

Investment study of gold and bitcoin based on time series forecasting models

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Abstract. Since investment products are subject to many influences, there is a great deal of uncertainty about their direction. Therefore, people try to predict their price trends by various methods to develop their own investment strategies. We predict the movement of gold and bitcoin by building mathematical models and develop investment strategies to profit. We build a price prediction model for bitcoin and gold. Then, based on the price prediction model, we built an investment model that considers investment commissions, returns and risks to maximize investment returns. Finally, we evaluated the model and proved that the model provides the best investment strategy. Using this model, with an initial capital of \$1,000, a gold trading rate of 1%, and a bitcoin trading rate of 2%, our mathematical model achieves an annual asset appreciation of over 250% for a final return of \$103,191 over a five-year period starting on September 11, 2016 and ending on September 10, 2021.

Keywords: Time series forecasting; gray forecasting; investment strategies; gold; bitcoin.

1. Introduction

Investing is an important aspect of the financial sector, and it includes a variety of investments with different characteristics. There are both long-established, strong investment products such as gold and new risk and reward investment products such as bitcoin.

Nowadays, we often make money by buying and selling assets that are volatile in value, such as gold and bitcoin. We often choose to buy assets at low prices and then sell them at high prices to maximize returns. However, we often don't know the trend of volatility of these assets, so we need to use mathematical models to help us predict the trend of asset price changes and guide us to make decisions accordingly [1].

Suppose on November 9, 2016, we have \$1,000. Now, we need to develop a mathematical model that will help us invest in gold and bitcoin in order to maximize our returns by October 9, 2021. On each trading day, the trader will have a portfolio of cash, gold and bitcoin $[C, G, B]$. The initial state is $[1000, 0, 0]$. The commission cost for each transaction (including buying and selling) is $\alpha\%$ of the transaction amount. Assume that $\alpha_{\text{gold}} = 1\%$, $\alpha_{\text{bitcoin}} = 2\%$ and there is no cost to hold the asset.

2. Model Design

2.1. Model Overview

Our model is divided into prediction model and investment model.

The predicted bitcoin and gold prices will be used in the investment model. Our rational investment model is as follows: in the initial state, the value of our portfolio $[C, G, B]$ is $[1000, 0, 0]$. At this time, we use 45% of the funds (US \$450) to invest in bitcoin, 45% of the funds (US \$450) to invest in gold, and the remaining 10% of the funds (US \$100) remain in our hands as standby funds. We will take 365 days as a cycle and adjust the portfolio at the end of each cycle. At the end of each cycle, there are several situations:

Case (1) Bitcoin and gold were both profitable, Case (2) Bitcoin and gold both lost money, Case (3) Bitcoin made a profit and gold lost money, Case (4) Bitcoin lost money and gold made a profit.

2.2. Time series forecasting models

Since the only data available in this modeling are the price parameters of bitcoin and gold obtained gradually over time. The available data is small and the increase of gold price and bitcoin has uncertainty. Therefore, the relationship between the increase of gold and bitcoin can be considered as a gray model and predicted using a time series based gray model.

2.2.1 Constructing a gray model

As the name implies, gray forecasting is the future prediction of gray system problems. Gray forecasting is based on the gray system model [2].

It is possible to build as complete a model as possible with incomplete information.

The use of cumulative generated numbers as the data used in forecasting, where the cumulative numbers often contain more information and more factors.

There is a better mechanism for accuracy analysis and credibility analysis of the prediction results.

Our team found a correlation coefficient of 0.0026 between bitcoin and gold[3], the impact is small and can be ignored. Therefore, in constructing the forecasting models, our team uses two GM(1,1) models to forecast the price movements of gold and bitcoin respectively.

