Applied Research Analysis of Game Theory and Markov chain

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Abstract. Since probability theory was put forward, it has been applied in all aspects of life and many science and technology. A good use of probability thinking can make a more rational, wise, and correct judgment and improve the probability of winning. Among them, game theory and Markov chain are the most influential. They have become one of the important theoretical foundations of various industries and fields, and gradually mathematize various fields, playing a vital role in their applied research field, promoting the development of society and science and technology. The prisoner's dilemma model of game theory was analyzed in various situations and explained how to apply it to economics, how to affect the social economy, and how to solve some social economic problems. By analyzing and utilizing the integrability, constant return, periodicity, and ergodicity of Markov chain, it is applied to signal transmission, traffic management, and bad debts of banks. Markov chain is used to model various transmission processes to facilitate the application field.

Keywords: Probability theory, game theory, Markov chain.

1. Introduction

Probability theory is now one of the most important parts of mathematics. It not only has a systematic theoretical development but also has become a practical analysis tool and an indispensable theoretical basis in many fields. Meanwhile, It plays a central role in physics, chemistry, economics, biology, information theory, and many other fields of study and effectively solves related problems [1]. Therefore, Probability theory is arguably the most widely used branch of mathematics.

In probability theory, the most representative and extremely concerned is the game theory and Markov chain. Markov chains are a really ordinary and comparatively easy statistical stochastic process that has been applied to a lot of different fields, and it is also a typical approach for probabilistic modeling and data analysis. Game theory is a very competitive and researched theory. At the same time, it is also a new branch of modern mathematics and has made necessary influence and improvements in many fields.

In order to study the application of probability theory, this paper introduces the elements, characteristics, theories, and models of the game theory and Markov chain to analyze and realize the applied research in economics and information theory respectively and apply them to solve the problems.

2. Methodology develop

2.1. Game theory

Game theory may be defined as the study of certain unions or organizations only applying the supplied information under beforehand designed conditions and rules to respectively weigh the advantages and disadvantages and make a rational choice that interacts with the individual results [2]. The main aim of studying Game theory is to demonstrably analyze how the results are affected when each individual in the game according to their own strategy decides what strategy can be adopted to achieve the optimal solution or equilibrium under certain circumstances.

The basic elements of a complete game include the player, strategy space, the order of play, and information. A player may be stated as a participator who is capable of thinking independently, making decisions, executing, and ultimately taking the consequences of the game. As long as an individual or a group of people can meet the above requirements, they can be regarded as a player. In general, the more players have, the outcome of the combination becomes variable, and the less predictable outcome of the game becomes. The strategy space is the set of options that the player has in the game. Each strategy corresponds to a result, so choosing a suitable strategy is indispensable. This also means that if the strategy space is larger, the game will be more complex. The order of play means the order in which the players make their choices which is a non-negligible factor, it will directly change the result. Although the players and the strategy space are the same, if the order of play becomes varying, the result will be completely different from the previous one. Before making a decision, the player needs to analyze and guess the opponent's strategy with the given information, and take actions that maximize its own profits after weighing the gains and losses. It's necessary to keep your strategy secret and prevent opponents from taking countermeasures and precautions in advance [3].

2.2. Markov Chain

2.2.1 Introduction

Markov chains are a type of statistical stochastic process that is widely used and relatively simple, and they have been used in a wide range of applications, from text generation to financial modeling. Markov chains are conceptually very intuitive and easy to implement, as they do not require the use of any advanced mathematical concepts, and are a classical approach to probabilistic modeling and data analysis.

2.2.2 Overview and features

Discrete-time Markov chain The Markov chain, named after Russian mathematician Andrei Markov, is a stochastic process that transitions from one state to another in state space. The process necessitates the property of "memorylessness": the probability distribution of the next state can only be determined by the current state, and events in the time series preceding it are irrelevant. This particular type of "memorylessness" is known as the Markov property. To study Markov chains, it is necessary to understand the Markov property. A stochastic process is Markovian if, Given the current state, it is conditionally independent of the previous state. (i.e., the historical path of the process). Markov chains have many applications as statistical models of real processes.

