

The progress of quantum anomalous Hall effect in graphene and the state-of-art applications

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Abstract. Contemporarily, the researches of the quantum Hall effect (QAHE) leads an intensive potentiality of topological insulators and corresponding quantum computing. The QAHE was considered as the last member of the family of Hall effect. In this paper, the history and development of QAHE will be introduced, together with the theoretical prediction and experimental observations. The relative application of normal QAHE and high temperature QAHE are discussed for the potential guiding meaning. The reveal of experimental observed QAHE may shed a light on guiding further explorations of QAHE and the state-of-art applications (e.g., quantum com).

Keywords: Quantum Hall effect, graphene, Quantum Anomalous Hall effect.

1. Introduction

The quantum Anomalous Hall effect (QAHE) is intensively researched in condensed matter physics due to its unique property. The Nobel prize physics of 1995, 1998 and 2010 were given to research and discoveries about quantum Hall effect, which were integer Hall effect, fractional Hall effect and half integer Hall effect in graphene, respectively. The quantized spin Hall effect is also a member of the HE system. As the last part of quantum Hall effect, the development of QAHE is still a hot spot around the world. In 1880, the Hall effect of ferromagnetic material in absence of external magnetic field was firstly discovered [1]. From Hall effect in normal conductor, the quantum properties are revealed from the increasing magnetic fields [2]. Since the working principle of AHE is different from ordinary Hall effect, the differences between the two would be discussed. The QAHE was theoretically demonstrated but its experimental observation is achieved till 1988 [3].

The graphene is important in the research about QAHE due to its single-layered structure, which could restrict delocalised charge carriers on its surface satisfying the requirements of QAHE occurring. The structure of π bond constituting the graphene provides the delocalised electrons with a possible 2D plane to accumulate the quantised anomalous Hall voltage.

In this article, the theory of QAHE and corresponding origin and quantised Hall effect will be illustrated, followed by the potential application and possible influence of QAHE in industry and academia. The limitations and prospects will also be constructed. The reminder of the paper will be separated as follows. The section 2 will illustrate the basic principles of family of Hall effect and briefly introduce graphene. In the third section, the application of QAHE in two-dimensional graphene would be demonstrated and discussed based on recent researches in the past few years. Following the discussion about limitations and prospects of QAHE in the future in section 4, the paper is concluded in section 5.

2. Basic Principles

2.1 QAHE

For a piece of conductor with current J_x placed in a magnetic field B_y (perpendicular to the direction of current flow), there will be a transverse voltage accumulated by the charge carriers exerting Lorentz force over the conductor at direction that perpendicular to both magnetic field and current, which is discovered by Hall in 1879. [4] As shown in Fig. 1, in the conductor of thickness t , the transverse Hall voltage V_H is

$$V_H = \frac{J_x B_z}{nte} \quad (1)$$

where J_x is the current, B_z is the applied magnetic field, n is the number of charge carriers and e represents the elementary charge. In the ordinary Hall effect, the Hall resistance is completely relying on the magnitude of applied magnetic field and current inside, which shows linear proportional relation with both. However, in the later research and experiments, when small magnetic field is applied, the relation between the current and magnetic field showed an unusual association, which is called the anomalous Hall effect. Followed by the theory of SHE and QHE, the differences between these members are revealed. The Techniques of QHE, QAHE and QSHE are shown according to the properties of charge carriers and ferromagnetic materials, which have no applied magnetic field to provide energy and corresponding Lorentz force differentiating the quantized properties from the ordinary Hall effect.

