Analyzation of the Application Scenarios of Ackerman Geometry based on Vehicle Steering Model

Donghao Sui[1, *, †], Yulai Zhang[2, †]

[1] School of Mechanical Engineering Pennsylvania State University-University Park State College 16802, PA, United State

[2] School of Mechanical Engineering Jiangsu University Zhen Jiang 212000, China

* Corresponding Author Email: dzs5936@psu.edu

†These authors contributed equally.

Abstract. A complete set of models about vehicle steering are established to testify the feasibility of Ackerman geometry applied in various scenarios. The relationship between different parameters from cars and how they affect the steering ability is studied by Python. Linear model and nonlinear model are built to adapt to small and big slip angle. The mechanism section is tested with data from two types of cars: BMW M3 and Chevrolet Cavalier. We find that M3 has a better steering capability since it can generate more lateral force with less sideslip. The relationship between slip angle and lateral force illuminates our research about Ackerman geometry. The result indicates that regular Ackerman principle is beneficial in the big wheel steering angle with low-speed conditions, because it can reduce the slip angle and tire-road interaction, making cornering smoother and reducing tire wear. However, in small wheel steering angle with high-speed conditions, the Ackerman principle has some limitations and should be amended, which adds to the slip angle and thus increase the lateral force to get better steering. That is basically the reason why Reverse Ackerman is widely applied in Formula 1.

Keywords: Ackerman geometry; slip angle; steering ability; lateral force; tire-road interaction.

1. Introduction

Ackermann steering geometry is the geometric arrangement of the connecting rods in the steering of cars or other vehicles [1]. It aims to solve the problem that the inside and outside wheels of a turn need to draw circles of different radii [2].

Ackermann’s steering mechanism was designed by Georg Lankensperger, a German carriage maker, and his business partner and agent Rudolf Ackermann (1764 -- 1834) submitted a patent in 1818, But Erasmus Darwin is said to have had a priori right to the invention in 1758 [3].

The Ackermann angles we're talking about are the two large and small angles between the steering wheels when a car turns [4]. Not the steering Angle. Ackermann refers to the Angle at which the inner wheels turn much more than the outer wheels when the vehicle turns due to the geometry of the steering system [5]. The inner wheel (relative to the center of bending) is usually at a larger angle than the outer wheel [6]. As you increase the steering angle, the angle increases. Ackermann Settings are usually achieved by changing the position of the steering push rod fixing hole on the steering cup. Turning the putter to a larger angle will produce more Ackermann angles and vice versa [7]. Because the inner and outer wheels have different driving radii when the vehicle turns, Ackermann can help the two front wheels point in the right direction for more grip [8].

In this paper, we model the BMW M3, and Chevrolet Cavalier based on our own experimental data. Moreover, we also carry out our own analysis on slip Angle and Ackermann steering mechanism [9]. In this paper, we will analyze the slip Angle and its impact on vehicle steering and its own steering mechanism. We will elaborate on the application of Ackermann steering mechanism.

2. Mechanism

2.1. Kinetic Model

Targeted at normal road cars with two-wheel steering system, following assumptions are made: no acceleration or breaking, no Ackerman geometry (two front wheels parallel, which is a good assumption to simplify the model), forces and velocities on right side and left side equal [10]. The main object of this study is tire-road interaction and how they affect vehicle kinematics. Therefore, only forces on tires and gravity are considered. The schematic of a car with two-wheel steering system is displayed in Figure 1.

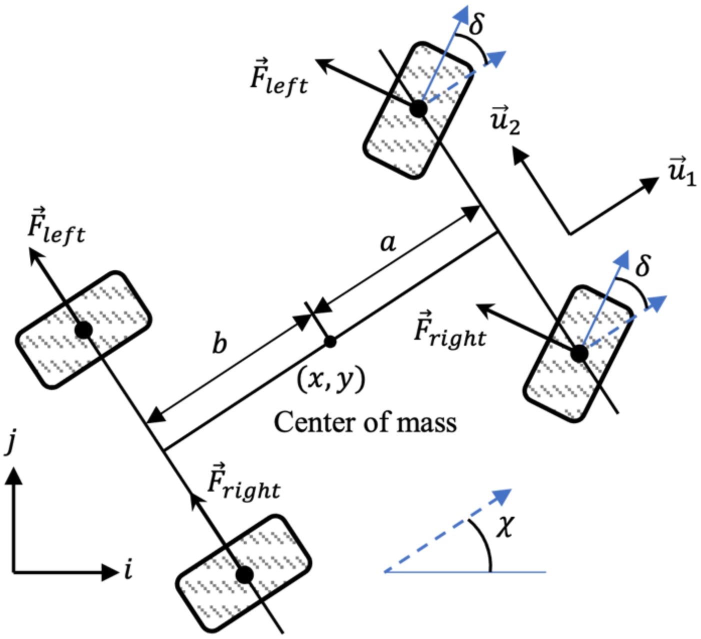

Figure 1. Schematic of the Car.

In this schematic there are two coordinate system: stationary frame (i, j) and car-fixed frame$(\vec{u}_1, \vec{u}_2)$. The former one is mainly used to describe the position and rotation angle of the car and the latter one is in orientation of the car's head, which makes it easier to calculate parameters on tires. a And b are respectively the distance between center of mass and the two axles. χ Is the angle at which the car deviates from its original position? δ Is the angle that the front wheel turns? Since both systems consist of unit vectors, we have:

$$\vec{u}_1 = \begin{bmatrix} \cos\chi \\ \sin\chi \end{bmatrix} \qquad \vec{u}_2 = \begin{bmatrix} -\sin\chi \\ \cos\chi \end{bmatrix} \tag{1}$$

Front force:

$$\vec{F}_f = -\|\vec{F}_f\| \sin\delta\, \vec{u}_1 + \|\vec{F}_f\| \cos\delta\, \vec{u}_2 \tag{2}$$

Rear force:

$$\vec{F}_r = \pm \|\vec{F}_r\|\, \vec{u}_2 \tag{3}$$

Gravity has zero component in this dimension, so we have only lateral forces actuating the car. According to Newton's second law, we have:

$$\vec{F}_f + \vec{F}_r = M_G \vec{a}_G \tag{4}$$

To simplify the model, forces and velocities on right and left tires are combined as a center wheel model so we don't have to differentiate between left and right (Shown in Figure 2).

