Simulation and Discussion of Vehicle U-turn Trajectory based on Four-wheel Model with Weight Transfer

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Abstract. A Formula Student race car is an open-wheel race car designed and manufactured by university students, and it is a good engineering practice. The Formula Student race cars are always raced on small tracks comprised of different corners including S-turn and U-turns. Turning performance is one of the key criteria to judge whether the car has a good overall handling performance or not. Therefore, it is necessary to study the turning performance of vehicles. This paper will mainly focus on the U-turn trajectory of a vehicle when the effect of weight transfer is counted, and the difference between the four-wheel model and the two-wheel model. Acceleration and breaking during the cornering will not be discussed in this paper to simplify the research. In other words, this paper will mainly discuss the condition of steady-state U-turn. A more realistic trajectory compared with the trajectory obtained by using a two-wheel model will be generated in this paper.

Keywords: Weight transfer; four-wheel model; tilting moment; lateral force; turning radius.

1. Introduction

We have consulted some references to provide evidence for our research focusing on the four-wheel model. In Ref. [1], the authors showed how lateral force changes with various slip angles when the car is turning. He also illustrated that lateral force distribution at tire-road contact area during a turning maneuver. These pieces of evidence demonstrate the rationality of the non-linear force model which is later frequently mentioned throughout this paper. Recently, Ref. [2] has found that variability in the data is affected by some factors which rely on the capabilities of the machine and service provider of indoor testing, and the sources of variability in the data and methods to eliminate or compensate for the variability was generated. It makes indoor testing a feasible way to verify the lateral force model of tires used for simulation by researchers. Not long ago, it was proven that the lateral force produced by a tire has a maximum value, and the total lateral force generated by an axle decreases when weight transfer occurs in Ref. [3]. In Ref. [4], the author solved the magnitude of weight transfer which is based on forces acting on two tires and gravity of one axle, and the moment equilibrium equation. This deduction process provided a feasible and relatively accurate method to calculate the value of weight transfer. The calculation formula of the value of weight transfer was also generated in Ref. [5]. In addition, the curve of maximum lateral force changing with the vertical load of a tire was produced in this reference. This curve is the main basis for our calculation of lateral force acting on the car. Recent research shows that. Ref. [6] displayed a figure which illustrates the type and relative value of forces acting on the car when it is turning intuitively. It contributes to our clear understanding of the origin and result of weight transfer. It also helps people unfamiliar with this phenomenon to realize the existence of the concept of weight transfer. This paper also shows the nonlinear relationship between the lateral force borne by the tire and the vertical load acting on the tire, which provides a theoretical basis for our research. This reflects the rationality of our research. It was generated that the more load on a tire, the more grip it has in Ref. [7]. Nevertheless, the relationship is non-linear. As it has more load put on it, it doesn't gain the grip as fast as the tire's workload increases. It means that the total lateral force produced by an axle when weight transfer occurs is less than when weight transfer does not occur. In Ref. [8], the author applied a tire force model which is the basis of our research to estimate the lateral tire force successfully. It proves the rationality of force analysis used in this paper. Not long ago, it was proven that the 'Magic Formula' tire model which was generated by Pacejka is well established [9]. We believe that researchers can replace the non-linear force model with the 'Magic Formula' to make the result more accurate and reliable for shorty's vehicle development. As for Ref. [10], it specifically described all parameters in vehicle dynamics and calculation of vehicle sideslip angle as well as lateral tire force, and it has also described the 4 different force models to help people better understand the calculation including the Pacejka model that we probably use in future research.