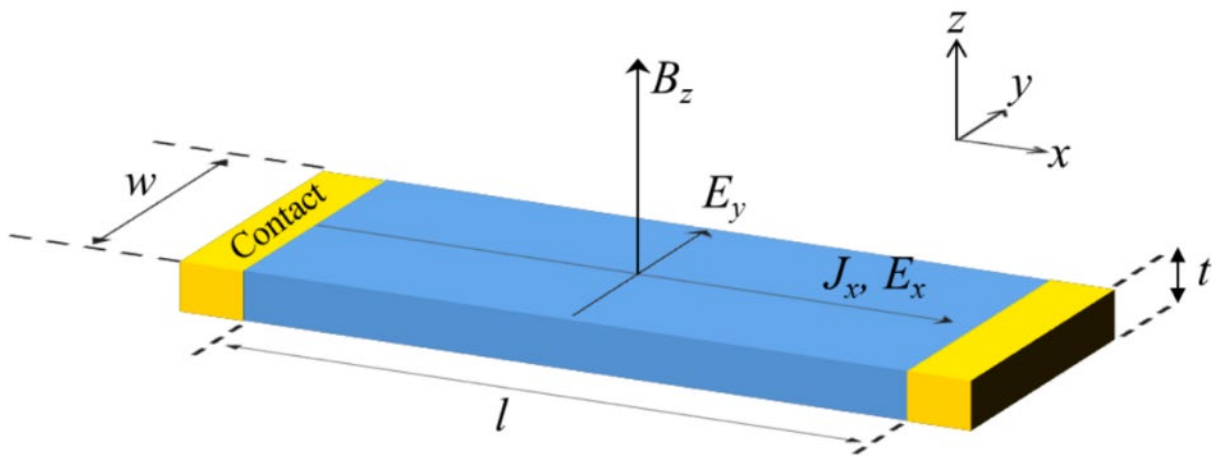


Figure 1. A sketch of Hall effect.

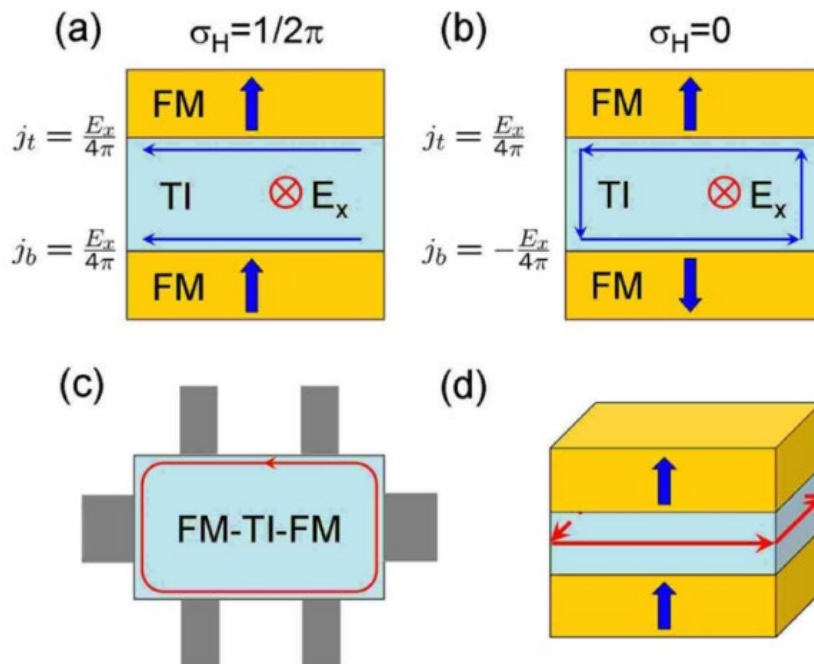


Figure. 2 Illustration of QHE in ferromagnet-topological insulator heterostructure [5].

The QHE is a quantized version of ordinary Hall effect which occurs at 2D electron systems, including the integer quantum Hall effect and fractional quantum Hall effect. The Hall resistance R_{xy} could be expressed as

$$R_{xy} = \frac{V_{Hall}}{I_{channel}} = \frac{h}{e^2\nu} \quad (2)$$

where V_{Hall} is the Hall voltage in the material, $I_{channel}$ is the channel current and e represents the elementary charge. The value of divisor ν could be integer or fraction, referring the integer QHE and fractional QHE, respectively. Predicted in 1975, quantum Hall effect was experimentally observed in graphene. The transverse integer quantum Hall conductivity σ is

$$\sigma = \nu \frac{e^2}{h} \quad (3)$$

Here, ν is the filling factor determined as the ratio between the density of states and the density of states in the well-defined Landau levels. In experiments, a plateau is observed between Landau levels indicating that there are electrical charge carriers present. If the magnetic field reduces to zero, the results then reach to classical results.