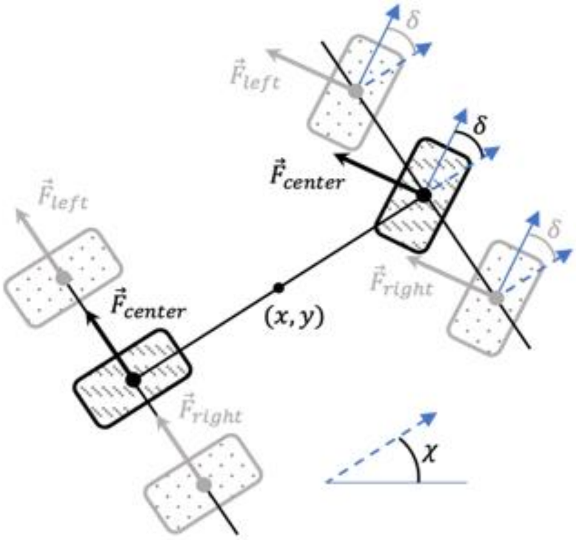

Figure 2. Two-wheel Model.

$$\vec{F}_{center} = \vec{F}_{left} + \vec{F}_{right} \tag{5}$$

This is established on the assumption that both tires are in the same condition. It's not hard to prove that total force and torque are the same, so this simplification is valid.

According to the fixed axis rotation law of a rigid body, we have this equation:

$$\sum_i (\vec{r}_i - \vec{r}_G) \times \vec{F}_i = \left(\sum_i m_i l_i^{\,2}\right)\chi'' \tag{6}$$

χ'' Is angular acceleration. $\vec{r}_i - \vec{r}_G$ Is the arm of force? $m_i l_i^{\,2}$ Is the moment of inertia, which is abbreviated toI_Z.

Then we have rotation motion for two-wheel model:

$$I_Z \chi'' = \pm a\|\vec{F}_f\| \pm b\|\vec{F}_r\| \tag{7}$$

2.2. Slip Angle and Lateral Force

Now we need a physical model for the front and rear tires. Since there is no breaking nor acceleration, the only force exerted is perpendicular to the tire. When the tire rubber is in contact with the road, the deformation increases due to the difference between velocity and direction of the tire, which caused a lateral force to balance the deformation [11]. And the lateral force provides the driving force needed to turn the car.

The generation of tire side forces (which exist only because of "friction" between the tire and the road) depends largely on the vertical load, but also on the tire width, size, and pressure, which is obviously related to the area of contact between tire tread and the road [12]. The forces generated at the tire-track interface are mainly generated by two mechanisms: adhesion and hysteresis. Adhesion is produced by the intermolecular bond between the tire rubber and the aggregate of the track surface.

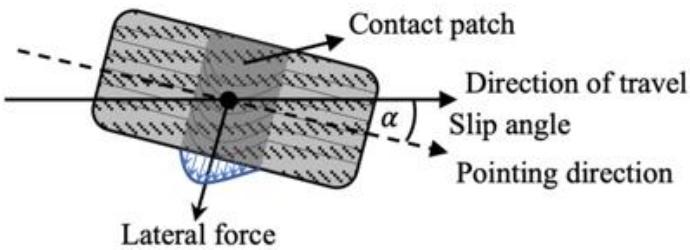

Figure 3. Tire-road Interaction.

General distribution of lateral force is revealed in Figure 3, and the total force is slightly behind the center of the contact patch. This creates a torque that tends to pull the wheel back to the straight position. In Figure 3, α is what we call slip angle, which is the angle between the velocity and orientation of the tire [13] basically, the slip angle is the main cause of lateral force.

For the velocities of tires, we have:

Velocity of the front tire:

$$\vec{v}_f = \vec{v}_G + a\chi'\vec{u}_2 \tag{8}$$

Velocity of the rear tire:

$$\vec{v}_r = \vec{v}_G - b\chi'\vec{u}_2 \tag{9}$$

$\vec{v}_G$ Is the velocity of the mass point, and χ' is the angular speed of the car. For the slip angles, we have:

Slip angle of the front tire:

$$\alpha_f = \delta - sin^{-1}\frac{\vec{v}_f \cdot \vec{u}_2}{\|\vec{v}_f\|} \tag{10}$$

Slip angle of the rear tire:

$$\alpha_r = -sin^{-1}\frac{\vec{v}_r \cdot \vec{u}_2}{\|\vec{v}_r\|} \tag{11}$$

On most occasions, lateral force reaches the peak when slip angle reaches around 6 degrees. Too much angle means sideslip and is out of our consideration in this model. General relationship between slip angle and lateral force is revealed in Figure 4 [14].

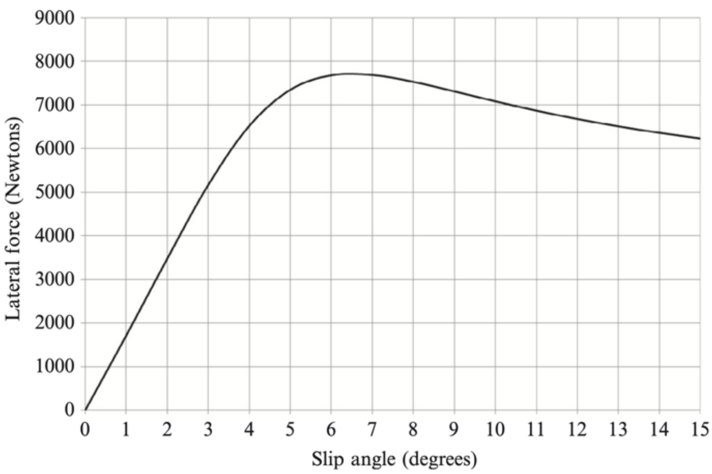

Figure 4. Lateral Force vs Slip Angle for a Typical Tire.