In the most common simplified estimation, the two-wheel model was always used to simulate the U-turn trajectory of a vehicle where weight transfer is not considered a factor. However, in reality, weight transfer happens when a car is turning. That means the two-wheel model based on the above ideal conditions is not accurate enough to precisely perform the realities' driving scenario. This leads us to overestimate the turning performance of the race car. It may cause the race car to rush out of the track. So, a more practical model is required in this research. The focus of this paper will be the influence of weight transfer on the turning performance of an automobile. This paper will involve some basic knowledge about the moment equilibrium equation to calculate weight transfer which is a critical factor contributing to the turning performance. Also, the non-linear lateral force model of a tire will be used to give a relatively accurate result. In this paper, the four-wheel model will be applied to estimate the capability of a vehicle to complete a U-turn more accurately. For the events in the FSAE competition such as the skip-pad event and autocross event, U-turn plays role in testing the vehicle's performance [11]. The data for the analysis comes from the design data of the Nanyang Venture Formula (NVF) car, which is a single-seat race car for the Formula Student competition, with aero parts uninstalled at this moment. However, we still assume there is no effect from the Ackermann angle, the intervention of the suspension will not be discussed. The numeric solver in python is used as well, and we find that the turning radius of this new model is bigger than the results shown in using the two-wheel model. That means the two-wheel model overestimated the turning performance of the car. We considered the weight transfer that would occur in the real situation. So, we believe that we have obtained a more accurate model.

2. Method

The basis of this research is that vertical load generates the lateral force of a tire. Vertical load, posted by Rose Bentley on the internet of his understanding of this physical concept, is defined as 'load that is put on the tire from the car's aerodynamics. He also said that there are more downwards load that caused by aerodynamics, and this leads to more grips generated. Here, the word 'grip' is a substitute for the force at the contact between the tire and the track [3]. In addition, it is also referring to weight transfer. This is a transition phase of load on the rear to front of the car when we apply a break during cornering or accelerating. Also, more importantly, the vertical load has a great influence on the lateral force. Others like, tire size, width and pressure are also important parameters for the contact area between the tire and the track.

The definition of lateral force is the rise in elastic force on the tire due to the lateral deflection, and this force acts in the center of the contact patch between the ground and tire [3]. Increasing in lateral force would cause a simultaneous increase in centrifugal force, which is directed outward to counter the centripetal force oriented inwards, and this would result in inelastic deformation and deflection on the tire, and hence, a larger slip angle. Moreover, a tire usually has three different states in turning, which are elastic, transitional, and frictional, which respectively refer to no sliding, some sliding, and complete sliding. The maximum lateral force is reached in the middle of the transition from transitional state to frictional state.

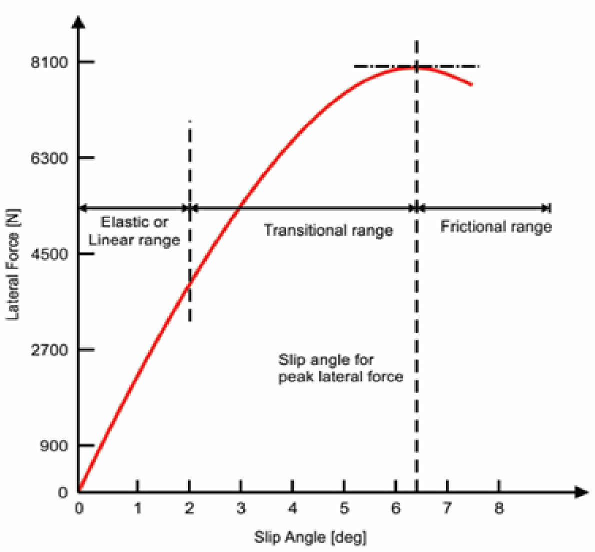

Figure 1. Lateral force VS. Slip angle for a normal load of 8000 N. (Santos, 2014, Fig 4).

This peak force is also called the limit of adhesion. The relationship between lateral force and the slip angle in different tire situations is shown in Fig.1 [3].

