Modeling and analysis of the entire process of hurricane loss based on probability theory

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Abstract. This paper is about a research on sustained wind speed pattern of a hurricane, involving wind speed distribution, hazard analysis as well as further evaluation on its relationship with storm surge heights. It establishes a simulative model base on Hurricane Jose in 2017 and relative data are collected from NOAA. Weibull distribution can be helpful on monitering sustained wind speed and a probability density distribution curve showing the likelihood of a hurricane with magnitude similar to Jose to strike New York is established by mathematical calculation software. The possible risk is approximately evaluated based on this likelihood to reach a final damage budget. This research can be helpful on damage prediction and prevention on coastal natural hazard, especially for a time that as climate change enhances. With the help of results this reseach has obtained, a damage forecast can be made before a hurricaneforms in order to benefit decision makers to construct corresponding level of defence system to reduce loss when a hurricane actually approaches.

Keywords: hurricane, hazard analysis, storm surge, natural hazard.

1. Introduction

The storm surge hazard is one of the greatest threats to coastal areas. Occurrence of hurricanes and the associated extreme storm surges are low-probability events; however, when they do occur, they can cause large-scale damage. Nearly half of the more than 2,325 deaths directly caused by hurricanes that have made landfall in the United States since the mid-20th century have been caused by storm surges (Rappaport, 2014). The impact of several devastating hurricanes has revealed the vulnerability of much of the United States to this catastrophic natural hazard. The wind speed and frequency of the most intense hurricanes are also expected to increase in the future (Emanuel, 2020; Gutmann et al. 2018; Knutson et al. 2010; Walsh et al. 2016). In addition, the coastal population of the United States is expected to increase by more than 50% by 2060 (Neumann et al., 2015), putting an increasing number of people at imminent risk from storm surges in the future. Currently, studies link the hazards caused by storm surges to the Saffir-Simpson Hurricane Scale (SSHWS) (Camelo et al., 2021), but studies have demonstrated that the correlation between wind intensity and SSHWS is not always linear (Bloemendaal et al., 2020; Irish et al., 2010; Kantha, 2006; Powell et al., 2007) and the development of storm surges is actually a complex result of multiple factors interacting with each other, which is difficult to quantify and analyze using the hurricane scale alone. In this paper, a new geometric model is proposed to simulate the hurricane trajectory and thus obtain the wind speed probability distribution function from the perspective of hurricane motion trajectory, and the mathematical correlation between the probability distribution function of hurricane wind speed and other parameters of hurricane and hurricane storm surge risk is investigated from a new perspective other than SSHWS, in order to assess hurricane risk more accurately and reliably. With increasing importance of effective and comprehensive prediction and control of storm surge risk, the purpose of this paper is to provide a new insight to more accurately understand and quantify the magnitude of the storm surge. With these information coastal cities will be able to take preventive measures in advance, such as evacuation and construction of water retaining facilities, thus reducing damage and casualties.

This paper aims to find a statistical method to quantify storm surge heights by first analyzing the influencing factors that affect storm surge heights and identifying a few of these variables that can be quantified for subsequent analysis. The methodological part talks about getting the probability distribution function of hurricane wind speed in three steps: selecting the mathematical model, that is, selecting the distribution type; then performing a geometric analysis to simulate the shape of the hurricane and the city in actual problems with simple geometric shapes respectively to do simplification; and finally performing a probability calculation with Bayesian formula to get the probability distribution function of the studied hurricane wind speed. In the data analysis section, Hurricane Jose in 2017 was used as the study object, and the above steps were followed, which finally allowed the probability distribution function of wind speed for Hurricane and Jose, the plot, and the mathematical expectation of the damaged buildings in New York City in this hurricane and the probability of different levels of structural damage to the buildings could be obtained by calculations as well. Then, the data of hurricanes that occurred in New York City from 1980 to the present were counted, and the values of the factors affecting the height of storm surge were inputed as independent variables, and the height of storm surge due to hurricanes were entered into the fitting model of MATLAB as outputs, and the fitting results were observed and evaluated by using the highest R2 fit. In the conclusion section, the methods presented in the paper are summarized, and several suggestions are made for more subsequent in-depth studies.