For the specific relationship between lateral force and the slip angle, we build following models:

$$F = C\tan\alpha f(\lambda) \tag{12}$$

$$f(\lambda) = \begin{cases} (2-\lambda)\lambda & \text{if } \lambda < 1 \\ 1 & \text{if } \lambda \geq 1 \end{cases} \tag{13}$$

$$\lambda = \left|\frac{\mu F_N}{2Ctan(\alpha)}\right| \tag{14}$$

C Is a constant remaining to be ensured? F_N Is the vertical force exerted on the tire, which is related to the mass of car? μ Is friction coefficient, determine by the quality of tire. When α is small. $\lambda \geq 1, f(\lambda) = 1$. $\tan\alpha \approx \alpha, F = C\alpha$. When α is big. $\lambda < 1$. $f(\lambda) = (2-\lambda)\lambda \approx 2\lambda$ (Compared with 2, λ is a far too small)$F = \mu F_N$. So, we made such simplification by separating equation (12) into two models:

$$F = \begin{cases} C\alpha & \text{linear model } (\alpha \text{ is small}, \lambda \geq 1) \\ \mu F_N & \text{nonlinear model } (\alpha \text{ is big}, \lambda < 1) \end{cases} \tag{15}$$

When α is small, the lateral force is linearly related to the slip angle. But when α is big, the lateral force is mainly determined by the vertical force [15].

2.3. Two Types of Turns

We experimented with two types of turns: lane change (16) and U-turn (17). That is achieved by specifying the function $\delta(t)$ (Figure 5) which indicates how the wheels are being turned by the driver.

Lane change:

$$\delta = \begin{cases} 0 & t < t_{start} \\ \delta_0 sin\left(\frac{t - t_{start}}{t_0}\right) & t_{start} \leq t < t_{start} + 2\pi t_0 \\ 0 & t_{start} + 2\pi t_0 \leq t \end{cases} \tag{16}$$

For some parameters in lane change, we have: $\delta_0 = \frac{\pi}{50}$ rad, $t_{start} = 0.1$ sec, $t_0 = 0.5$ sec, $t_{end} = t_{start} + 2\pi t_0$

Equation for U-turn is displayed in (17), and for some unknown parameters in U-turn, we have: $\delta_0 = \frac{\pi}{24}$ rad, $t_{start} = 0.1$ sec, $t_0 = 1$ sec, $t_{end} = 5.45$ sec.

The plots for both models are displayed in Figure 5.

U-turn:

$$\delta = \begin{cases} 0 & t < t_{start} \\ \frac{\delta_0}{2}\left[1 - cos\left(\frac{\pi(t - t_{start})}{t_0}\right)\right] & t_{start} \leq t \leq t_{start} + t_0 \\ \delta_0 & t_{start} + t_0 \leq t < t_{end} \\ \frac{\delta_0}{2}\left[1 - cos\left(\frac{\pi(t_{end} + t_0 - t)}{t_0}\right)\right] & t_{end} \leq t < t_{end} + t_0 \\ 0 & t_{end} + t_0 \leq t \end{cases} \tag{17}$$

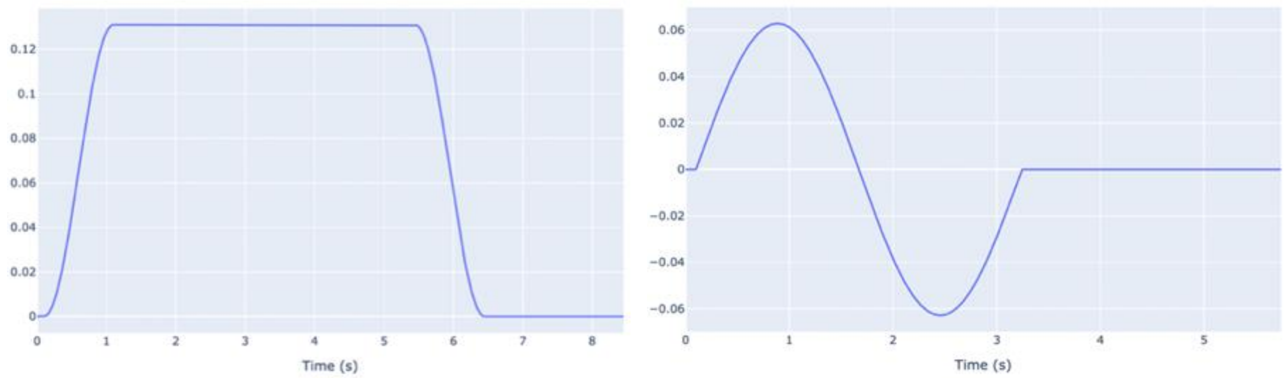

Figure 5. Function for U-turn (left) and Lane Change (right).

3. Result: Simulation of the Model

Two types of cars are tested for the model: BMW M3 and Chevrolet Cavalier. Their data is given in Table 1.

In this table,a,b corresponds to the distance between axels and mass point in schematic of the car. M3 is only a bit longer than Cavalier but its mass is more evenly distributed, and its total mass is larger, which contributes to its yaw moment of inertial. Generally frictional coefficient is determined by the quality of tire, so we assume that they are the same.