The object of study is a stochastic process. Stochastic processes are processes that use statistical models to predict and process something. The change in the state of a random variable in a Markov chain over time steps is called evolution or shift. For example, stock price prediction by the rise and fall of the stock today, but predict the rise and fall of the stock tomorrow and the day after tomorrow; weather forecast by whether it rains today, predict whether it rains tomorrow and the day after tomorrow. All these processes can be quantified by mathematical formulas. By the probability of rain, stock rise, or fall, the formula can be used to derive the situation after N days.

To summarize the Markov chain in one sentence, the probability of a state shift at a given moment depends only on its previous state.

As a simple example, if the weather is a state every day, whether it is sunny today depends only on yesterday's weather and has nothing to do with the previous day's weather.

3. Application of game theory and Markov Chain

3.1. Application of game theory

There are many types of games, so usually have different classifications according to different standards. In the beginning, the games were simply divided by the number of people, but now games generally divide into cooperative games and non-cooperative games. The essential difference between the two is whether the players have a strong desire to cooperate and have negotiated and threatening rules to restrict each other's behavior. If they have, for the former, whereas for the latter. According to the order of the game, it can be categorized as a static game and a dynamic game. When players make choices at the same time, it is a static game. However, it is usually difficult to realize that players act at the same time. Therefore, most of the time static games make decisions when both sides are not clear about the choices of the other side and cannot be changed once confirmed. The Prisoner's Dilemma is a typical case that is about two people who conspired to commit a crime together when they are caught and then chose whether to confess to the police and give up their partner without being able to communicate with each other. Games can also be classified depending on other factors mentioned above, such as complete information games, incomplete information games, and so on [4].

Game theory has been applied in many fields and has some connections. For example, economics, military, politics, and even the natural sciences and biology. Among them, it is most commonly used in economics and has the most significant impact. Game is ubiquitous in all kinds of economic activities, which made the field of economics realize that decisions between people affect outcomes respectively and was focused on research.

The Prisoner's dilemma is a model that is often used in economics. In this model, if either one pleads guilty and the other stays silent, The confessor is immediately released, while the silent one receives a ten-year prison sentence. If they both remain silent, they will be sentenced to one year in prison, maximizing the overall interest; this is the Pareto optimal solution. If they both plead guilty, they both get eight years in prison, it's the Nash equilibrium [5]. It obviously reflects the absence of team interests due to individual interests. That is to say, if players make decentralized decisions based on individual interests, without discussion and coordination of all interests, it is difficult to maximize individual and collective interests at the same time. From the perspective of social and economic activities, "Prisoner's Dilemma" is a strategy that requires both sides of the game and the whole society to consider the gains and losses of profits together, and then choose whether to cooperate or not. For example, in order to prevent oligarchs from monopolizing the market and damaging social interests, we can construct a prisoner's dilemma model to break the cooperative relationship between players and make oligarchs compete with each other, thus reducing the price and improving the quality of goods to satisfy the requirements of the whole society and promoting the social economy. On the contrary, if the cooperation of game players can improve the social economy, it is necessary to try to get rid of the prisoner's dilemma model [6].

3.2. Application of Markov Chain

3.2.1 Mathematical definition:

Suppose our sequence of states is... ...Xt-2, Xt-1, Xt, Xt+1..., then the conditional probability of the state at Xt+1 depends only on the state Xt at the previous moment, i.e., P(Xt+1｜...Xt-2, Xt-1, Xt)=P(Xt+1｜Xt) [7]

Since the probability of a state transfer at a certain moment depends only on its previous state, the Markov chain model is determined as soon as we can find the probability of a transition between any two states in the system.

3.2.2 Theoretical properties

Markov chains have four properties: integrability, constant return, periodicity, and ergodicity [8]. These properties are not necessarily possessed by Markov chains and are different from Markov properties. All four properties are properties to be labeled on the state during the transfer process. "Irreducibility": If a Markov chain has only one connected class in the state space, i.e., all members of the state space, then the Markov chain is irreducible, otherwise the Markov chain has irreducibility. "Perpetuity": If a Markov chain can return to a state repeatedly in its evolution after reaching it, then the state is a perpetual return state. "Periodicity": A normally returning Markov chain may be periodic, i.e., in its evolution, the Markov chain can return to its state at a period greater than "ergodicity" means that the Markov process gradually converges to a stable state over a long period, regardless of the current state, and that the stable state is independent of the initial state.