2. Analysis On Factors Generating Storm Surge

A storm surge is an abnormal rise in water produced by a storm that exceeds predicted astronomical tides (NOAA, 2021). This rise in water levels can cause extreme flooding in coastal areas, especially when the storm surge coincides with normal high tides, resulting in storm surges reaching 20 feet or more in some cases. Storm surge heights are caused by three main forcing mechanisms, namely direct wind pressure, barotropic water level adjustment, and wave radiation stress (Irish et al., 2008). Of these, direct wind stress and barotropic water adjustments are responsible for the majority of the total surge (National Hurricane Center, NOAA) and its prediction is highly dependent on the accuracy and lead-time of atmospheric variables predicted by numerical weather models (Bernier and Thompson, 2015). Meteorological parameters include storm intensity, which is given by central pressure deficit and maximum wind speed, storm size, which can be regarded as the radius of maximum winds), and storm path near the landfall have been found to significantly influence the potential for hurricanes to produce storm surge (Weisberg and Cheng 2006; Irish et al., 2008; Resio et al. 2009; Khalid et al. 2021). Among these factors, wind speed is relatively less complex to quantify and simulate with statistical methods, so the focus will be on near surface wind speed probability function in the follow-up methodology.

3. Method

Figure 2.1 shows the flow chart of the proposed method in this paper. Starting from collecting hurricane data, including hurricane track maps, near-surface wind speeds, and wind pressures, the probability distributions of wind speeds for different hurricanes are analyzed using the Weibull distribution. After obtaining the wind speed probability distribution function of the hurricane, the shape of the hurricane is approximated as an elliptical shape. The elliptical model of the hurricane area can be used to simplify the calculation of the conditional probability of a disaster caused by a hurricane. Considering that the hurricane occurs in a city, the conditional probability distribution function event of the above result is applied to Bayes' theorem. In this way, when a hurricane does occur in a city, the probability distribution function of urban wind speed can be obtained and subsequent analysis can be performed.

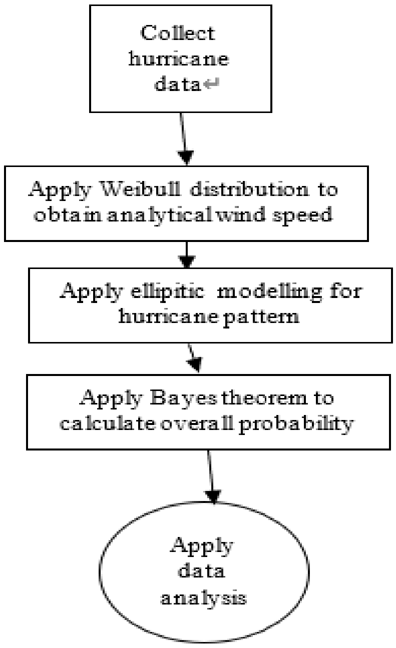

Fig. 1 Flow chart of analyzing process

Mathematical model:

Weibull distribution is s elected for wind speed distribution function (Bhattacharya P. et al., 2009). Usually, the wind speed distribution fits the shape of Weibull's distribution. So it is chosen in this case to apply data analysis.

The probability density function of a Weibull random variable is:

$$f(x;\lambda,k)=\begin{cases}\frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1}e^{-\left(\frac{x}{\lambda}\right)^{k}}, x\geq 0\\ 0, \quad \text{otherwise}\end{cases} \tag{1}$$

k and λ are the characteristic parameters. Its cumulative distribution function is

$$F(x;\lambda,k)=\begin{cases}1-e^{-\left(\frac{x}{\lambda}\right)^{k}}, x\geq 0\\ 0, \quad \text{otherwise}\end{cases} \tag{2}$$

Geometrical analysis: Elliptic approximation

In order to make use of the wind speed distribution from data and the result given by Weibull distribution, a model of the hurricane effective area should be set up.

After observing the hurricane track and shape information published in NOAA, the effective area of the hurricane can be treated as a ellipse. This is because the hurricane is often formed on the ocean without obvious obstacles. As it approaches to the land, its atmosphere pattern will be likely to get disturbed by the blockages on land, causing the contours of wind speed within the hurricane to get dispersed and the gradient of wind speed becomes smaller. In this case, the boundary of the hurricane can be concluded as an ellipse with its center located on one of its focus.