The GM (1, 1) based gray forecasting method is described below[4,5] .

$$\begin{aligned} \text{Let } X^{(0)} &= (X^{(0)}(1), X^{(0)}(2), \Lambda, X^{(0)}(n)), \text{ for } 1 - AD \text{ get} \\ X^{(1)} &= X^{(1)} = (X^{(1)}(1), X^{(1)}(2), \Lambda, X^{(1)}(n)) \\ &= (X^{(1)}(1), X^{(1)}(1) + X^{(0)}(2), \Lambda, X^{(1)}(n-1) + X^{(0)}(n)) \end{aligned} \quad (1)$$

Establishing differential equations in whitened form

$$\frac{dX^{(1)}}{dt} + aX^{(1)} = \mu \quad (2)$$

Let $\hat{a} = (a, \mu)^T$, according to the least squares idea to get

$$\hat{a} = (B^T B)^{-1} B^T Y_1 \quad (3)$$

Among them

$$B = \begin{bmatrix} -\frac{1}{2}(X^{(1)}(1) + X^{(1)}(2)) & 1 \\ -\frac{1}{2}(X^{(1)}(2) + X^{(1)}(3)) & 1 \\ \vdots & \vdots \\ -\frac{1}{2}(X^{(1)}(n-1) + X^{(1)}(n)) & 1 \end{bmatrix} \begin{matrix} M \\ M \\ M \\ M \end{matrix} Y_1 = \begin{bmatrix} X^{(0)}(2) \\ X^{(0)}(3) \\ \vdots \\ X^{(0)}(n) \end{bmatrix} \quad (4)$$

The solution of the equation is obtained as

$$X^{(0)}(k+1) = (X^{(0)}(1) - \frac{\mu}{a})e^{-ak} + \frac{\mu}{a} \quad (5)$$

2.2.2 Parameter selection of the model

According to the GM(1,1)gray prediction method, the prediction gap G between the day price and the true price predicted by using data from the previous ϕ Bit and ϕ Gold days is tested using a Python program for Bitcoin, Gold respectively. And the variance and mean of the prediction gap G are used to measure the final choice.

Since bitcoin prices tend to be highly volatile, Therefore, we test the predicted values for the upcoming day when ϕ_{Bit} takes values from 3 to 7, and try to see if better results are obtained when the variable weights are mixed with predicted data from multiple days.

It is found that the GBit obtained when ϕ_{Bit} takes the values of 3,4 and 5 has the best effect through the test. We try to adjust the weights of 3,4 and 5days results for the overall evaluation and try to perform a mixed evaluation. Though testing we found that the GBit obtained when ϕ_{Bit} took the values of 3, 4 and 5 had the best effect, tried to adjust the weight of the results of 3, 4 and 5 days for the overall evaluation, tried to conduct a mixed evaluation, and found that when $\phi_{Bit} = 3$ accounted for 43%, $\phi_{Bit} = 5$ accounted for 57% and $\phi_{Bit} = 4$ accounted for 0%, $E(\text{GBit}) = 1.70\%$ and $D(\text{GBit}) = 0.027\%$ has obvious comparative advantages, so such hybrid prediction parameters are chosen as the parameters used in the gray prediction of the bitcoin price $GM(1,1)$, and the specific prediction results are compared with the real results as shown in Fig 1.

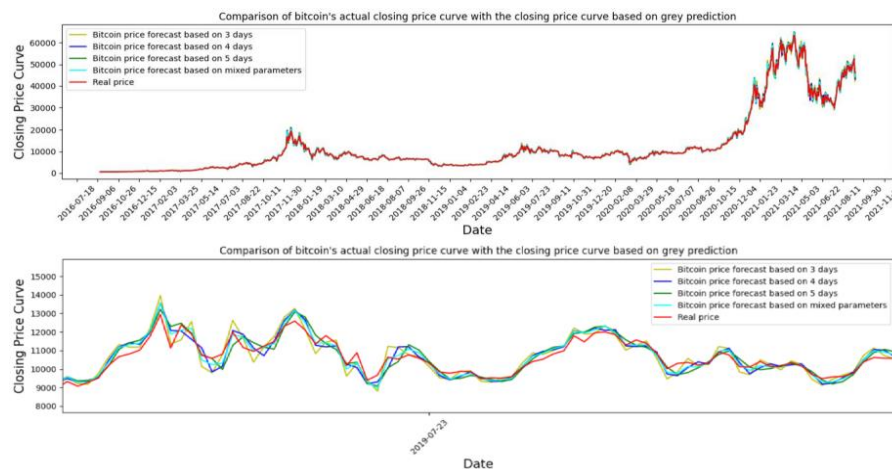


Fig. 1 Comparison of bitcoin's actual closing price curve with the closing price curve based on grey prediction and partial enlargement

Unlike Bitcoin, the volatility of gold's price tends to be influenced by past medium to long- term prices. Therefore, the test ϕ_{Gold} with values from 10 to 30 is more informative for the forecast of the upcoming day.