We compiled and built a series of models in Python using data above to facilitate our project research. In python, modules like Dataclass and NumPy are imported. We set classes for series like turn type and position type (front tire or rear tire). Then we define functions for important arguments like slip angle and force to bridge all parameters together. By using solve_ivp we can solve second order differential equations to get the results for the car motion. At last, we use Plotly to plot the results we want (like the relationship between slip angle and lateral force through time).

Table 1. Data for Cars.

Parameters	BMW M3	Chevrolet Cavalier
Front wheel (a)	1.36 m	0.98 m
Back wheel (b)	1.37 m	1.66 m
Mass (m)	1549 kg	1187 kg
Yaw moment of inertia(I)	2886 $kg \cdot m^2$	1928 $kg \cdot m^2$
Frictional coefficient (μ)	0.9	0.9
Front tire cornering stiffness (C_f)	194,000 $N/rad/$axle	58,000 $N/rad/$axle
Rear tire cornering stiffness (C_r)	240,000 $N/rad/$axle	58,000 $N/rad/$axle

3.1. Lateral Force vs Slip Angle

The relationship between slip angle and lateral force is a significant criterion of a car’s turning capability as well as drivability. Considering that both cars are similar in the trend of trajectory due to identical frictional coefficient, we plot the figure for BMW M3 as an example (Shown in Figure 6). As is revealed in the picture, linear model and nonlinear model are basically the same when α is small (below 1 degree). However, for big slip angle, lateral force tends to stabilize at around 6kN for the nonlinear model, while linear model just remains its trend. Generally, more force is generated on the rear tire.

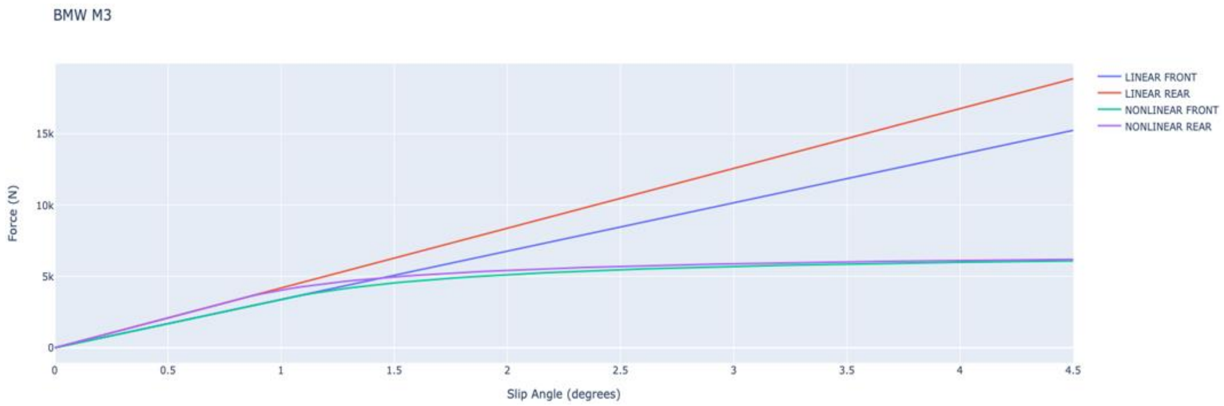

Figure 6. Lateral Force vs Slip Angle for BMW M3.

3.2. Trajectory of Travel

In our experimental assumption, we first let the two models under test enter sharp U-turn and use the known formula to build a model in Python and plot the movement track of the two cars. Considering that trails of either car in the same model are similar, we choose a few representative trajectories to analyze.

Figure 7 displays the trajectory of BMW M3 in a U-turn of linear model. The rotation angle rises steadily with time and the turning process moves smoothly, lasts for 5 seconds. The whole trajectory looks like U, and the radius is around 2 meters. It is very similar to the scenario when we turn around in the crossroads. In this condition, the steering wheel should remain the same during turning process, which means total torque and slip angle is fixed since it makes a uniform motion. This is basically the same condition for non-linear model if the speed remains low. Even if the speed rises, the only difference should be that the rotation radius may probably be bigger.

Figure 8 displays the trajectory of BMW non-linear sharp lane change. It is also a very common motion on the road. During this process the steering wheel keeps changing to correct the orientation of the vehicle, which means the lateral force and slip angle are no longer constant. The rotation angle reaches the highest at 1.7 seconds, which is 37 degrees. This is also the same point that angular velocity reaches the lowest and angular acceleration reaches the highest. Still the linear model for this shouldn't be much different, but the peak point of rotation angle should be a bit higher.

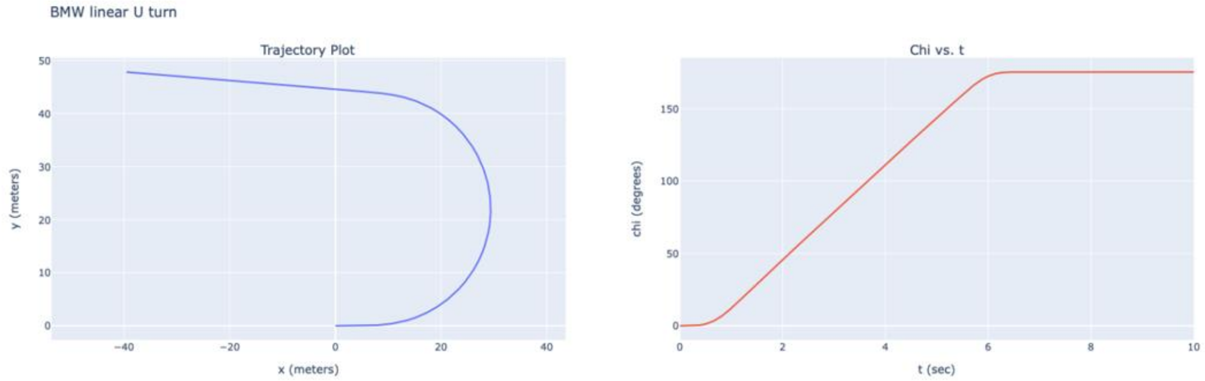

Figure 7. BMW Linear U-Turn.