For example, if the current state is u(x) = (0.5, 0.2, 0.3), then the next matrix state is u(x)T = (0.18, 0.64, 0.18), and the system converges to a steady state (0.22, 0.41, 0.37) according to this transition matrix. It turns out that no matter which point you start from, it will converge to this point after a long Markov Chain.

3.2.3 Application results

Markov chains are widely used in many fields. For example, language modeling, signal transmission, population migration, genetic inheritance, bad bank debt, traffic management, and production planning [9]. For example, why is signal transmission related to Markov chains? Consider the problem of transmitting signals along telephone lines or radio waves. Each piece of data must go through a multi-stage process before it can be transmitted, and at each stage, there is the possibility of data corruption due to transmission errors. This is consistent with its nature. Then we model the transmission process by a Markov chain. Set the states to 0 and 1 and the transfer matrix. p is whether the probability of passing through the stages is constant. Another common application is random wandering, including random wandering on graphs. This provides us with great convenience for modeling the motion of masses, involving money classes [10].

4. Limitations & Future outlooks

4.1. game theory

4.1.1 Summary of research status

Since the 1980s, game theory has undergone an extraordinary expansion, with Kreps, Fuldberg, Troyer, Robinstein, and Wilson among the more significant names to emerge during this time. Designing auction systems were already being done using game theory in the late 1990s.[11] Smith developed a two-way verbal auction process, and his essay that served as the basis for experimental economics won the 2002 Nobel Prize in Economics. By developing the idea of evolutionarily stable strategies, Smith and Plessy significantly advanced the field of evolutionary game theory. Freedman has extensively researched how evolutionary games might be used in economics.

4.1.2 Limitations

An analysis of the nature of game theory clearly shows that the number of problems that can be solved by game theory is relatively limited. The only problems that can be solved within the game theory and that can be solved analytically in some way to achieve equilibrium are game-theoretic solutions. There are many problems that can be solved by game theory, even in the last 100 years of research. For example, the prisoner's dilemma, the smart pig game, the hand grenade problem, and countless others, but in very few cases the outcome of the game can be consistently predicted by mathematical analysis.

4.1.3 Future outlooks

Simulated and experimental investigations based on finite rationality, behavioral hypotheses, and simulation. Due to the imperfect rationality that distinguishes human decision-making, traditional game theory, which is founded on the premise of perfect rationality, struggles to anticipate or explain the behavior and phenomena that take place in actual economic management systems. Studying the traits and mechanisms of interaction between actors, the evolution of cooperation, and its laws will become a significant area of research in the field of game science.

4.2. Markov Chain

4.2.1 Summary of research status

Marcov chain contributed to the development of probability theory, in particular the new field of stochastic process theory.

In stochastic process theory, Markov processes represent a class of stochastic processes of general importance and are widely used in modern physics, biology, utilities, geology, water, and atmospheric science [12].

In recent years, Markov chain prediction theory has been applied in fields such as education, economics, financial investment, biology, crop cultivation, geological ... ... and other applications. In particular, the science of water resources is very widely applied.

4.2.2 Limitations

The primary benefits and drawbacks of Markov analysis

1) The ability to estimate the likelihood of a system with numerous deterioration states and maintenance capability

2) Restrictions.

(1) Because all variables are statistically independent, no two states can be statistically related to one another unless they are physically adjacent to one another;

(2) All matters are statistically independent;

(3) The various probabilities of state changes need to be known

(4) Knowledge of matrix operations is complex and challenging for non-specialists to comprehend.

4.2.3 Future outlooks

Areas that are currently being investigated or that have not yet been investigated include Markov processes, Markov random fields, infinite particle Markov processes, measure-valued branching processes, and Markov processes on manifolds. processes, measure-valued branching processes, Markov processes on manifolds, etc., are some of the areas that are being investigated further or that have not yet been investigated.

5. Conclusion

To varying degrees, mathematical statistics theories and methods are applicable to all areas of human activity. By studying game theory and Markov chains, we are shown how to collect and analyze data. The problem of deriving and studying the regularity of stochastic phenomena is investigated, as are a variety of applications in social life.

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