The following table shows some of the typical data of GGold values tested at 10 to 30 for ϕ_{Gold} .

Through the test, it was found that $E(\text{GGold})$ and $D(\text{GGold})$ were also increasing as the value of ϕ_{Gold} was taken higher and higher. Considering gold's own property of relatively slow price change, $\phi_{Gold} = 10$ is chosen as the parameter used in the gray prediction of gold price $GM(1,1)$, and the specific prediction results are compared with the real results as shown in the figure 2.

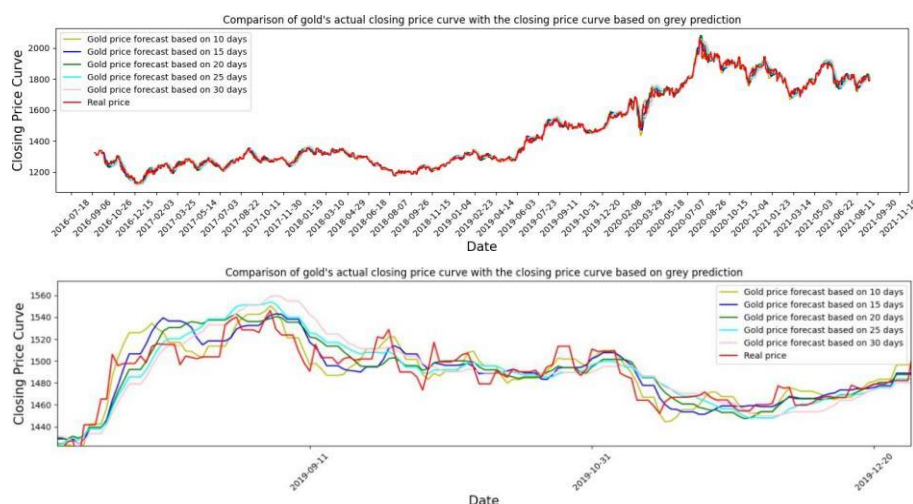


Fig. 2 Comparison of gold's actual closing price curve with the closing price curve based on grey prediction and partial enlargement

2.3. Rational Investment Model

2.3.1 The results

In our decision, we put 45% of the initial \$1,000 in bitcoin, 45% in gold, and 10% in reserve. The following results are obtained under our model, as shown in table 1.

Table 1. Three Scheme comparing

	Cash(C)	Gold(G)	Bitcoin(B)
The initial state	\$1000.0000	0.0000	0.0000
End of period 1	\$1987.2039	0.0000	0.0000
End of period 1	\$118.6626	0.3557	0.5129
End of period 1	\$300.5595	0.3298	0.6939
End of period 1	\$1171.9208	0.0000	1.3435
End of period 1	\$103191.1932	0.0000	0.0000

2.3.2 Bitcoin Investment

(1) General Idea

The price of Bitcoin in the five years from 2016 to 2021 has a large increase and fluctuation. Bitcoin has a volatile price curve, making it a risky investment for a short term. We need to invest carefully.

We first adopt the grey prediction method to predict the price of bitcoin in the future through the price of bitcoin before the purchase day. Since the price of Bitcoin fluctuates, the price data of 3 to 5 days before the purchase date can be used to predict the price of the day, so that a more accurate forecast price can be obtained. Since it is difficult to prepare for long-term investment by only predicting the price of the day, we use the method of recursive prediction to predict the bitcoin price of the next 5 days (including the purchase day). The specific method of such prediction is to first predict the price of the day, then calculate the price of the day into the known data to predict the price of the next day, and so on to predict the bitcoin price of the next five days (including the purchase day). We will decide our investment strategy on that day based on the price trend of bitcoin in these five days.