The Landau quantization driven 2D electron systems form a unique insulating phase, which leads to a dissipation less states appearing at edges, induced by spontaneous magnetization. By breaking the time reversal symmetry of a suitable topological insulator by introducing ferro magnetism usually leads to quantum anomalous Hall effect, in which the plateau of Hall conductance σ_{xy} of e^2/h is expected to be observed even without an external magnetic field as illustrated in Fig 2. Another cousin of quantum Hall effect is quantum spin Hall effect (QSHE) which exists in special 2D semiconductors without necessary of application of large magnetic field. The similarity and differences are schematically demonstrated in Fig. 3.

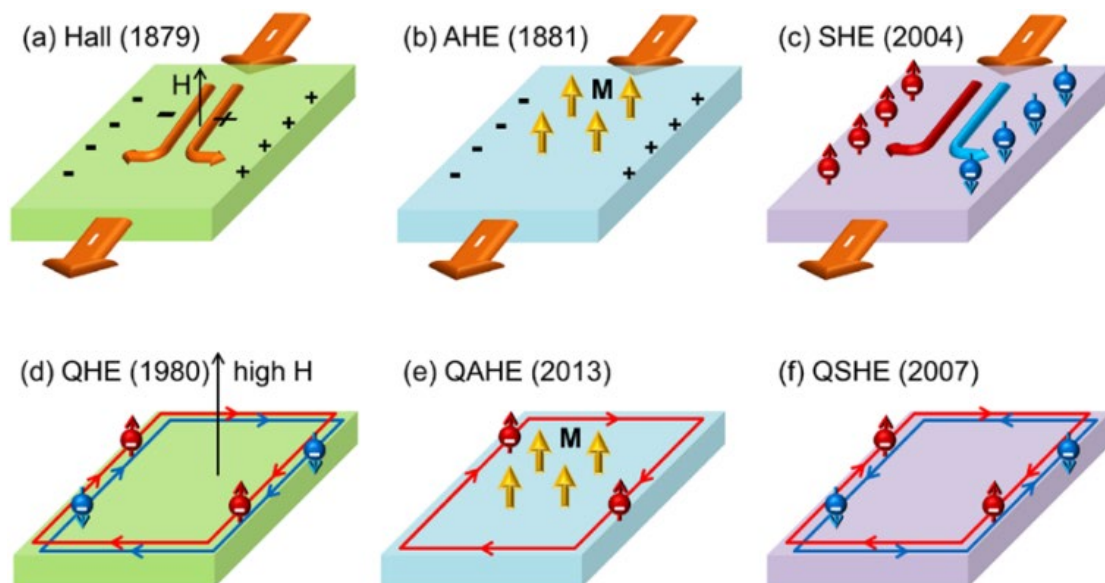


Figure. 3. Categories of the Hall effects [6].

2.2 Graphene

Since the QAHE requires a 2D environment, graphene plays a crucial role in the investigation of QAHE. Graphene, as the foundation of all carbon graphitic forms, consists of carbon atoms held in a single layer by overlapping sp^2 hybrid bonds. Originating from the $2p$ orbitals, the characteristics of graphene are relying on the delocalizing π state bands, which provides graphene with high mobility of charge carriers, thermal conductivity and optical transparency [7]. With these properties, graphene has comparative advantages in both industrial and academic fields, including the research in QAHE.

With consideration of aid of the properties discussed in the previous section, the advantages of graphene used in the research of QAH insulator are evident.

3. Applications

According to the properties exhibited in QAHE, the probability of potential applications of it is always attractive. Owing to its property, high temperature QAHE is intensively required for the development of topological insulators. In the past few years, the intrinsic topological insulators are intensively studied in material science and condensed matter physics [8]. As a new state of quantum matter, topological insulators allow current-carrying edge state with insulating inside the material, which is depending on the QAHE. The thermal properties of topological insulator material MnBi_2Te_4 is illustrated in Fig. 4 [9]. The material undergoes a transition of paramagnetic-to-antiferromagnetic order at $T_N = 25\text{K}$, characteristic of the onset of antiferromagnetic transition. Below T_N , field induced metamagnetic transition occurs at the critical field (H_{c1}) which depends on the temperature. In time reversal symmetry topological insulator, the charge carrying edge state could be protected so that the information is possible to be transported through spin of electrons instead of charge carrier. Considering the contribution of topological insulators, the principles and experimental reveal of QAHE are regarded as fundamentals of quantum computers cooperating with quantum Hall effect [10].