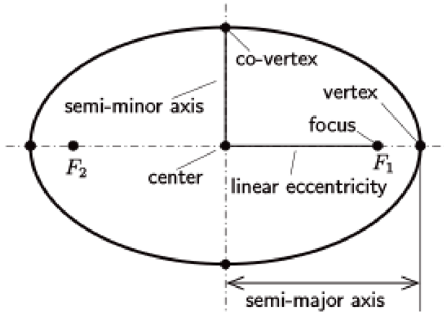

Fig. 2 Ellipse used to simulate hurricane shape

An ellipse is the summation of points with equal sums of distances to focus F_1 & F_2. The orientation of the hurricane will be described as the line starts from one focus F_1, passing through the other focus F_2, and finally reaches the city center.

Probability calculation: Bayes theorem

The modelling of ellipetic hurricane area can help calculate a conditional probability of damage prediction. To transfer this result to overall event without extra conditions provided, Bayes theorem should be applied. Bayes' theorem is written as following:

$$P(A|B) = \frac{P(B|A) * P(A)}{P(B)} \tag{3}$$

A & B are two different events while $P(B) \neq 0$. It provides the relationship between two condiational probabilities $P(A|B)$ & $P(B|A)$ of thw events with their marginal probabilities $P(A)$ & $P(B)$.

4. Data Analysis

Hurrican Jose in 2017 is adopted as the object of study in this section.The contour plot figure can be obtained from NOAA, and analysing it helps with extracting the data of sustained wind speed generated by the hurricane. The colour bar provides criteria about the probability of receiving a specific wind level, so an expection of wind speed can be calculated.

The data collected contains data indicating the magnitude of sustained winds. It may be shown in direct figures or probabilities. This will help estimate the wind speed within the range we interested in.

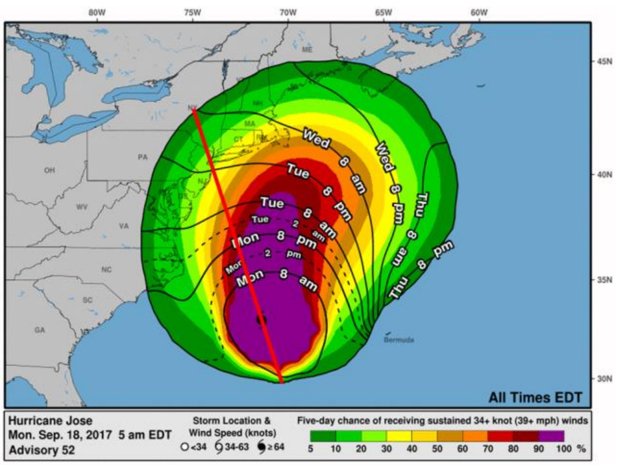

Fig. 3 Most Likely Arrival Time of Tropical-Storm-Force Winds

In order to find the mathematical relationship between sustained wind speed and storm surge, data collection was first conducted. This table provides the major hurricanes that New York City has experienced after 1980. Since Hurricane Ida occurred recently, there are no exact storm surge figures yet, so an estimate is used.