(2) Trading Strategies

Bitcoin is traded in two situations: buy and sell. In our model, when the price of bitcoin is predicted to rise, we buy it before the rise. When you predict a drop in bitcoin prices, sell before the drop. Bitcoin can't be bought and sold very often because of the impact of commissions and ups and downs, so we judge before buying and selling. When considering buying, if the price of bitcoin on the last day of the price increase is not significantly higher than that of the day, there is no need to buy, because frequent buying will increase the commission cost significantly. When considering selling, you should sell bitcoin if the predicted decline is too steep. We can also calculate the value of our bitcoin holdings on that day and compare them with the value when we bought them, and sell them if the gains meet our expectations. In short, it is to decide whether to buy or sell by judging the size of the expected return and weighing the degree of risk.

As for how to measure the degree of risk and expected return of Bitcoin, we can solve it by setting parameters. We set the magnification ratio as S and the maximum allowable loss as P . The amplification ratio S controls the amount of revenue users expect to get. The maximum loss tolerance P controls the loss rate that the user can tolerate. These judgments make it possible to buy bitcoin on a rally. Sell bitcoin when it is about to fall sharply or when the net gain from selling is large. As bitcoin rose significantly, it is suitable for short-term investment.

When the initial investment in bitcoin is \$450, the following table shows the trading operations through this model:

Table 2. Bitcoin purchase method obtained after running the model

Way	Date	BTC	Cash	Date	BTC	Cash	Date	BTC	Cash
Sell	2018/9/13	0.000	2585.82	2019/7/21	0.000	5910.01	2020/3/23	0.000	6543.54
Buy	2018/9/16	0.389	0.000	2019/7/22	0.547	0.00	2020/3/25	0.948	0.00
Sell	2018/11/11	0.000	2429.52	2019/8/10	0.000	6358.95	2020/5/8	0.000	9297.35
Buy	2018/11/24	0.547	0.00	2019/8/13	0.548	0.00	2020/5/9	0.928	0.00
Sell	2018/12/1	0.000	2207.58	2019/9/21	0.000	5276.49	2020/8/31	0.000	10648.01
Buy	2018/12/5	0.561	0.00	2019/9/25	0.605	0.00	2020/9/4	1.025	0.00
Sell	2019/1/7	0.000	2218.72	2019/10/21	0.000	4878.40	2021/2/22	0.000	75691.73
Buy	2019/1/11	0.597	0.00	2019/10/22	0.582	0.00	2021/2/24	1.518145	0.00
Sell	2019/2/22	0.000	2306.30	2019/11/18	0.000	4847.50	2021/5/12	0.000	84431.63
Buy	2019/2/23	0.568	0.00	2019/11/23	0.652	0.00	2021/5/18	1.898698	0.00
Sell	2019/5/14	0.000	4351.69	2019/12/14	0.000	4635.36	2021/5/19	0.000	79847.76
Buy	2019/5/16	0.520	0.00	2019/12/15	0.643	0.00	2021/5/21	1.919417	0.00
Sell	2019/6/1	0.000	4357.77	2020/2/23	0.000	6093.11	2021/6/18	0.000	79847.76
Buy	2019/6/2	0.489	0.00	2020/2/27	0.680	0.00	2021/6/20	1.976204	0.00
Sell	2019/6/27	0.000	6195.99	2020/3/5	0.000	5836.43	2021/9/6	0.000	102019.3
Buy	2019/6/29	0.491	0.00	2020/3/11	0.726	0.00	Total Cash: 102019.27239321964		
Sell	2019/7/11	0.000	5820.98	2020/3/12	0.000	5643.95			
Buy	2019/7/15	0.560	0.00	2020/3/14	0.986	0.00			

In order to show the transaction process more directly, a schematic diagram of bitcoin transaction is drawn. The yellow shaded part in the diagram represents that all the funds invested in bitcoin are invested in the market to hold bitcoin. The blank represents selling all bitcoin and holding cash, as shown in figure 3:

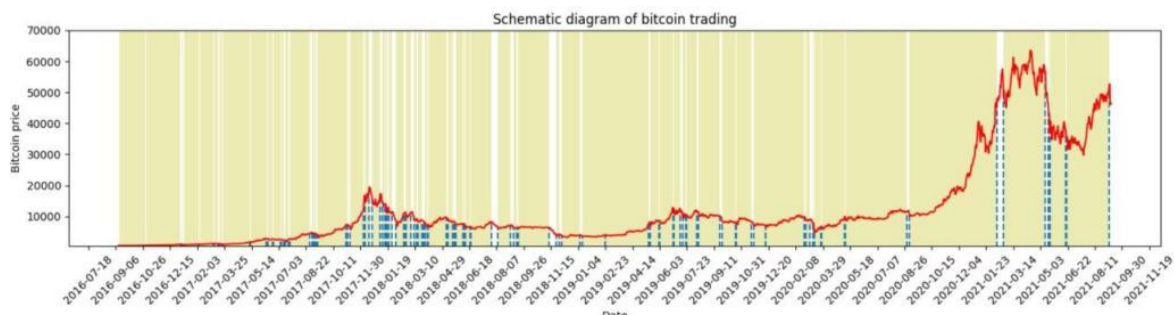


Fig. 3 Schematic diagram of bitcoin trading

2.3.3 Gold Investment

(1) General Idea

From the gold price trend chart in the title, it can be seen that during the five years from 2016 to 2021, the gold price is relatively stable in the early stage, and will experience a large upward stage in the later stage. The overall price of gold is in a relatively stable state, so gold investment is a long-term investment with low risk. We can take a sound, long-term approach to gold to make profits.

We first use the grey forecasting method, using the gold price before the purchase date to predict the gold price in the future. Because the gold price is relatively stable, the price data of the 10 days before the purchase date can be used to predict the gold price of that day, so that a more accurate forecast price can be obtained. Since it is difficult to prepare for long-term investment by only predicting the price of the day, we use the method of recursive prediction to predict the gold price of the next 15 days (including the purchase day). The specific method of such prediction is to first predict the price of the purchase day, then calculate the predict price of the purchase day into the known data to predict the price of the next day, and so on to predict the gold price of the next 15 days (including the purchase day). We will be based on the 15-day gold price forecast to determine the purchase day's investment strategy.

(2) Trading Strategies

When the initial investment in gold is \$450, the following table shows the trading operations through this model:

Gold is traded in two categories: buy and sell. In our model, when gold is expected to rise, buy before the rise. When we anticipate a decline in the price of gold, sell before the decline.

Given the impact of commissions and fluctuations, gold can't be bought or sold very often, so we judge it before buying or selling. When considering buying, If the price of gold on the last day of the forecast price rise is not significantly higher than that of the current day, there is no need to buy, because frequent buying will cause a large increase in commission cost. When consider selling, If the price of gold on the last day of the forecast decline is not significantly lower than that of the current day, it means that the gold price will not fall significantly and there is no need to sell. We can also calculate the profit by comparing the value of gold held that day with that of the cost of buying the gold. By comparing the two aspects, we find that the price of gold does not fall by much, and the price of gold cannot gain much profit by selling it, so we do not sell it. Trade only when one of the two conditions is met. In short, it is to decide whether to buy or sell by judging the size of the expected earnings.

As for how to measure the rise and fall of gold and expected revenue, we can solve it by setting parameters. We judge the fluctuation of gold price by dividing the predicted gold price in 15 days by the gold price on that day and the size of parameter β . β represents the expected change in the gold price. Similarly, we measure the net return by calculating the gain from selling gold that day divided by the cost of buying gold and the parameter γ , γ controls the expectation of profitability. These judgments have led to buying gold only when the price has risen significantly, selling gold when the price is about to fall sharply or when the net gain from selling is large. This can significantly reduce the cost of paying commissions and make a better profit. And the optimal value of the parameters can be determined by trial and error.