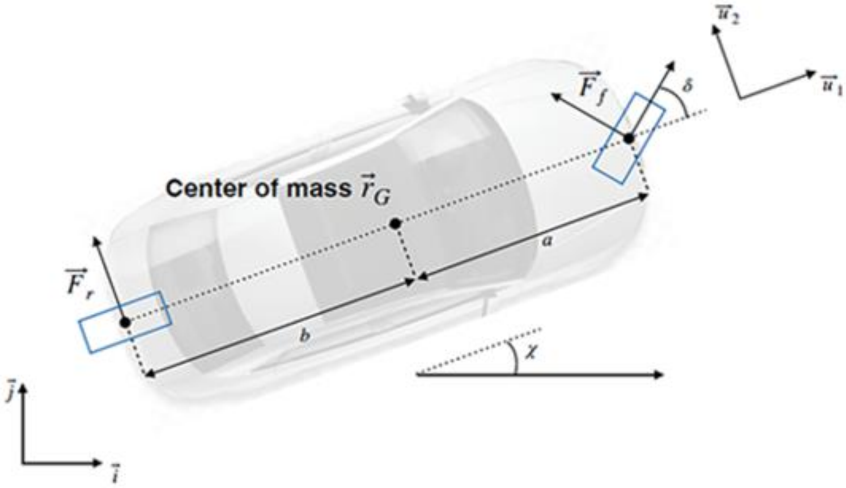

Figure 2. Two-wheel model.

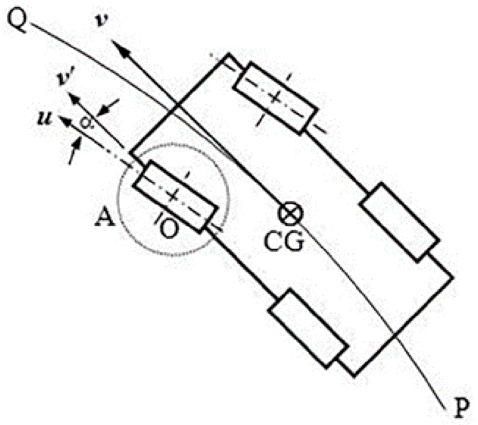

Figure 3. Vehicle under cornering maneuver with slip angle.

In Fig.2, the two-wheel model without weight transfer is displayed. Compared with the former, what is displayed in Fig.3 is a four-wheel model used in this paper, where CG is the assumed center of gravity, and v is the direction of a vehicle traveling. The differences in results originating from these two models will be discussed.

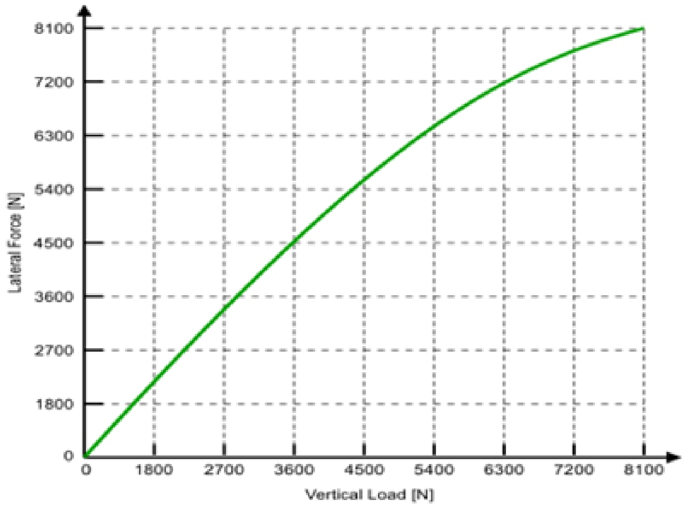

Figure 4. Lateral force vs vertical load for a single slip angle value (Santos, 2014, Fig 8).

From analyzing Fig.4, it is possible to gain an essential conclusion. Under centripetal acceleration and four-wheel model analysis, there will be a weight transfer on the axle from the inside tire to the outside one. Although the overall vertical load on the axle will remain the same, the total lateral force provided by the axle will be smaller than if no weight transfer occurs [3]. This conclusion is crucial for this research. In this paper, we will utilize it to obtain a rational U-turn trajectory of a race car.

Next, the specific research methods will be explained. The vertical load of a car at a steady state for the front tire and rear tire can be known by knowing the weight distribution of the car. Based on that, we can know the vertical loads for the front tire and rear tire without the effect of weight transfer. In the two-wheel model, the car is moving in a 2D plane. So vertical forces and vertical torques need to sum up to 0.

$$mg=F_{Nf}+F_{Nr} \tag{1}$$

$$af_{Nf}-Bf_{Nr}=0 \tag{2}$$

According to (1) and (2), the vertical loads on the front and rear wheels and the weight distribution of the front and rear wheels can be solved. Fig.3 illustrates the non-linear behavior of a typical tire used with Formula SAE racing cars. This figure provides the relationship between vertical load on a tire and the maximum lateral force generated by a tire.