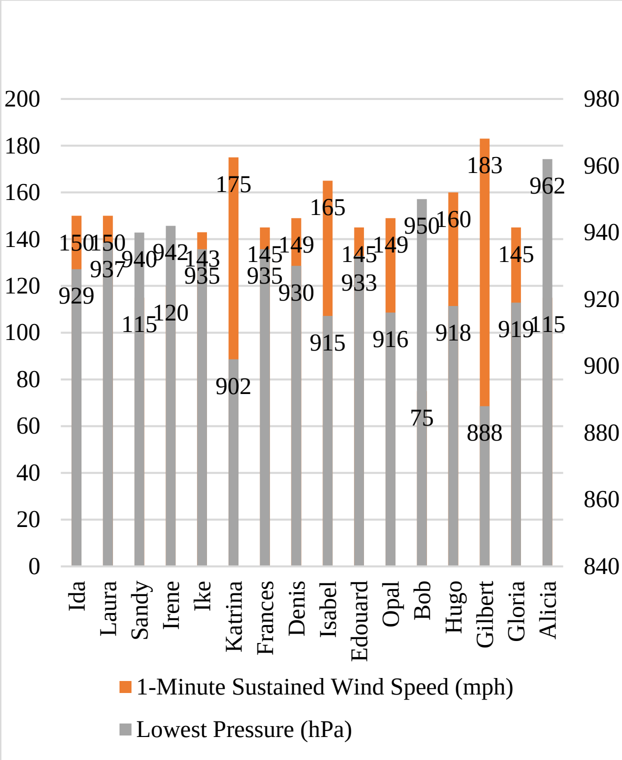

Fig. 4 Statistics of hurricanes in the United States after 1980

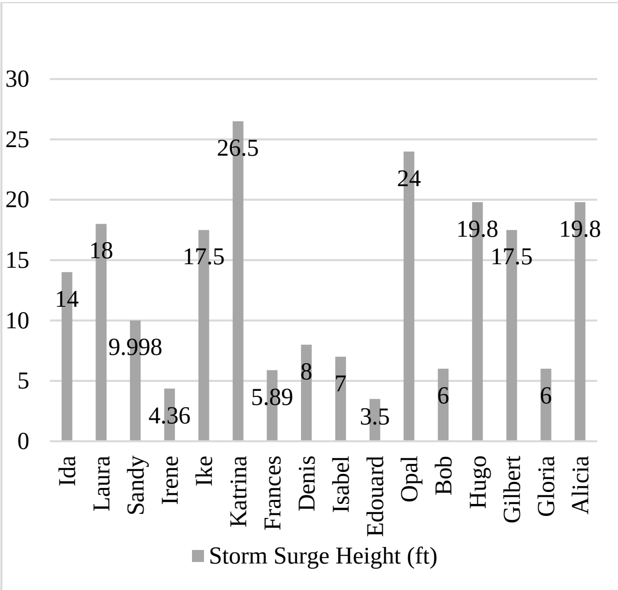

Fig. 5 Storm Surge Heights of hurricanes in the United States after 1980

Wind speed distribution function can be obtained by inputting data into MATLAB with Weibull distribution fitting. This will generate a probability density distribution of wind speed provided the hurricane hits the city. Table 1 shows the parameters corresponding to the probability density function obtained above. The expectation of sustained wind speed provided the hurricane hits the city has the probability density function:

$$f(x) = \begin{cases} \frac{1.786}{27.239}\left(\frac{x}{27.239}\right)^{1.786-1} e^{-\left(\frac{x}{27.239}\right)^{1.786}}, x \geq 0 \\ 0, \ otherwise \end{cases} \tag{4}$$

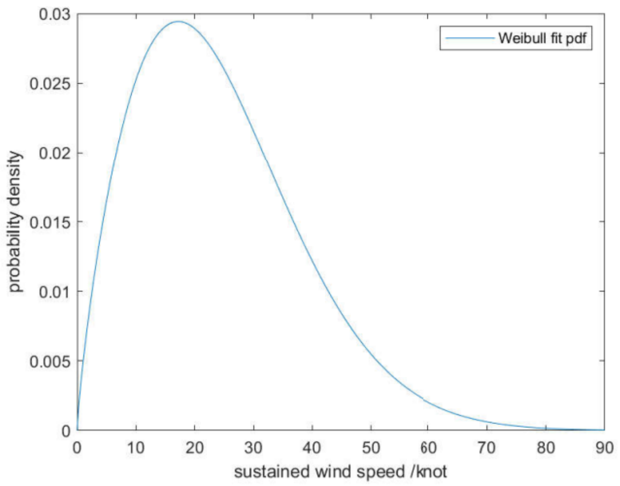

Fig. 6 Probability Density Function Plot of wind speed

Table 1. Parameter for the wind speed probability density function

α	β	Likelihood	AIC
27.239	1.786	431	138

The model is applied as a quarter circle with presented radius is used to represent the interested region and a circle with a approximated radius is used represent the boundary of city. The quarter shaped is chosen because inland areas are less important for analysing hurricanes and this sector shape can effectively reduce the inland area involved.

In real cases, the shape of the model can be changed and modified to fit actual circumstances. The distance between them is located along the 45 degree line. The red boundary represents the edge of the hurricane.

