

# Dynamic Model Based Research on Intelligent Vehicle Steering and Obstacle Avoidance

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**Abstract.** In order to improve road safety, promote the development of autonomous driving, and realize the stability of unmanned vehicles during steering and obstacle avoidance, this paper establishes a lateral dynamic model and a cyclotron curve model based on the 3-degree-freedom vehicle dynamics model. Taking no collision between cars and timely obstacle avoidance as the optimization goal, this paper discretizes the model, a prediction model is established, a soft constraint solver is designed, and a relaxation factor is added to the performance index function. Based on the model prediction and control principle, the obstacle avoidance trajectory planning and tracking of unmanned vehicles is integrated and controlled, and the solution is converted into a quadratic programming problem according to the constraints and rolling optimization characteristics. Finally Matlab/Prescan joint simulation is used to verify the steering and obstacle avoidance quality of the model. The results show that the model has excellent safety, stability and comfort in non-highspeed situations.

**Keywords:** Lateral Dynamic Model, Clothoid Curve, Steering, Obstacle Avoidance, Simulation.

## 1. Introduction

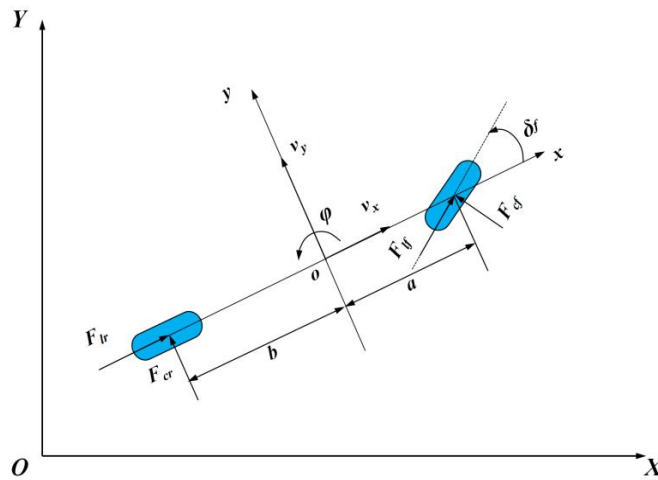
In the new generation of scientific and technological revolution with artificial intelligence as the core, intelligence will gain more and more attention in the automotive industry. Intelligent driving technology can not only reduce the driver's operation intensity, alleviate driver fatigue, but also improve people's travel efficiency and quality[1-2], thus make the entire transportation system more orderly. What intelligent driving can't avoid is the problem of steering and obstacle avoidance, and some scholars have also studied this. Based on the Markov decision process of reinforcement learning, Cao Jie et al.[3] established a vehicle U-turn dynamic model and constructed a hierarchical proximal policy optimization algorithm, which can learn the strategy faster and have a higher U-turn success rate. Wang Yilin et al. [4] designed and optimized the enhanced tangent speed constraint model, and then based on the TORCS microsimulation platform, the steering control effect of the method and the traditional tangent method was compared and verified, which improved the safety of automobile autonomous driving. Piao Changhao[5], Wang Jian[6], Demirli [7]respectively designed path tracking controllers based on PID, LE-SO, adaptive fuzzy control. Lan Nan et al. [8] studied and interpreted the European standard ECE R79 autonomous driving steering, analyzed the functions and test process of R79 and classified them, then concluded that it has strong guiding significance for the development of intelligent driving in China. Zhai Li et al. [9] proposed a local obstacle avoidance path planning algorithm based on artificial potential field method, used the dynamic window approach to carry out real-time dynamic obstacle avoidance planning and applied Bezier curve to smooth the trajectory. Xiu Caijing et al. [10] designed the obstacle avoidance system of self-driving cars based on the artificial potential field method. Sheng Pengcheng et al. [11] used the velocity obstacle method to detect the collision threat in real time, and established a mathematical model of the shortest obstacle avoidance time and safety distance to achieve efficient dynamic obstacle avoidance, and finally used Bayes' theorem to calculate the probability of the hazard level of the candidate path. Zhao Haipeng et al. [12] introduced obstacle avoidance constraints based on the triangle area method, designed the cost function to meet the most predicted trajectory output,

meanwhile introduced the nonlinear model prediction controller with rollover constraints for online optimal path tracking.