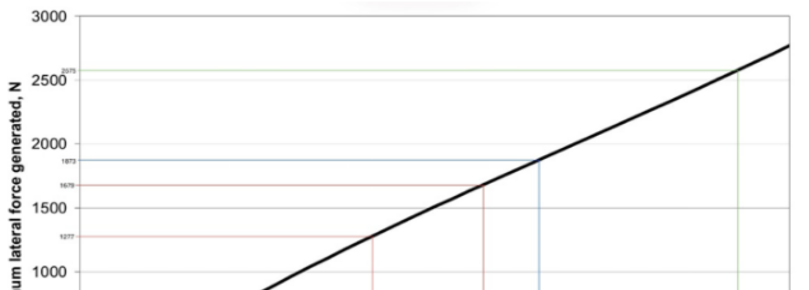

Figure 5. Non-linear behavior of a typical Formula
SAE tire, max lateral force produced for a vertical load.

According to Fig.5, the maximum lateral force provided by each of the front tires and rear tires is obtained. In the two-wheel simplified model, it is easy to tell that the maximum lateral force provided by the front axle and rear axle are two times the maximum lateral force provided by a front tire and a rear tire. In practical terms, the car in the circular motion experiences a centripetal acceleration. In the imaginary equilibrium state, there is a centrifugal acceleration, and the car experience a centrifugal force. We let the car travel at a linear speed of 10m/s, assuming at the corner the car experiences a lateral acceleration of 1.2g, which is (1.2mg) N, we get an estimated corner radius which is around 8.495m. After the above calculation, that radius will be the target of the steering parameter for the two-wheel model to be achieved. The centrifugal force produces a tilting moment on the car, and the magnitude of the moment equals the magnitude of centrifugal force times the vertical distance between the height of the center of mass on the corresponding axle and the ground. That moment creates the load difference between the vertical load between the inner side tire and the outer side tire, and the load difference for each side due to the effect of the tilting moment can be obtained. As the result, the vertical load for the inner side and outer side tire can be obtained by adding or subtracting the load change to the vertical load in steady-state.

$$WT = 1.2mg\ h_{CG}/t \tag{3}$$

WT represents the weight transfer for an axle, h_{CG} is the height of the center of gravity and t is the track width. In the calculation, we set out the moment equilibrium equation at the midpoint of the contact line between the tire and the ground on one side of the front axle and the rear axle respectively [4]. We find there is a total of 470*2=940N weight transfer at the front axle, and a total of 717*2=1434N weight transfer at the rear axle (3). According to the graph obtained from the experiment, we know the maximum lateral force provided by the tire at each axle. Then we combine this essential outcome with some motion parameters which was used in the past research, and we found the loss in grip at the corner for the front axle is 80N, and the loss for the rear axle is 80N. The trajectory of the car in a specified U-turn is then generated with help of a numerical solver.

Fig.6 below briefly illustrates the steps we applied to approach the final result. We first calculated the lateral acceleration and then derive the centrifugal force acting in the opposite direction of the centripetal force. After the tilt moment calculation, we combine the numerical value with the help of the suspension system, we get the weight transfer on the axle from the inside tire to the outside one under the centripetal acceleration and four-wheel model's precondition. Then, we blend in the concept of lambda which is contained in the nonlinear force model to get the final force result. Finally, this analytical result would then be done by observing the difference between the result and the two-wheel model.