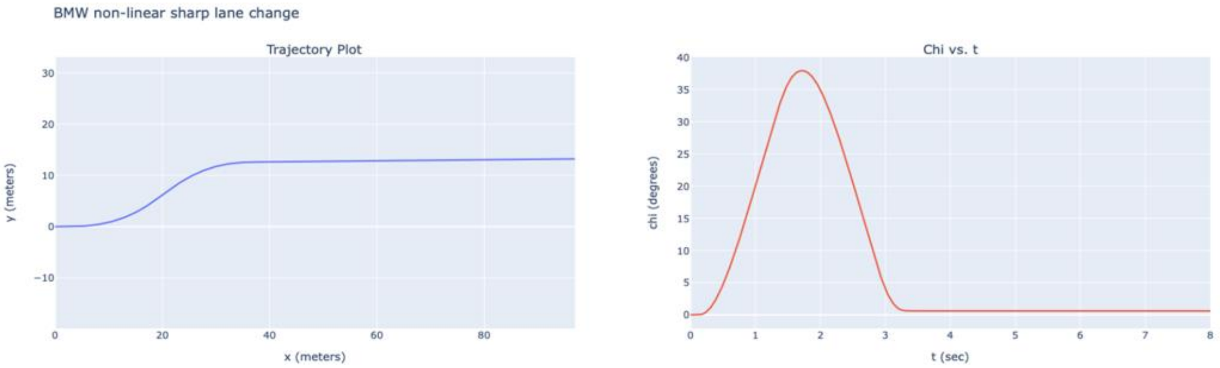

Figure 8. BMW Non-linear Sharp Lane Change.

We repeat the above plots and commentary of trajectories under non-linear force model for chevy, except for a sharper turn. Figure 9 shows Chevy's trajectory in a sharp turn. Total rotation angle is much lower than BMW U-turn and it shows a smaller rotation radius. Generally, its rotation velocity is lower than that of BMW linear U-turn. Since they share the same frictional coefficient and Chevy is in a lighter weight, the maximum of lateral force generated by Chevy is lower than that of BMW. Besides, Chevy is shorter in size, which means it has a lower yaw moment of inertia, less capable of sharp turning under the same lateral force. Which is why Chevy turns at a smaller angle under the same steering condition.

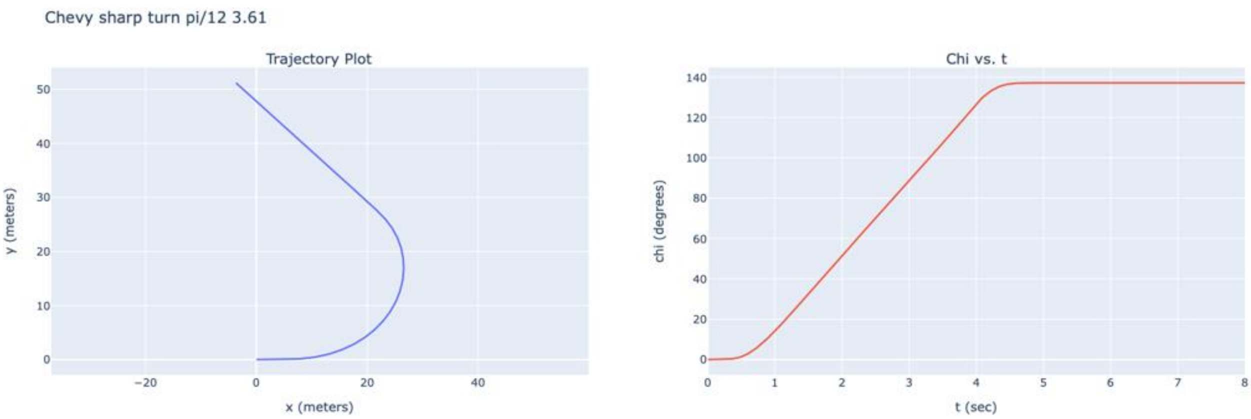

Figure 9. Trajectory (left) and Rotation Angle (right) of Chevy in a Sharp Turn

3.3. Motion of Sharp Non-Linear Turn

According to the two images below (Figure 10 and Figure 11), the force generated by the front wheels of BMW M3 model can reach more than 6000N when it makes a sharp turn. But the Cavalier's front wheels generate less than 6, 000N of force when he makes a sharp turn. Therefore, we believe that the M3's front tires and steering mechanism generate more friction to provide centripetal force and help steer during sharp turns.

We also note the contrast of the sideslip Angle on the right. We found from the data that the M3's front tires were able to produce a smaller sideslip Angle if two cars were going into a tight turn in the same situation. According to our experimental data and images, the sideslip Angle of BMW M3 during sharp turn is about 10 degrees but not more than 11 degrees, and the maximum Angle is about 2s. But the Cavalier's sideslip was greater than 12 degrees when he made a sharp turn, reaching his maximum sideslip around 2s. So, it's reasonable to assume that the M3 will be more controlled when steering. BMW M3 has a smaller sideslip Angle when it makes a sharp turn, and a smaller sideslip Angle will make the vehicle respond faster when it turns. This is what we usually experience as turning away from "pushing".