In this paper, a 3-degree-of-freedom vehicle dynamics model is established based on the tire model of the magic formula. Then, the two dimensions of lateral deviation and yaw angle deviation are added to obtain the lateral dynamic model and then it was discretized. On this basis, the clothoid curve model is introduced to optimize the steering and obstacle avoidance paths. Under the constraint, the performance index function with the relaxation factor is designed and the feedback correction is provided. Finally, simulation tests is applied to convey the excellence of the proposed model.

## 2. Car Dynamics Model

A car is a very complex system that requires precise control, and many factors can affect its state of motion. Accurately translating the characteristics of a car into a mathematical model through a mechanical system is not only an extremely difficult task in itself, but the additional calculation time generated by complex mathematical models will greatly reduce the real-time safety of automotive control. As shown in the Figure 1, the vehicle only considers the movement of the three degrees of freedom directions, longitudinal, lateral and sideways, and to simplify the calculation, the following assumption will be made: ① The car is symmetrical left and right, and above the wheels is a rigid body. ② The car moves on a horizontal road surface and ignores air and road resistance. ③ The vehicle tires are identical and the lateral deviation characteristic curve is linear, ignoring the influence of the steering system.



**Figure 1.** Schematic diagram of the 3-degree-of-freedom vehicle dynamics model

Assume the front wheels are steering wheels,  $XOY$  is the inertial coordinate system,  $xoy$  is the automotive coordinate system.  $m$  is the mass of the car;  $a, b$  are respectively the distance from the center of mass to the front and rear axles;  $\varphi, \dot{\varphi}$  are the body yaw angle and angular acceleration;  $\Delta_f$  is the steering angle of forewheel;  $I_z$  is the moment of inertia around the vertical vehicle plane  $z$ -axis;  $\dot{x}, \ddot{x}$  are the velocity and acceleration in the  $x$ -axis direction;  $\dot{y}, \ddot{y}$  are the velocity and acceleration in the  $y$ -axis direction;  $F_{lf}, F_{lr}$  are the longitudinal forces of the earth on the front and rear wheels;  $F_{cf}, F_{cr}$  are the transverse forces of the earth on the front and rear wheels;  $F_{xf}, F_{xr}$  are the  $x$  forces of the earth on the front and rear wheels;  $F_{yf}, F_{yr}$  are the  $y$  forces of the earth on the front and rear wheels;  $C_{cf}, C_{cr}$  are the side stiffness of the front and rear wheels;  $C_{lf}, C_{lr}$  are the longitudinal stiffness of the front and rear wheels;  $\alpha_f, \alpha_r$  are the slip angles of the front and rear wheels;  $s_f, s_r$  are the longitudinal slip rates of the front and rear wheels.

According to Newton's second law, the following equation is obtained:

Longitudinal dynamic equation

$$m\ddot{x} = m\dot{y}\dot{\varphi} + 2(F_{lf} \cos \delta_f - F_{cf} \sin \delta_f) + 2F_{lr} \quad (1)$$

Lateral dynamic equation

$$m\ddot{y} = -m\dot{x}\dot{\varphi} + 2(F_{cf} \cos \delta_f + F_{lf} \sin \delta_f) + 2F_{cr} \quad (2)$$

Sideways dynamic equation

$$I_z \ddot{\varphi} = 2a(F_{cf} \cos \delta_f + F_{lf} \sin \delta_f) - 2bF_{cr} \quad (3)$$

According to assumption ③, the tires are identical and the lateral deviation characteristic curve is linear, so when the tire is in contact with the ground attachment, it is affected by the lateral and longitudinal forces of the earth respectively as:

$$\begin{cases} F_{lf} = C_{lf} s_f \\ F_{lr} = C_{lr} s_r \\ F_{cf} = C_{cf} \alpha_f \\ F_{cr} = C_{cr} \alpha_r \end{cases} \quad (4)$$

This paper applies the tire model  $Y(x)=D\sin(\text{Carctan}(Bx-E(Bx-\arctan(Bx))))$  based on the magic formula proposed by H.B.Pacejka, which considers the tire to be linear in the vertical and lateral directions, and the damping is constant. It has high fitting accuracy for conventional tires under the common range of lateral acceleration  $\leq 0.4g$  and the slip angle  $\leq 5^\circ$ . In addition, since the magic formula is based on test data, it can be used to a certain extent in addition to high accuracy in the test range, and even to a certain extent beyond the limit value, it is possible to extrapolate to the limited working conditions with good confidence.[13]