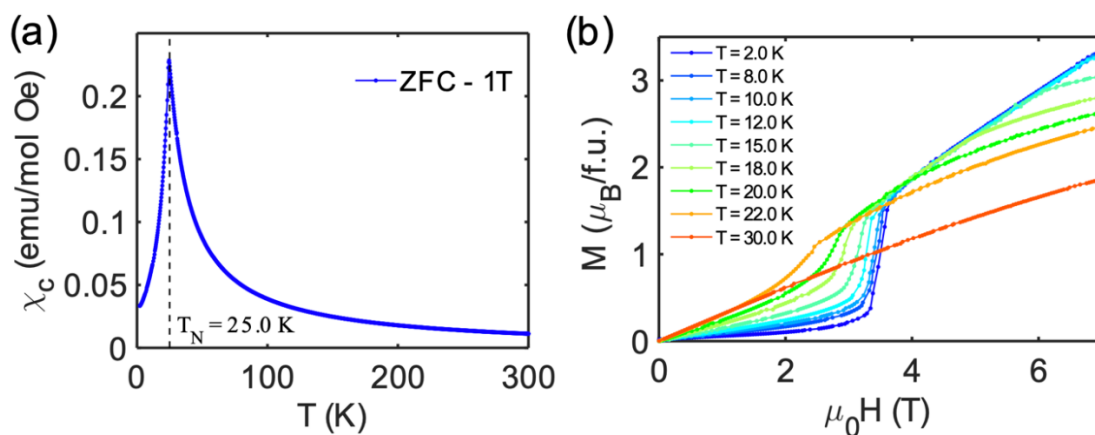


Figure. 4 Magnetic susceptibility as a function of temperature (left panel) and Isothermal magnetization measurement as a function of magnetic field (right panel) [9].

In addition, since QAHE does not require additional external magnetic field, the corresponding Hall resistance could be used as a new practical standard measurement of resistance of materials. For the same reason, quantum anomalous Hall state generated a path to development of topological electronic and spintronic devices, leading to the topological superconductors [11]. The study of topological insulator is potentially developing the superconductivity called topological superconductor. By novel synthetic approaches, the proximity-induced superconductivity are investigated in heterostructures in Dirac surface states of 5-10 quintuple-layer $(\text{Bi,Sb})_2\text{Te}_3/\text{graphene}/\text{gallium}$ as illustrated in Fig. 5 [8]. According to the experimental results, there is an obvious superconductivity gap at Dirac surface states at low temperature.

Moreover, the QAHE could be used for designing intrinsic magnetic topological materials, which is lying in the centre of condensed matter and gives the possibility of interplay of magnetism and topology. With the process of QAHE in 2D space, the topological insulators could be potentially studied. With fabricated magnetic topological graphene nanoribbons the topological edge bands were

observed, and by tuning the its length the transition from trong anti-ferromagnetic to a paramagnetism-like weak coupling could be controlled. [13]