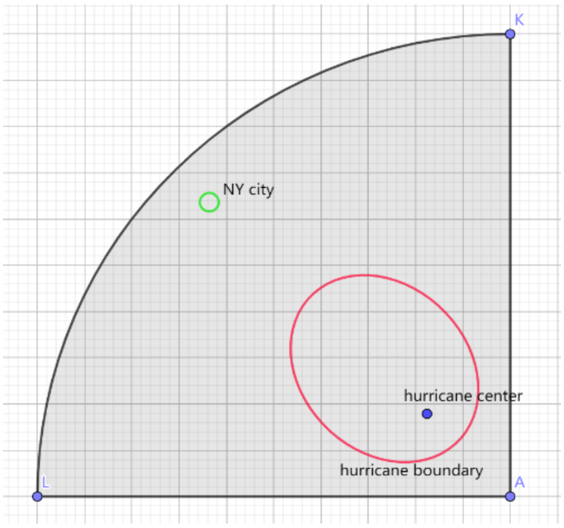

Fig. 7 Diagram of an example of area interested

With all the distances provided, the distance between centre of the city and the centre of the hurricane can be calculated.The possible area will be the intersection between the quarter circle and another circle which is centered at the same position as the city center. The radius of the circle is the combination of a semi-major axis a, and a linear eccentricity c of the ellipse as mentioned in the ellipse model. The possible area will be the intersection between the quarter circle and another circle with radius 1200 km and center the same as the city center. The shape bounded by orange curves will be the targeted area. Its area is calculated, which is 2438003.28 km^2. Also the size of the quarter is 3141592.65 km^2.

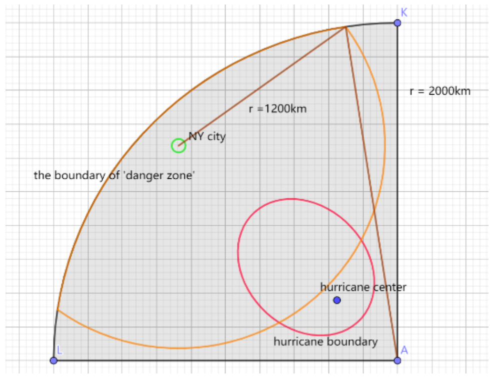

Fig. 8 Simulation diagram of target city and the track and shape of the hurricane

We are interested in the maximum damage that will occur on the city so we focused on the point when hurricane is closest to the city center. If we assume the closest point will fall into the quarter equally, the probability

$$P(the\ hurricane\ hits\ the\ city) = \frac{2438003.28}{3141592.65} = 0.776 \quad (5)$$

This is a very big probability based on the area interested in. If we enlarge the gray area, the probability will decrease correspondingly.

The process of applying Bayes' theorem to calculate the unconditional probability about whether the hurricane will hit the city or should be: the probability of receiving a 34+ knot sustained wind in five years under the condition that the hurricane hits the city should equal to the probability of receiving a 34+ knot sustained wind in five years when the hurricane hits the city multiple the probability of receiving a 34+ knot wind in current wind pattern divided by the probability of the hurricane hits the city:

We are interested in the maximum damage that will occur on the city so we focused on the instants when hurricane is closest to the city center along the overall track which the hurricane travells. The shape bounded by orange curves will be the targeted area which will cause the city under hurricane influence when the centre of the hurricane is located inside. The two probabilities can be assumed to be statically independent then the probability distribution of wind speed will be the multiplication between them.

5. Results And Discussion

The source of the table is National Weather Service, NOAA, and in the complete table, the speeds of Beaufort from 0 to 12 winds are listed in order with the corresponding damage effects. Since the damage to the building structure is considered here for a certain value of wind speed, only winds of magnitude 9 to 12 are shown here, since it is from magnitude 9 that the structure starts to show damage.

Table 2. The classification of winds and their possible damages

Beaufort Number	Description	Speed	Visual Clues and Damage Effects
9	Strong Gale	47 to 54 mph	Structural damage occurs, such as chimney covers, roofing tiles blown off, and television antennas damaged. Ground is littered with many small twigs and broken branches.
10	Whole Gale	55 to 63 mph	Considerable structural damage occurs, especially on roofs. Small trees may be blown over and uprooted.
11	Storm Gale	64 to 75 mph	Widespread damage occurs. Larger trees blown over and uprooted.
12	Hurricane Force	Over 75 mph	Severe and extensive damage. Roofs can be peeled off. Windows broken. Trees uprooted. RVs and small mobile homes overturned. Moving automobiles can be pushed off the roadways.

Use the result obtained in the case study of hurricane Jose, the culmulative distribution function of the wind speed should be $F(x;k,\lambda)=1-e^{-(\frac{x}{27.239})^{1.786}}$. *With* 1 knot=1.15078 Mph, table 3 shows the probability of different level of structural damage occurs.