Tire slip rate can be expressed as:

$$s = \left| \frac{\dot{x} - R\omega}{R\omega} \right| \quad (5)$$

Let  $\alpha + \delta_f = \theta$ , then  $\tan \theta = \frac{\dot{y} + a\dot{\varphi}}{\dot{x}}$  and  $\tan \alpha_r = \frac{b\dot{\varphi} - \dot{y}}{\dot{x}}$  follow. The model requires that the front and rear wheel slip angles and front wheel rotation angles are small enough, so the slip angles of the front and rear wheels can be expressed as:

$$\alpha_f = -\delta_f + \frac{\dot{y} + a\dot{\varphi}}{\dot{x}} \quad (6)$$

$$\alpha_r = \frac{\dot{y} - b\dot{\varphi}}{\dot{x}} \quad (7)$$

Based on the above derivation, combined with the previous assumptions, the 3-degree-of-freedom dynamic model of the obtained car is shown in Equation (8).

$$\begin{cases} m\ddot{y} = -m\dot{x}\dot{\varphi} + 2\left(C_{cf}\left(\delta_f - \frac{\dot{y} + a\dot{\varphi}}{\dot{x}}\right) + C_{cr} \frac{b\dot{\varphi} - \dot{y}}{\dot{x}}\right) \\ m\ddot{x} = m\dot{y}\dot{\varphi} + 2\left(C_{lf}s_f + C_{cf}\left(\delta_f - \frac{\dot{y} + a\dot{\varphi}}{\dot{x}}\right)\delta_f + C_{lr}s_r\right) \\ \dot{Y} = \dot{x} \sin \varphi + \dot{y} \cos \varphi \\ \dot{X} = \dot{x} \cos \varphi - \dot{y} \sin \varphi \\ I_z \ddot{\varphi} = 2\left(aC_{cf}\left(\delta_f - \frac{\dot{y} + a\dot{\varphi}}{\dot{x}}\right) - bC_{cr} \frac{b\dot{\varphi} - \dot{y}}{\dot{x}}\right) \end{cases} \quad (8)$$

### 3. Automotive Steering Controller Design

#### 3.1. Sideways Dynamic Model

The sideways dynamic model is a model that adds two dimensions, lateral deviation and lateral angle deviation, on the basis of the 2-degree-of-freedom dynamic model. Based on the above analysis, the dynamic equation can be obtained:

$$\begin{cases} \dot{v}_y = \ddot{x} = -\dot{\varphi}v_x + \frac{1}{m}(F_{cf} + F_{cr}) \\ \ddot{\varphi} = \frac{1}{I_z}(aF_{cf} - bF_{cr}) \\ e_\varphi = \dot{\varphi} - v_x\rho \\ e_d = v_x e_\varphi + v_y \end{cases} \quad (9)$$

Here  $e_d$  is the lateral distance deviation of the center of mass from the centerline of the road;  $e_\varphi$  is a yaw angle deviation; Road curvature  $\rho$  is an additional variable.

Take the state variables as  $\xi = [v_y, \dot{\varphi}, e_d, e_\varphi]$ , control variables as  $u_1 = \delta_f, u_2 = \rho$ , then obtain a predicting model considering road curvature:

$$\begin{aligned} \dot{\xi} &= A(t)\xi(t) + B_1(t)u_1(t) + B_2(t)u_2(t) \\ \dot{\eta} &= C\xi \end{aligned} \quad (10)$$

Here  $A(t)$ ,  $B_1(t)$  and  $B_2(t)$  satisfy Equation (11).

$$A(t) = \begin{bmatrix} \frac{C_{yf} + C_{yr}}{mv_x} & \frac{aC_{yf} - bC_{yr}}{mv_x} & 0 & 0 \\ \frac{aC_{yf} - bC_{yr}}{I_z v_x} & \frac{a^2 C_{yf} + b^2 C_{yr}}{I_z v_x} & 0 & 0 \\ 1 & 0 & 0 & v_x \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad (11)$$

$$B_1 = \begin{bmatrix} -\frac{C_{yf}}{m} \\ \frac{aC_{yf}}{I_z} \\ 0 \\ 0 \end{bmatrix}; B_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -v_x \end{bmatrix}; C = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

Real time is continuous, but the observed time series are discrete. In order to better analyze the model, this paper linearizes and obtains the discrete state equation of the car dynamics model:

$$\begin{aligned} \xi(k+1) &= A_d \xi(k) + B_{d1} u_1(k) + B_{d2} u_2(k) \\ \eta(k) &= C_d \xi(k) \end{aligned} \quad (12)$$