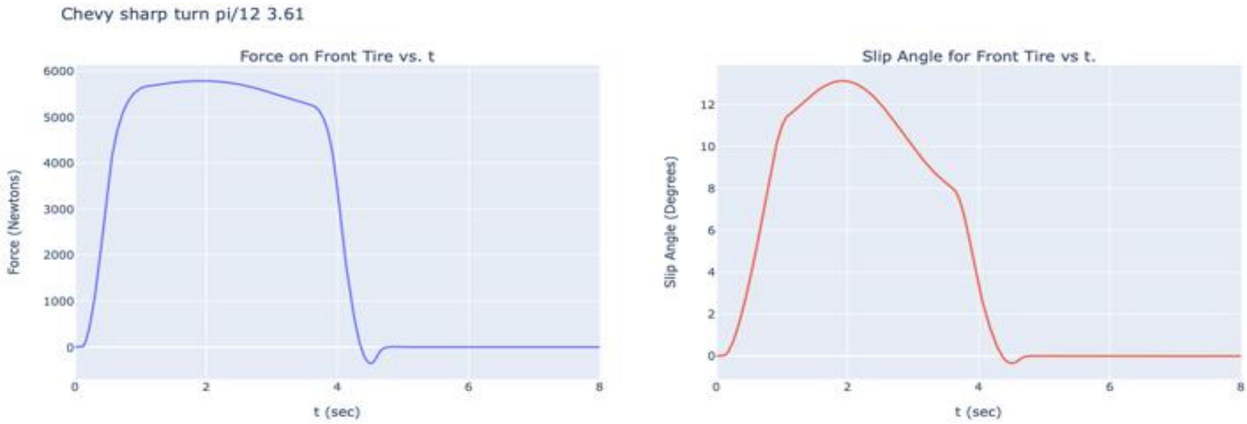

Figure 10. Chevrolet Cavalier Sharp Turn.

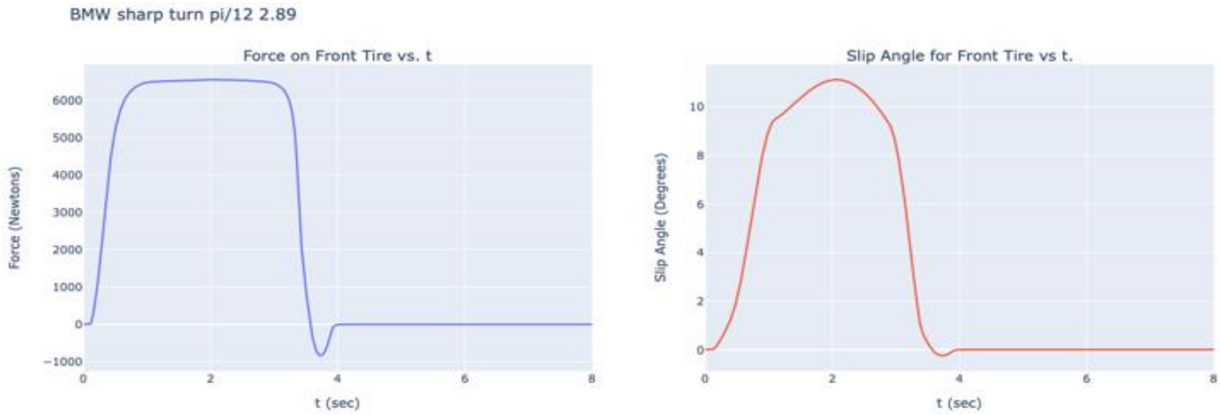

Figure 11. BMW Sharp Turn.

3.4. Summary of Results

Through the above model construction and data analysis, the BMW M3 and Chevrolet Cavalier data models are now in front of the reader. Our analysis concluded that the M3 had better grip and handling on tight turns than the Chevrolet Cavalier. First, through the analysis of the above data, we believe that the BMW M3 series is heavier, which is convenient to provide more downforce when turning, and thus can generate more friction to provide centripetal force when the friction coefficient is constant. Also, based on the front wheel steering stiffness shown in the data, it's not hard to see that the M3's front tire cornering stiffness is significantly higher than the Cavalier's. To some extent, the front wheel of BMW M3 will not easily deform when it enters the turning state at high speed. In fact, this is not conducive to the maneuverability of the vehicle. But the M3 turns much better than the Cavalier, mainly because the M3 has a smaller steering radius. So, we think the M3 has been designed with a lot of focus on handling during cornering, possibly using a different steering suspension than the Cavalier to make it more adaptable to cornering at high speed and low speed.

4. Discussion: Ackerman Geometry

In the previous model we assumed that two front wheels are in the same condition (parallel). As a matter of fact, modern cars are designed differently. For a car on road, when it turns, its tire follows a circular trajectory, and the outer wheel makes a bigger turn than the inner wheel [16]. Wheels are perpendicular to the radius defining the trajectory. When the four radiuses intersect at the same point inner side, we call it a 100% Ackerman. The angle between two front tires is what we call Ackerman angle (θ in Figure 12).

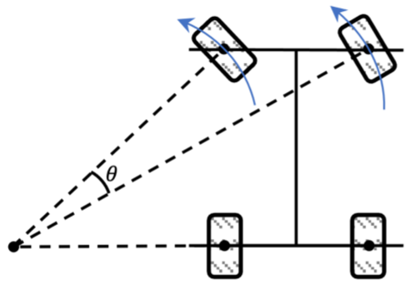

Figure 12. 100% Ackerman Geometry.

In this condition, the inner wheel turns slightly more than the outer wheel, four wheels travel around concentric circles. All four tires have pure rolling motion when its velocity approaches 0, which can make the turning process smoother and reduce tire wear [17]. Meanwhile, lateral forces tend to be equal, which is a preferred situation to keep driving steady.

4.1. Ackerman Application on Road Cars

In practice, cars are designed between 100% Ackerman and parallel condition (normally 40%~60%, displayed in Figure 13). As the speed of cars increases, slip angle rises, which creates lateral force (explained in 2.2). According to modern automobile steering theory, 100% Ackerman steering is not the best condition for a cornering car because it sacrifices the slip angle and lateral force to keep trajectory in a circular track. Normally a speeding car contains great momentum forward depending on mass and velocity. Lateral force is necessary if the momentum is to be changed to another direction. In Figure 12, the orientation of tire is in the same direction of its velocity, which means the slip angle equals to 0. That is impossible for a vehicle model in motion since we all know that centripetal force is the activation of circling motion. The faster a car travels, the more lateral force is required to keep its trajectory in a circular track, which means a bigger slip angle. 100% Ackerman geometry is only an ideal model [18].