When the initial investment in gold is \$450, the following table 3 shows the trading operations through this model:

Table 3. Gold purchase method obtained after running the model

Transaction date	Way	Quantity of gold(G)	Cash(C)
2017/1/9	Buy	0.3781	450.0000
2017/4/27	Sell	0.0000	472.6409
2018/1/4	Buy	0.3560	0.0000
2019/7/4	Sell	0.0000	498.6667
2019/8/8	Buy	0.3301	0.0000
2020/3/4	Sell	0.0000	527.9248
2020/3/27	Buy	0.3197	0.0000
2020/8/14	Sell	0.0000	615.4225
2020/12/9	Buy	0.3308	0.0000
2021/1/13	Sell	0.0000	608.8376
2021/3/11	Buy	0.3496	0.0000
2021/6/11	Sell	0.0000	651.0389

In order to show the transaction process more directly, a schematic diagram of gold transaction is drawn. The green shaded part in the diagram represents that all the funds invested in gold are invested in the market to hold gold. The blank represents selling all gold and holding cash, as shown in figure 4:



Fig. 4 Schematic diagram of gold trading

2.4. Aggressive Investment Model

When investors are not rational enough, they will make riskier buys and sells in the expectation of greater returns. In this case, we would put as much money as possible into high- risk, high-return bitcoin investments and not buy gold at all. At this point at the end of five years, the total gain would be over \$8.55 million. While it may seem that aggressive investments would yield better returns, the returns from this ultra-risky, unpredictable investment are not informative, so this model will only be discussed here and will not be used in the actual modeling application.

3. Model Evaluation

Transaction costs are a big factor in bitcoin and gold investment. Generally speaking, transaction costs affect the number of trades and then the strategy of trades by affecting the returns. Different investment projects have different sensitivities to transaction costs. From a practical point of view, investment products with stable price fluctuations, such as gold, are generally long-term investments, and their trading frequency is not high, so the transaction cost has little influence on them; As for investment products like Bitcoin, due to its volatile price, high risk and high return characteristics, it often needs to seize opportunities in time, so the transaction frequency is relatively frequent, and the transaction cost has a great impact on it.

In order to more directly reflect the impact of transaction cost on trading strategy, we divided the commission from 0 to 10% into 20 groups, each group with an interval of 0.5%, and calculated the respective returns of gold and Bitcoin and the changes in these 20 cases within 5 years. The specific results are shown in the following table. (All start at \$450).

Table 4. The relationship between revenue and transaction costs

$\alpha(\%)$	Bitcoin Earnings	Gold Earnings	Bitcoin Profitability	Gold Profitability
0.0	1608189.894	524.8094908	3573.755319	1.166243313
0.5	678027.5003	566.0906491	1506.727778	1.25797922
1.0	230957.9958	601.8382774	513.2399907	1.337418394
1.5	127994.2059	592.921737	284.4315687	1.31760386
2.0	75092.78429	533.8705335	166.872854	1.186378963
2.5	39136.1742	512.9267354	86.96927599	1.13983719
3.0	20688.50232	492.7996305	45.97444959	1.09511029
3.5	9310.893569	73.45661578	20.6908746	0.163236924
4.0	4604.587762	454.8664551	10.23241725	1.010814345
4.5	2973.328411	440.3120196	6.607396469	0.978471155
5.0	1393.994579	406.9507712	3.097765731	0.904335047
5.5	957.3889577	390.9507463	2.127531017	0.868779436
6.0	978.9253023	377.6566401	2.175389561	0.839236978
6.5	586.6651722	371.6852128	1.303700383	0.82596714
7.0	673.2254688	459.1951142	1.496056597	1.020433587
7.5	627.2763759	445.5531839	1.393947502	0.990118186
8.0	537.9006981	439.3579372	1.195334885	0.976350971
8.5	417.0384029	426.2852724	0.926752007	0.947300605
9.0	280.3728447	418.0621916	0.623050766	0.929027092
9.5	257.0278319	375.4429248	0.57117296	0.834317611
10.0	326.8027359	364.241996	0.726228302	0.809426658