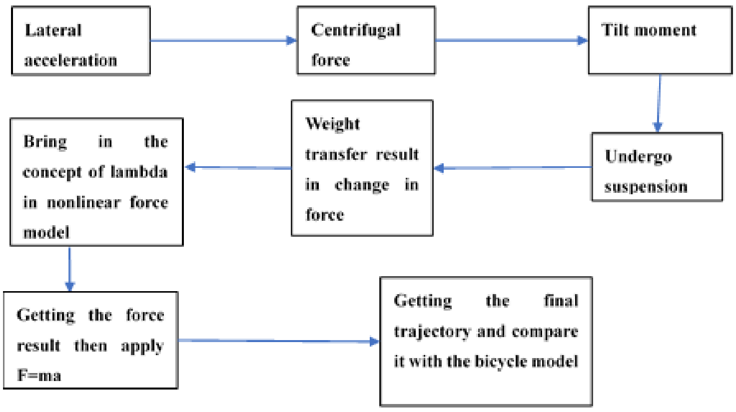

Figure 6. Complete flow chart of the Method section.

The weight of a Formula Student race car is 400kg. We assumed the weight distribution of the front axle and the rear axle of the car is 42% and 58%. So, the lateral forces of the front and rear axles can be obtained according to the following calculation process:

$$\text{Front axle: } 0.42\times m\times g = 0.42\times 400\times 9.81 = 1648.08\text{N} \tag{4}$$

$$\text{Rear axle: } 0.58\times m\times g = 0.58\times 400\times 9.81 = 2275.92\text{N} \tag{5}$$

With calculating both front and rear axle force distribution, the corresponding force distributed respectively on two of the fronts and two of the rear tires would then be:

Front tire:

$$F_{zf} \times 0.5 = 824.04\text{N} \tag{6}$$

Rear tire:

$$F_{zr} \times 0.5 = 1137.96\text{N} \tag{7}$$

3. Result

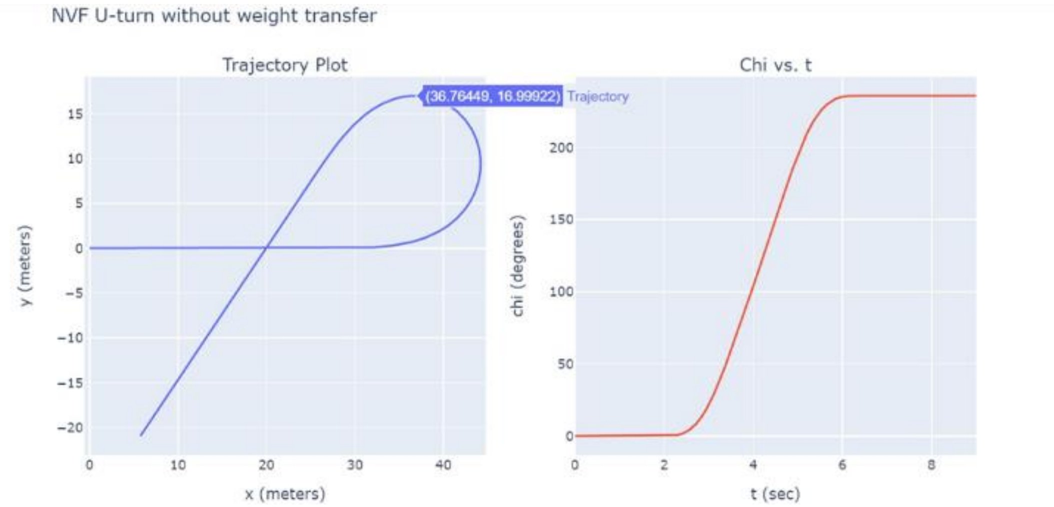

Figure 7. Trajectory diagram and time (t) VS car angular position (chi) diagram obtained from the two-wheel car dynamic model.