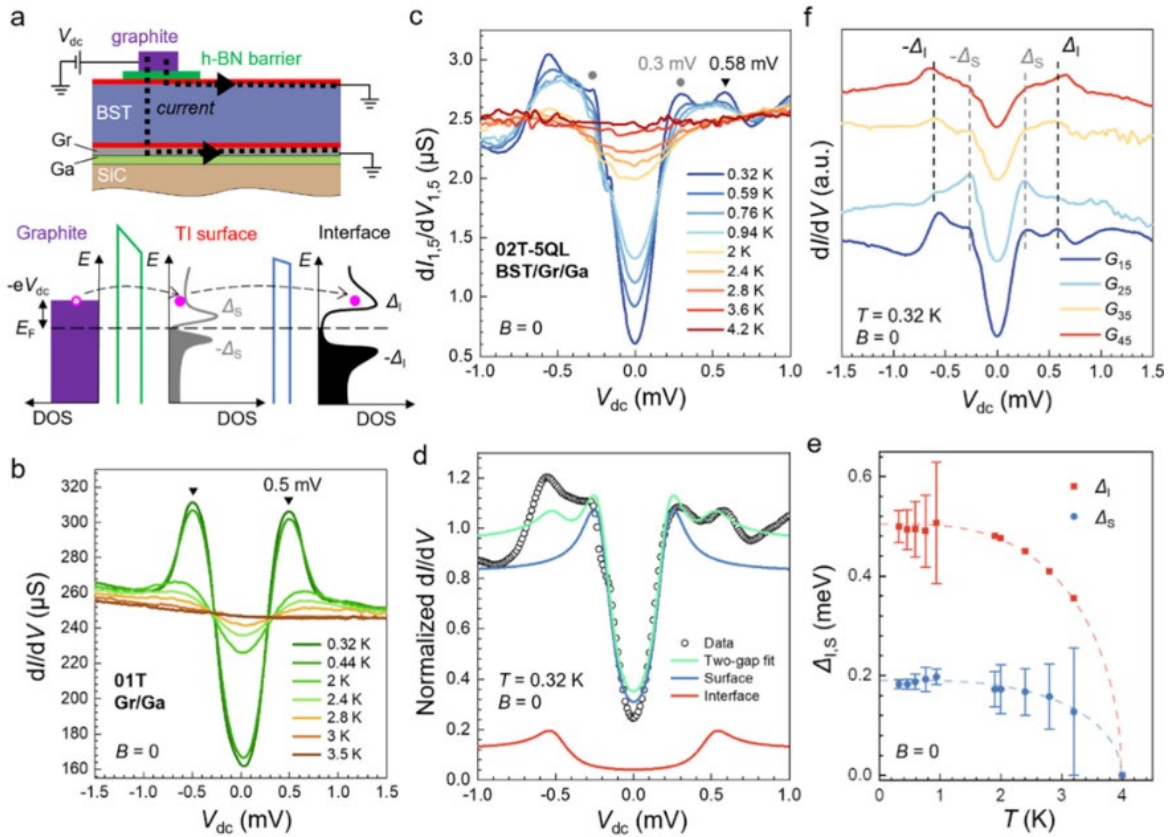


Figure. 5 The proximity-induced superconductivity in BST/Gr/Ga heterostructures [12].

4. Limitations and Future prospect

QAHE and its corresponding possible applications are attractive and appealing as a recent developing trend. There are, however, still some drawbacks of current research about QAHE. To generalize the advantages of QAHE in different types of materials, the major challenge to solve is the limitation of low temperature. At current stage of research, the QAHE only occurs at extremely low temperature in most of ferromagnetic materials. To reach the quantized plateau and pure dissipation less chiral edge state, ultralow temperature is needed. Contractively, high temperature QAHE was only discovered experimentally in graphene, which restricted the potentiality of application of relative discoveries mentioned in the previous sections. More relative research is deserved to observe the QAHE occurring at higher temperature. Additionally, the fractional QAHE is also limited. As the analogue of fractional QHE, the possibility of fractional QAHE was demonstrated but still at the stage of theoretically research, crucially requiring the material system [14]. Compared to quantum Hall effect, the QAHE occurs without external magnetic field, which greatly promotes the superconductor proximity effect. The futural development of QAHE with the aid of relative high temperature QAHE in graphene could provide direction of new research about superconductor [15].

5. Conclusions

In summary, this paper discussed the QAHE starting from ordinary Hall effect, including the integer and fractional quantum Hall effect. As a convincing supporting material of research and experiment about high temperature QAHE, the structure and properties of graphene is also discussed in this article. The family of Hall effect are introduced and functions of QAHE and relative

topological insulators are discussed with potential application for the high-level precise and low-power-consumption electronics. Experimental observation of QAHE in graphene revealed its topological properties, which paves a path for the development of quantum computers together with the topological insulators. In the future, the QAHE in graphene would remain as an intensive hot spot in condensed matter physics and relevant material science. Overall, these results offer a guideline for potentially clearer understanding of topological insulator and thus quantum computing.

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