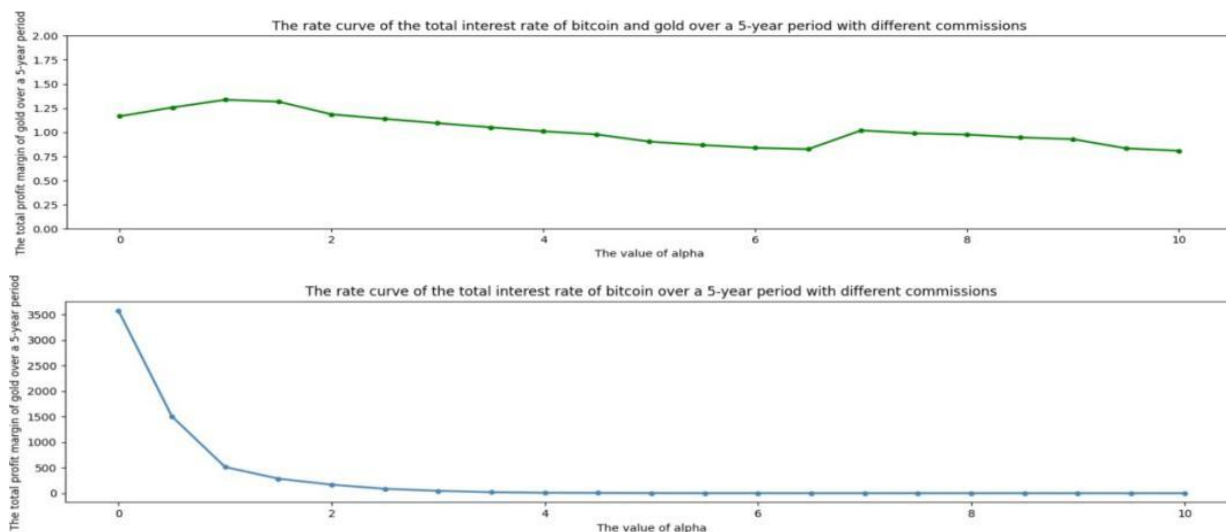


Fig. 5 The rate curve of the total interest rate of bitcoin and gold over a 5-year period with different commissions

As can be seen from the table 4 and figure 5, the rate of return of Bitcoin is very sensitive to changes in transaction costs. With the increase of transaction costs, the rate of return of Bitcoin declines exponentially. The yield of gold will also decrease with the increase of transaction cost, but the decline trend is more moderate, and when the transaction cost increases to a certain extent, the yield of gold also increases slightly. This is because with the increase of transaction cost, the transaction frequency of gold decreases to a certain extent, so that the model will only buy and sell when there is a lot of profit, and finally can get profit. It can be seen that when the transaction cost is low, investment in products with high risks and high returns such as Bitcoin can achieve good returns. When the transaction cost is high, investment in high-risk products should be reduced and investment in stable products such as gold should be increased to ensure stable returns.

4. Summary

First, we have built two investment models based on the characteristics of gold and bitcoin respectively. They can predict the market conditions in the coming days and perform certain operations based on the given initial capital and commissions. Moreover, the prediction accuracy is high and the curve fit is good. In general, both models always make the right decisions. The gold investment model avoids huge losses due to its low risk, by predicting whether the gold price will plummet at the time of investment to decide whether to sell or not. The Bitcoin investment market is volatile, so when running the Bitcoin investment model, if the profit or loss reaches the expected value, it will stop running to stabilize the return and reduce the risk. In addition, the investment scheme of the bitcoin investment model can also be adjusted to different initial funds to make the model more robust and scalable.

Our model is relatively scientific and rigorous. However, the quoted gray forecasting model predicts linear changes in days and has a lag time to actual changes, which makes it difficult to buy or sell bitcoin or gold at the optimal time and is insensitive to small continuous changes in product prices. A better time series forecasting model could greatly improve this problem.

In summary, with this model, we can invest in two products with different properties as a rational investor and see how amazing the returns can be if we make an accurate risky investment as a risk taker. However, the model is not realistic after all, as it can be predicted more accurately without the influence of various environmental factors, and the reality is even more unpredictable. Therefore, there are risks and investments should be made with caution.

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