Set controlling time domain  $T_m <$  predicting time domain  $T_p$ , and  $\Delta\xi(k)$  is the predicting starting point, then the predicted state of  $k+1$  moment is:

$$\Delta\xi(k+1|k) = A_d \xi(k) + B_{d1} \Delta u_1(k) + B_{d2} \Delta u_2(k) \quad (13)$$

Here,  $k+i | k$  represents the prediction of  $k+i$  moment when in  $k$  moment. According to the definition of increment  $\eta(k) = \eta(k-1) + C_d \Delta\xi(k)$ , iterate over the sampling time of the predicting time domain  $T_p$ , and finally get the predicted output as:

$$\begin{aligned}
 & \eta_p(k+1) \\
 &= [\eta(k+1|k) \quad \eta(k+2|k) \quad \dots \quad \eta(k+T_p|k)] \\
 &= \sigma_x(k)\Delta\xi(k) + \sigma_{u1}(k)\Delta U(k) + \sigma_{u2}\Delta u_2(k) + I\eta(k)
 \end{aligned} \tag{14}$$

Here each parameter satisfies Equation (15).

$$\begin{aligned}
 \sigma_{u1}(k) &= \begin{bmatrix} C_d(A_d+1)B_{d1} & C_dB_{d1} & \dots & 0 \\ \dots & \dots & \dots & \dots \\ \sum_{i=1}^{N_m} C_dA_d^{i-1}B_{d1} & \sum_{i=1}^{N_m} C_dA_d^{i-1}B_{d1} & \dots & C_dB_{d1} \\ \dots & \dots & \dots & \dots \\ \sum_{i=1}^{N_r} C_dA_d^{i-1}B_{d1} & \sum_{i=1}^{N_r} C_dA_d^{i-1}B_{d1} & \dots & \sum_{i=1}^{N_r-N_m+1} C_dA_d^{i-1}B_{d1} \end{bmatrix} \\
 \sigma_{u2}(k) &= \begin{bmatrix} C_dB_{d2} & C_d(A_d+1)B_{d2} & \dots & \sum_{i=1}^{T_r} C_dA_d^iB_{d2} \end{bmatrix} \\
 \sigma_x(k) &= \begin{bmatrix} C_dA_d & C_d(A_d^2+A_d) & \dots & \sum_{i=1}^{T_r} C_dA_d^i \end{bmatrix}; \quad I = [1 \quad 1 \quad \dots \quad 1]^T \\
 \Delta U(k) &= [\Delta u_1(k) \quad \Delta u_1(k+1) \quad \dots \quad \Delta u_1(k+T_m-1)]^T
 \end{aligned} \tag{15}$$

### 3.2. Clothoid Curve Model

A transition curve is a curve in which the curvature is continuously changed between a straight line and a circular curve, and between a circular curve and a circular curve. The clothoid curve is a kind of transition curve, which is not only beautiful in line shape, but also consistent with the trajectory line of the driver's uniform rotation steering wheel from the circular curve to the straight line or from the straight line into the circular curve. Let the length of the clothoid curve be \$L\$ and the curvature be \$1/R\$, then the basic expression of the clothoid curve is  $R \cdot L = A^2$ .

The literature [14] gives the calculation formula for the clothoid curve:

$$\begin{cases} dR = y + R \cos d\tau = R \\ dx_M = dx - R \sin d\tau \end{cases} \tag{16}$$

Here (x,y) is the coordinate of the point and  $\tau$  is the tangent angle of the point. Through the derivation of the formula of the cyclotron curve, it is fully proved that the arbitrary coordinates of the entire section of the clothoid curve can be obtained, and the curvature changes continuously. And the bibliographic also proves that the car rotates the steering wheel at a uniform speed and equiangular speed, and the trajectory driven by the control point meets the clothoid curve.