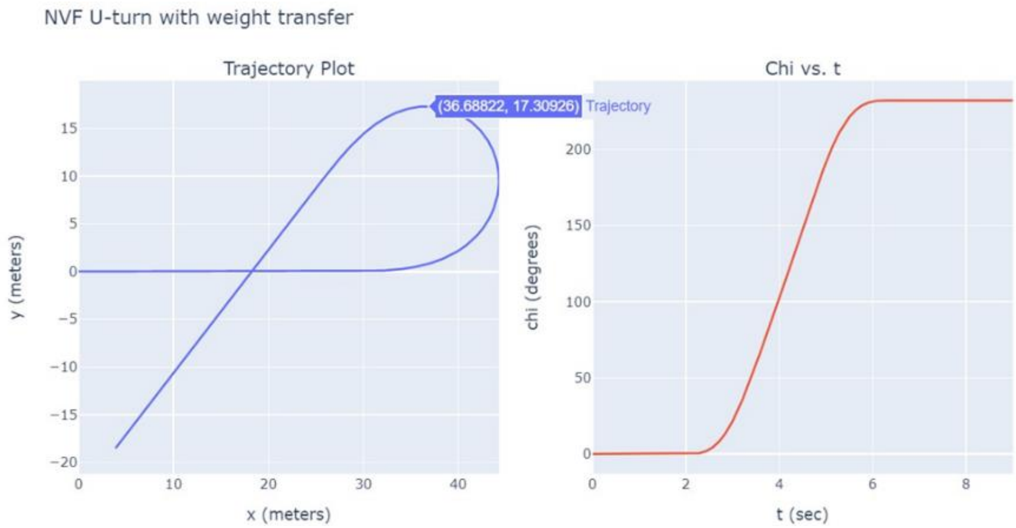

Figure 8. Trajectory diagram and time (t) VS car angular position (chi) diagram obtained from the four-wheel car dynamic model with weight transfer effect.

From Fig.7 and 8, the turning radius for the four-wheel model with weight transfer (17.309m) is larger than the result using the two-wheel model (16.999m) which does not count in the weight transfer. We find we overestimate the performance of the car if we do not count on the effect of the weight transfer.

From the chi vs t diagrams in Fig.9 and 10, we can tell that the gradient is higher in the two-wheel model. The actual meaning of the gradient for chi vs diagram is the angular speed of the car related to the origin in an inertial frame. With the same steering input, the two-wheel modeled car turns for a larger angle.

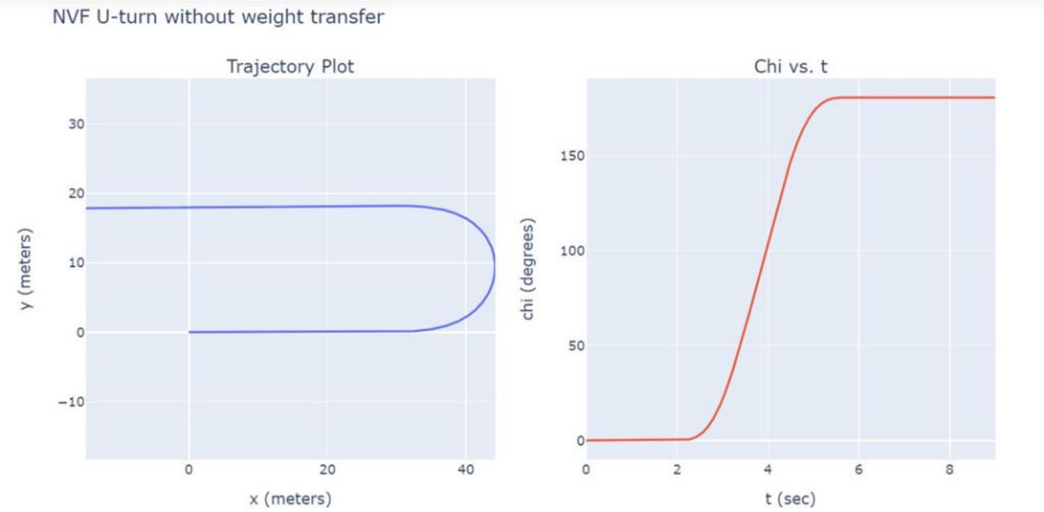

Figure 9. 180-degree U-turn Trajectory diagram and time (t) VS car angular position (chi) diagram obtained from the two-wheel car dynamic model.