Table 3. Probability of the occurrence different level of structural damage

Structural Damage Level	Wind Speed Range In Mph	In Knot	Probability
Structural Damage Occurs	47	54. 1	0.01183
Considerable Structural Damage Occurs	54	62.1	0.00838
Widespread Damage Occurs	63	72.5	0.005135
Severe And Extensive Damage	75	86.3	0.003486

There are currently approximately 1084722 buildings in New York City. (Building Footprints from NYC Open Data) Assume that the damage degree of each building under the wind is independent and has equal probability. According to the previously calculated probability of damage to the building, the probability of damage is 2.883%. The mathematical expectation of the amount of structural damage caused by wind in the New York City is 3135.

In order to find out the mathematical relationship between storm surge height and wind speed, we input data into MATLAB curve fitter to find a best fit with R^2 closest to 1 (by iteration). From the chart above, it can be found that the wind speed and storm surge height are not simply linear.

Table. 4 Correspond R2 to different fitting types

Example equation	$p_1 * x^3 + p_2 * x^2 + p_3 * x + p_4$	$a_1 \sin(b_1 x + c_1) + \cdots + a_5 \sin(b_5 x + c_5)$	$* x^4 + p_3 * x^3 + p_4 * x^2 +$ / $* x^3 + q_2 * x^2 + q_3 * x +$
R^2	0.2046	0.7507	0.3275

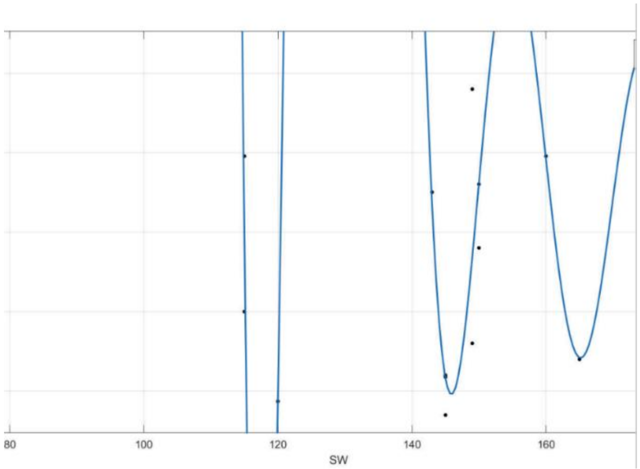

Fig. 9 Fitted plot of relationship between storm surge height and sustained wind speed

Table. 4 Corresponding coefficients of sine5 fitted function

Coefficients (with 95% confidence bounds)	a	b	c
1	a_1=683.5 ($\pm 9.495e+17$)	b_1=0.8223 ($\pm 1.797e+13$)	c_1=2.743 ($\pm 2.986e+15$)
2	a_2=507.9 ($\pm 1.642e+18$)	b_2=2.162 ($\pm 3.227e+13$)	c_2=-0.8285 ($\pm 5.064e+15$)
3	a_3=128.3 ($\pm 1.477e+17$)	b_3=6.221 ($\pm 1.21e+13$,)	c_3=0.1937 ($\pm 1.712e+15$)
4	a_4=308.7 ($\pm 1.049e+17$)	b_4=5.277 ($\pm 4.832e+13$)	c_4=-2.149 ($\pm 7.16e+15$)
5	a_5=375.2 ($\pm 1.158e+18$)	b_5=3.807 ($\pm 1.197e+14$)	c_5=1.838 ($\pm 1.791e+16$)
	SSE=212.5	RMSE=14.58	R^2=0.7507

This provides the sin5-fitted relationship between sustained wind (SW/mph) and storm surge height (SSH/ft) with the maximum R^2. If more data is provided, the result will be more precise and may be different.

6. Conclusion

In this study, a computational model of the probability function of hurricane wind speed is obtained. And in the case study of Hurricane Jose in 2017, the probability values of different levels of damage to buildings and the mathematical expectation of the damaged structures in New York City in this hurricane are calculated by this method in the case study of as 3135. This paper provides a preliminary insight for the future hurricane research, that is, the hurricane trajectory can be directly abstracted into a mathematical model, therefore, can be quantified. Along with the wind speed data, the probability function of the wind speed can be obtained and the damage of any city can be calculated by using the same process proposed in this paper.

There is a statistical relationship between wind speed and storm surge. A large number of data are needed to fit the statistical relationship between wind speed and storm surge height, so that the flood on urban inundation. This study only considers the hurricanes in New York City after 1980, and only gives a preliminary fitting result. Also, there are many properties of hurricanes that are influencing storm surge. To take these factors into account the parameters of storm surge, research other than statistical ones on hurricanes is needed to enrich this study.

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