### 3.3. Constraints

In order to ensure the safety, stability and passenger comfort on the road, and to improve the smoothness and reliability of the car when steering, the following constraints are set:

† Pavement adhesion condition constraints

$$|a_y| \leq \rho_e g, a_{ymin} \leq a_y \leq a_{ymax} \tag{17}$$

† Front wheel steering angle constraints

$$u_{1min} = \delta_{fmin} \leq u_1 \leq \delta_{fmax} = u_{1max} \tag{18}$$

† Tire slip angel constraints

$$\beta_{fmin} \leq \beta_f \leq \beta_{fmax} \tag{19}$$

† Centroid slip angel constraints

$$\beta_{\min} \leq \beta \leq \beta_{\max} \quad (20)$$

† Transverse displacement constraints

$$-\frac{d_{road}}{2} \leq \eta \leq \frac{d_{road}}{2} \quad (21)$$

† Sideways angle constraints

$$\varphi_{\min} \leq \varphi \leq \varphi_{\max} \quad (22)$$

## 4. Optimization and Solution

### 4.1. Objective Function

The car dynamics model used in this paper is relatively complex, with many parameters and constraints, and may not be able to find the optimal solution within the specified period. In order to ensure that the car can drive according to the desired trajectory, the literature [15] gives a soft constraint method to add a relaxation factor to the performance index function:

$$J(k) = \sum_{i=1}^p \|\xi(k+i|k) - \xi_{ref}(k+i|k)\|_Q^2 + \sum_{i=1}^m \|\Delta u(k+i|k)\|_R^2 + \rho \varepsilon^2 \quad (23)$$

Here  $\rho$  is the weight coefficient,  $\varepsilon$  is the relaxation factor,  $R$  is the control input weight matrix, and  $Q$  is the control output weight matrix. The error function is:

$$\varepsilon(k) = \sigma_x \xi(k) + I\eta(k) - \eta_{ref} \quad (24)$$

The reference output matrix for the predicted time domain is:

$$\eta_{ref}(k) = [\eta_{ref}(k+1|k) \quad \eta_{ref}(k+2|k) \quad \dots \quad \eta_{ref}(k+T_p|k)] \quad (25)$$

According to the constraints and rolling optimization, the controller Equation(26) can be summarized as a quadratic programming optimization problem. Quadratic programming is a typical mathematical optimization problem, its optimization goal is a quadratic real function with linear or nonlinear constraints, and the commonly used solution is the effective set method or the interior point method.

$$\begin{aligned} & \min J(k) \\ & s.t. \begin{cases} |a_y| \leq \rho_e g, a_{y\min} \leq a_y \leq a_{y\max} \\ u_{1\min} = \delta_{f\min} \leq u_1 \leq \delta_{f\max} = u_{1\max} \\ \beta_{f\min} \leq \beta_f \leq \beta_{f\max} \\ \beta_{\min} \leq \beta \leq \beta_{\max} \\ -\frac{d_{road}}{2} \leq \eta \leq \frac{d_{road}}{2} \\ \varphi_{\min} \leq \varphi \leq \varphi_{\max} \end{cases} \end{aligned} \quad (26)$$

### 4.2. Feedback Correction

After solving the above equation in each control cycle, according to the prediction model, the first element of the control sequence is taken as the actual control input increment, and acts on the system, obtaining:

$$u_1(k) = u_1(k-1|k) + \Delta u_1^*(k) \quad (27)$$

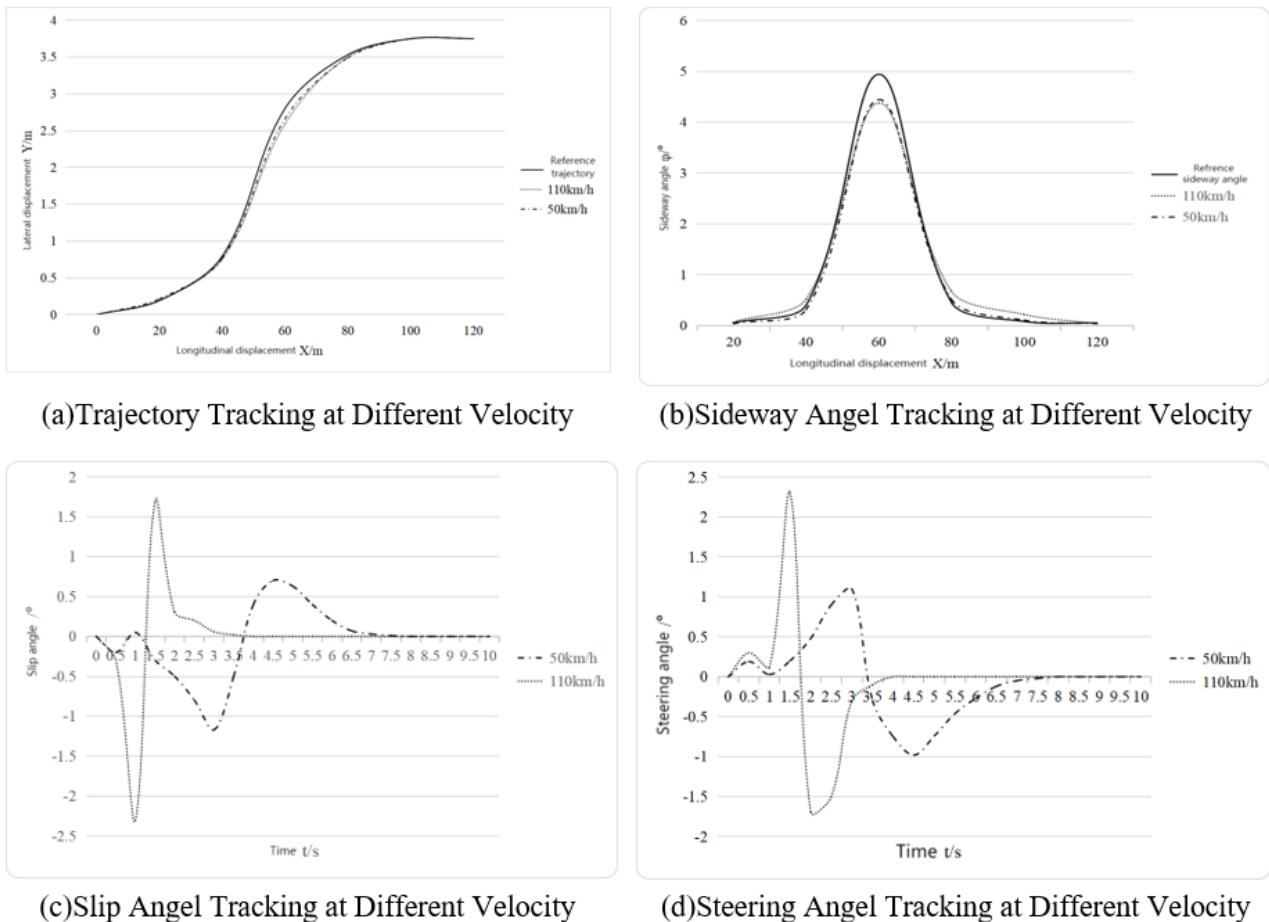
### 5. Simulation and Analysis

In order to verify the effectiveness of the steering and obstacle avoidance model, tracking control experiments are carried out on the joint simulation platform built by Matlab/Prescan. The main parameters of the car and the MPC control parameters are shown in Table 1, where  $T_s$  is the sampling period.

**Table 1.** Car and Controller Parameters

Car Parameters	Value	Controller Parameters	Value
m/kg	1895	$T_p$	50
a/mm	1171	$T_m$	5
b/mm	1770	$T_s$	0.02
$d_{road}/mm$	3750	$\delta_{\dot{r}}/rad$	$\pm 0.028$
$I_z/(kg \cdot m^{-2})$	3750	$\Delta\delta_{\dot{r}}/rad$	$\pm 0.002$
$C_{cf}/(N \cdot rad^{-1})$	72650	$\Phi/rad$	$\pm 0.028$
$C_{cr}/(N \cdot rad^{-1})$	121450	$A_y/g$	$\pm 0.4$

Figure. 2(a) shows the comparison of tracking and reference trajectory, and the actual trajectory of the car at high and low speed has a small error with the reference one. Figure. (b) shows the tracking and reference sideways angle, conveying approximately the same shape, with a maximum deviation around 60m, about  $0.5^\circ$ . Figure. 2(c) shows the comparison of tracking and reference slip angle, conveying it is much less than  $5^\circ$  at any time whether high or low speeds, and the faster the speed, the faster slip angle response and the larger the peak. Figure. 2(d) shows the comparison of tracking and reference steering angle, conveying it is much less than  $2.5^\circ$  at any time whether high or low speeds, and the faster the speed, the faster steering angle response and the larger the peak.



**Figure 2.** Simulation results

## 6. Conclusion

In this paper, the car is tracked and controlled by the sideways dynamic model and the clothoid curve model, and the conclusion is obtained:

(1) Integrate obstacle avoidance and tracking functions into the same control system, reducing calculation time, improving stability and real-time performance.

(2) At both high and low velocities, it can quickly and effectively steer and avoid obstacles, meet dynamic constraints, and improve safety and comfort.

(3) By adding the objective function with relaxation factor and feedback correction, simulation experiments show that the prediction trajectory is very accurate.

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