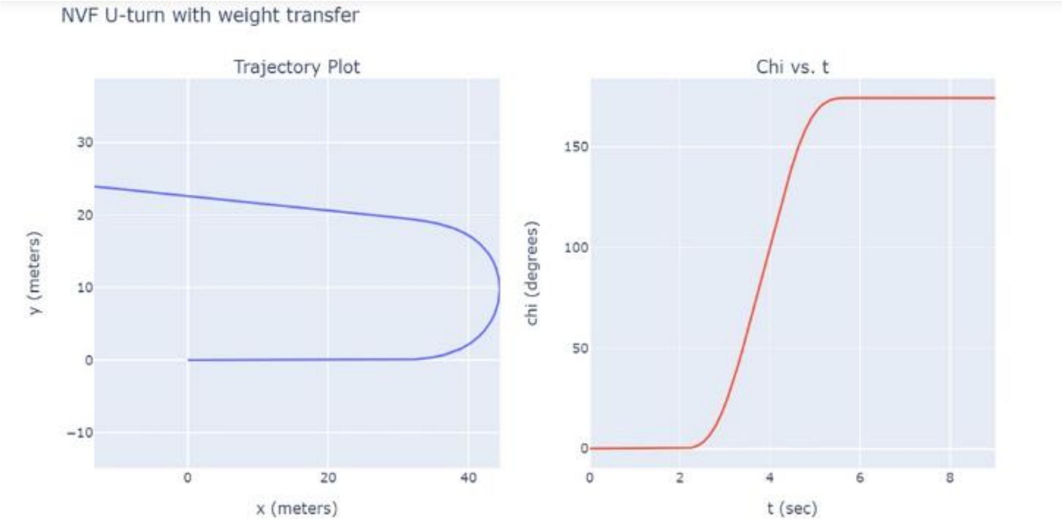

Figure 10. 180-degree U-turn Trajectory diagram and time (t) VS car angular position (chi) diagram obtained from the four-wheel car dynamic model with weight transfer effect.

In case we reduce the time of the steering input to make the two-wheel modeled car complete the 180-degree U-turn, from the comparison of Fig.9 and 10, we can see that the four-wheel modeled car with the effect of weight transfer cannot finish this specific U-turn as the finishing angle did not reach 180°.

4. Discussion

The results shown above detail explain the weight transfer reduces turning performance as the turning radius becomes larger. That is reasonable since the extra grip gained by the wheel on the outer side is less than the grip loss at the inner wheel and the vertical load on the tire is not linear to the maximum lateral force the tire can provide. Therefore, the total centripetal force is less in the four-wheel model with a weight transfer effect compared with the two-wheel model. That means the result of this research is more credible and realistic than the research which simplified a car into a two-wheel model. However, the model created in our model may still deviate from the actual situation due to the assumptions we made. For example, we assume the chassis is a rigid body and we neglect the effect of suspension as the roll stiffness is quite high. For a real race car, the suspension plays a role in influencing the weight transfer, in terms of the time of completing the total weight transfer. In a real case, the total process of weight transfer takes time with the effect of suspension, which means at the beginning of the cornering, the performance of the car is slightly better than the prediction from our model because the weight transfer does not intervene fully and the turning radius will be smaller. Other factors can influence the effect of weight transfer. One of these factors is the total mass of the car. A car with less mass is going to suffer less loss of the lateral grip caused by weight transfer because the Vertical Load VS Maximum Lateral Force property shows when the vertical load is relatively small, the relationship between vertical force and maximum lateral force tends to the linear relationship.

5. Conclusion

We have obtained the U-turn trajectory of a car based on weight transfer and the four-wheel model. We have also shown the difference between the two-wheel and the four-wheel model by using the non-linear force model. The method of python programming has been used in this research. A more realistic trajectory compared with the trajectory obtained by using a two-wheel model has been generated in this paper. Acceleration and breaking during the turning have not been discussed in this paper to simplify the research. However, in the real situation breaking or acceleration sometimes occurs with turning simultaneously when the driving technics such as trailing break are used.

For further possible research in the future, the condition of acceleration and braking could be considered, and the effect of suspension could also be added. Our current research is based on the non-linear lateral force model which is practical and feasible, but at this moment we also assumed that the camber and longitudinal slip does not influence the tire performance during the cornering. Thus, probably the tire force model with more parameters could be utilized for more accurate tire force for further research. The Magic Formula proposed by Pacejka, for example, could be a rational choice for the tire model to help researchers to obtain the U-turn trajectory precisely [10].

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