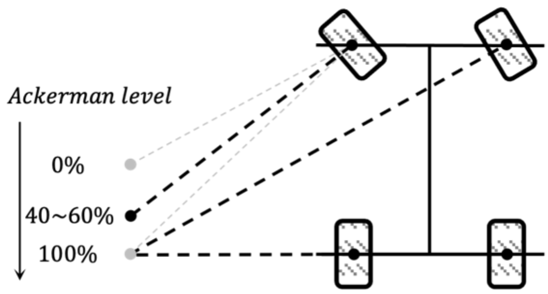

Figure 13. Ackerman Level.

To satisfy the requirements for both steering ability and reduction of tire wear, we make compromise for Ackerman as between 100% and 0% to get the best overall performance. Its specific

Ackerman level depends on the type of cars. For example, if it's a business car, the Ackerman level should be higher to cater to the feelings of passengers and drivers. If it's a sport car, the Ackerman level should be lower to get better steering ability in high speed.

4.2. Ackerman Application on Racing Cars

Normal Ackerman geometry may not be an appropriate design for racing cars (like F1). Ackerman's original intention is to make steering smooth and reduce tire wear. However, that is not the priority for an automobile race. On a racecourse, tire wear is inevitable, drivers put more emphasis on acceleration performance and controllability. For a Formula 1 racing car, Reverse Ackerman is a common design (displayed in Figure 14). When the car turns, the outer wheel turns more than the inner, contrary to the formal design [19]. The perpendicular lines of two front tires intersect outside the cornering center.

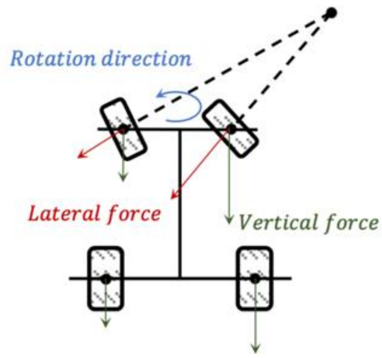

Figure 14. Reverse Ackerman on F1.

In the kinetic model we built in 2.1, the vehicle is supposed to travel at a very low speed while cornering so that the mechanical property differences between two front tires are negligible. But that principle is broken when the vehicle makes fast turns. A high-speed cornering process causes a shift of weight, which means the outer wheel takes more pressure than the inner wheel. In Figure 14, the vehicle is turning left so more vertical force is exerted on the right tire than the left. The faster the speed, the bigger the differences between.

Still, we can see the steering process as a circling motion. High-speed rotation requires large centripetal force, which means more lateral force is needed to maintain the trajectory in track. If lateral force isn't enough to support cornering, the vehicle may experience sideslip and even slip out of racecourse. In the tire-road interaction model we built in 2.2, lateral force shows positive correlation with slip angle and vertical force. Briefly, we need both slip angle and vertical force to be as large as possible to ensure enough lateral force [20].

In the previous mode, lateral force reaches the peak when slip angle is about 6.5 degrees. After that, the car begins to slip and loses lateral force. Research has it that the vertex moves when a car takes more load. As is shown in Figure 15, when a larger vertical load is exerted on a car, more slip angle is required to get the largest lateral force [21]. Meanwhile, the maximum lateral force goes up.

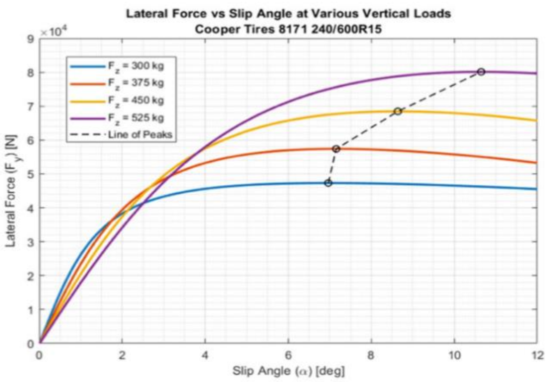

Figure 15. Lateral Force vs Slip Angle at Various Vertical Loads.

In Formula 1, engineers put much emphasis on achieving maximum dynamical performance. Since the outer wheel takes more load than the inner while steering, it requires more slip angle to achieve maximum lateral force. This is basically why the outer wheel turns more.

Another reasonable and brief explanation is based on geometric principles. Previously we saw the steering process as a circling motion. The radius of circle and the direction of tires are perpendicular. This rule is broken when the cornering speed goes up. As is shown in Figure 16, the corner center moves forward as speed rises [22].

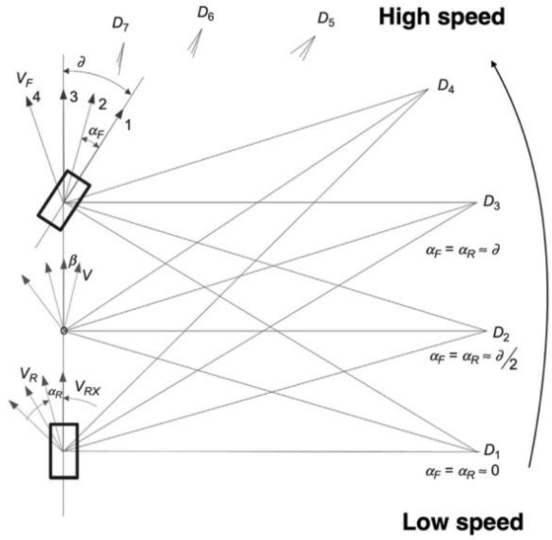

Figure 16. Migration of Corner Center with Speed

The main reason of this is that slip angle grows. In fact, turning radius is perpendicular to the velocity of tires. During a high-speed steering process, the velocity of tires deviates from the direction of tires. Basically, the tire is more slipping than rolling. Migration of corner center is detrimental to vehicle because it causes sideslip and understeer. To ensure its steering ability, we need to turn the wheel at a larger angle to overcome the effect of slip angle [23].

