}-P_yS_1C_{23}}{P_zC_{23}-L_2S_3+P_xC_1S_{23}+P_yS_1S_{23}}\right) \tag{25}$$

According to the elements (3, 1) and (3, 2) on both sides of the equation (23):

$$\begin{cases} \mathrm{r_{11}S_1 - r_{21}C_1 = S_5} \\ r_{12}S_1 - r_{22}C_1 = C_5 \end{cases} \tag{26}$$

Solving θ_5 by equation (26):

$$\theta_5 = Atan2(\mathrm{r_{11}S_1 - r_{21}C_1}, r_{12}S_1 - r_{22}C_1) \tag{27}$$

2.2. Simulation and Calculation Verification of Robotic Arm based on Matlab

2.2.1. Robotic arm modeling based on Matlab

For the robotic arm's kinematic simulation, the kinematic model of the robot arm can be obtained by writing a program using the functional algorithm in the Matlab Robotics Toolbox [9].In the robotics toolbox, the modeling procedure specified by the standard DH method is as follows, and ‘L’ represents the link of the robotic arm: L= ([θ, d, a, α],’standard’); According to the DH parameters in Table 1, the mathematical models of the five links (L1~L5) are established in turn:

L1=Link ([0, 0, 0, pi/2], 'standard');
L2=Link ([0, 0, 0.249, 0], 'standard');
L3=Link ([0, 0, 0.14, 0], 'standard');
L4=Link ([0, 0, 0, pi/2], 'standard');
L5=Link ([0, 0.098, 0, 0], 'standard');

Connecting each joint in turn to make them a robotic arm and name it Service Robotic Arm_5DOF: robot=SerialLink ([L1, L2, L3, L4, L5]);

Figure 3 show the results of robotic arm model.

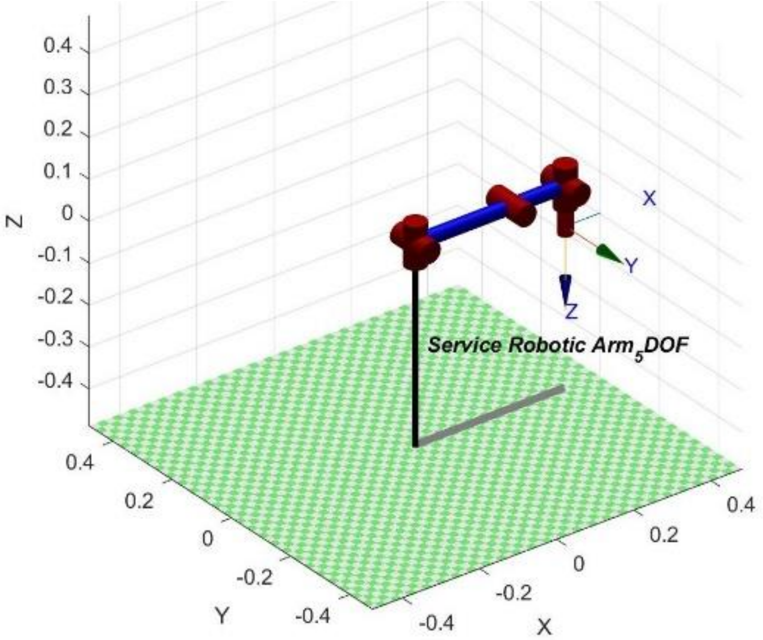

Figure 3. Robotic arm kinematic model

2.2.2. Robotic arm forward kinematics verification

This paper verifies the correctness of the robotic arm's forward kinematics by comparing the positional matrix of end-effector obtained by solving Matlab-based fkine function with the positional matrix obtained by the standard DH method in the previous paper. As an example, random angles are selected at five target joints, picking (θ1, θ2, θ3, θ4, θ5)=(0, π/4, -π/4, π/3, π/2) and the results of the forward kinematics using the Kine function in Matlab are:

$$T_5^0=\begin{bmatrix} 0 & -0.5000 & 0.8660 & 0.4019 \\ -1 & 0 & 0 & 0 \\ 0 & -0.8660 & -0.5000 & 0.1271 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{28}$$

Using the kinematic equations derived in equation (2) above, substituting the selected angles for the matrix operations. There are round-off errors between the calculated results and the results obtained through the fkine function, but it does not affect the correctness of the forward kinematic model.

2.2.3. Robotic arm inverse kinematics verification

The robotic arm used in this paper has five degrees of freedom, so the inverse solution about the robot arm's positional matrix is solved using the ikine function in the Matlab robotics toolbox. Set the matrix of Equation 15 as a known quantity and use the ikine function to solve for the joint angles.The results obtained using the ikine function are not the same as the original joint angle because the robot inverse kinematics can find multiple sets of joint angle solutions, while Matlab robotics toolbox only select one set of solutions.

3. Conclusion

In this paper, a five-degree-of-freedom robotic arm of service class is designed and its mechanical model is constructed by Solidworks. The robotic arm's mathematical model has been constructed using the DH method, and the DH parameters have been derived. The forward and reverse kinematics of the robotic arm has been analyzed and solved, and verification was completed using Matlab Robotic Toolbox. The work done in this paper may provide a theoretical basis for the subsequent trajectory planning and control of the robotic arm.

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