It is worth mentioning that steering mechanism of an F1 is strictly adjusted according to the racecourse. Reverse Ackerman may not be applied on a racecourse with too many tight corners. There are many other ways to generate large lateral force. Car spoiler is a common design on an F 1, which may not only reduce the impact of wake flow but also generate vertical force on it [24]. Tires on an F1 are specially made with large frictional coefficient. They are frequently changed during a race to maintain its best performance.

5. Conclusion

In the above paper, we discuss and study the problems of Ackermann Angle and slip Angle and lateral forces related to The Ackermann steering mechanism. The slip Angle and lateral force are analyzed and studied, and the relationship between them and Ackermann steering geometry is obtained. In our studies on specific models of the BMW M3 and Chevrolet Cavalier, we found that slip angles and side forces had an impact on the efficiency of the Ackermann steering mechanism. Under the premise of the same type of Ackermann steering mechanism, the larger the lateral force (or slip angle) the vehicle is subjected to during steering, the more sluggish the vehicle's steering response will be. Therefore, we need to transform the Ackermann steering structure to make the steering more responsive. The purpose of our research on Ackermann steering is to prevent the tire from slipping along the curve. The Ackermann steering mechanism was redesigned with this geometric solution in mind, which was to arrange the axes of all wheels as a radius of a circle with a common center point. Since the rear wheel is fixed, this center point must be on a line extending from the rear axle. Intersecting the front wheel axis on this line also requires the inside front wheel to rotate at a greater angle when turning than the outside front wheel. Instead of the previous "turntable" steering, the two front wheels revolve around a common pivot, and each wheel gets its own pivot, close to its own hub. Although more complex, this layout enhances controllability by avoiding the need to impose a large amount of pavement variation input at the end of a long lever arm and greatly reducing the back-and-forth travel of the steering wheel. The practical significance of this device is to make the vehicle respond faster and faster during the steering process, thus providing the driver with a better handling feel. In a race, properly adjusting the Ackermann steering mechanism will allow the car to steer more flexibly, which will also allow the driver to achieve better results.

References

[1] Li Chen, Fu You-bing, Zhang Jun-wei, Wan Fang, Zuo Xia (2018) Research on Modified Ackerman Principle Corner Considering Tire Side Properties.

[2] Shahram Azadi, Reza Kazemi, Hamidreza Rezaei Nedamani (2021), Vehicle Dynamics and Control, chapter 7, pp 261-317.

[3] SIMIONESCU P A, BEALE D. Optimum Synthesis of the Four-bar Function Generator in its Symmetric Embodiment: The Ackermann Steering Linkage [J]. Mechanism and Machine Theory, 2002, 37 (12): 1487-1504.

[4] Joop P. Pauwelussen. Vehicle Dynamics. ScienceDirect: Essentials of Vehicle Dynamics, 2015.

[5] Hans B. Pacejka. Tire and Vehicle Dynamics. Sciencedirect, 2012.

[6] Wen Youdong. Steering trapezoidal optimization based on Adamsa/Car. Beijing Automotive: 2016; (2): 37-39.

[7] Yu Zhisheng. Vehicle theory. 5th ed. Beijing: Machinery Industry Press, 2009.

[8] Wu D L. Influencing factors of tire wear. China Rubber Industry, 1991, 38(4): 237~244.

[9] Guo Konghui. Vehicle handling dynamics. 3rd ed. Nanjing: Jiangsu Science and Technology Press, 2011.

[10] Jazar R N. Vehicle Dynamics [M]. Springer Science Business Media New York, 2008.

[11] Russell Hibbeler. Engineering Mechanics: Dynamics, 14th Edition.

[12] Smain A T B, Faiza H, Nadia S. Experimentations on the adaptive sliding mode control for a trajectory tracking applied on a bi-steerable car. International Journal of Vehicle Design, 2015, 69(1): 285~303.

[13] Yu Zhisheng. Automotive theory (5th edition) [M]. Beijing: China Machine Press, 2009.

[14] Masato Abe. Vehicle Handling Dynamics. Sciencedirect: Theory and Application, Second Edition, 2015.

[15] Zhao Y Y. Finite element analysis of tread wear performance of radial tire [Master Thesis]. Qingdao: Qingdao University of Science and Technology, 2015.

[16] Zhao Jingchuan (1995) The Relationship between Inside and outside steering Wheel Angles of Automobile.

[17] Lu Fengyi, Zhang Wenjie, Xu Gening (1997) Discussion on abnormal Wear of automobile front Wheel tire.

[18] Rodrigo de Oliveira Santos (2014) The Absolute Guide to the Racing Types, part 1, Lateral Force.

[19] Wang Xiaofeng. Automotive chassis design [M]. Beijing: Tsinghua University Press, 2010.

[20] Peng Mo, Zhou Liangsheng, Yue Jingtao, et al. multi-axle vehicle [M]. Beijing: China Machine Press, 2014.

[21] Dale Thompson. Ackerman steering geometry. Racing Car Technology.

[22] Jonathan Vogel. Tech Explained: Ackerman Steering Geometry. Racecar engineering.

[23] James Balkwill. Performance Vehicle Dynamics. Sciencedirect: Engineering and Applications, 2018.

[24] Qin Gang, Sun Ying. Research on steer angle for double front axles steering systems considering with slip angle [J]. Journal of Huazhong University of Science and Technology (Natural Science Edition), 2011, 39(12): 18-21.