

Multi-Objective Optimization for Water Resource Allocation among Five States

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Abstract. In the face of worsening drought conditions in the western United States, how to deal with Cascade Reservoir Optimal Scheduling and rationally allocate water resources from two lakes to five states is a good problem worth discussing. This paper provides a rational method of allocation of water improves resources from Lake Powell and Lake Mead to five states. At present, the best scheme is to draw 0.077km³ of water from Lake Powell and 0.83km³ from Lake Mead. The best scheme is to use 77.7% water in agriculture, 17.9% in residential, 2.8% in industry, and 1.5% in power generation.

Keywords: Linear programming, NSGAI, Multi-Objective Optimization, Cascade Reservoir Optimal Scheduling

1. Introduction

In the past, the Colorado River was turbulent with sufficient water. Nowadays, the problem of drought is becoming more and more serious. The surrounding states' demand for water resources far exceeds the supply of river water. [1] In the last century, several dams were built in the Colorado River Basin, which played a key role in hydroelectric power generation and water regulation. Among them, the Grand Canyon dam and the Hoover dam are particularly important, with a large amount of water storage and power generation. In the face of water reduction and water level decline, natural resource officials in the U.S. states of Arizona (AZ), California (CA), Wyoming (WY), New Mexico (NM), and Colorado (CO) currently need a reasonable and sustainable water resources allocation scheme to achieve the coordination of supply and demand of water resources.

In general, our task is to give a plan for water allocation that satisfies all parties.

First, the total amount of water withdrawn has to meet the needs of the cities that depend on the Colorado River for their water supply (general water use and hydroelectricity generation). So, for a given amount of water storage, we use a linear programming model to solve for how much water can be taken from the two lakes (Lake Powell and Lake Mead) to achieve an overall balance of supply and demand. As the amount of water stored in the lakes changes, our model can be used to calculate a reasonable amount of water to be withdrawn.

Second, for the allocation scheme among the states, we only take a mathematical approach so that the final combined benefit is obtained. In terms of benefits, our model not only improves the efficiency of each state's water estimate but also weighs the competing interests of each state to maximize the overall benefits of the Colorado River water supply. This defines our model as a multi-objective optimization model.

At the same time, the allocation of the withdrawn water to the various industries that consume it (here, only agriculture, industry and residential) is also required to maximize the comprehensive interests through the optimization model.

In addition to water withdrawal from reservoirs, the amount of hydroelectric power generation is also an aspect of the overall benefits. Hydroelectricity generation accounts for a certain percentage of urban electricity consumption. However, since electricity generation is delivered to cities by a unified operator, we do not divide it into states for calculation.

We specify the model according to the topic question. The model is applied and extended to answer the questions in different cases. The result is a complete solution to the current water shortage problem in the western United States. At the same time, our algorithm and model can be applied for a certain time range of water quantity changes.

2. Methodology

2.1. Data Selection

Some of the constraints in the model are referenced to various data from Glen Canyon Dam (Lake Powell) and Hoover Dam (Miller Lake). The data obtained from the query are as table 1 [2].

Table 1. Data Related to River Flow.

Objects	P	M
Initial water level	1.0755km	0.3253km
Minimum water level constraint(maximum value of power generation constraint, ecological constraint)	3490ft (1060m)	1050ft (320m)
Flood control constraint(maximum value of reservoir storage allowed for flood)	3661ft (1115m)	1229ft (375m)
Theoretical reservoir capacity	32.0748km ³	38.30km ²
Minimum reservoir capacity	4.26km ³	9.63km ³
Maximum reservoir capacity	22.53km ³	34.15km ³
Electricity generation flow constraint (to meet the five major states' hydroelectric)	>1.987×10 ¹² J	
Water supply constraint (to meet water use in the five states)	>1.5367km ³	

The data we chose for the model building was found as recent as possible. This allows us to make our model more applicable to the current situation. As the report says, this is nearly 20 years of unprecedented severe drought in the western United States. So our models use data primarily from 2000 onward for prediction and simulation assessment. Going to a large amount of data that is too early would affect the current viability of the model.

Water consumption data for the five states from 1965-2015 are as figure 1 [3].

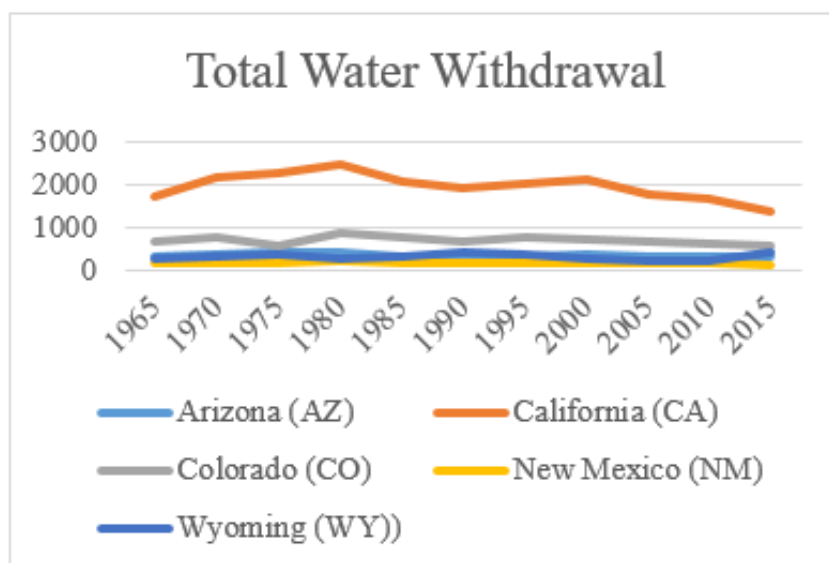


Figure 1. Total Water Withdrawal from 1965 to 2015.

The three areas of water use data help us to calculate the relationship between them and GDP to establish the objective function. We visualize this in a bar chart as figure 2 [4].

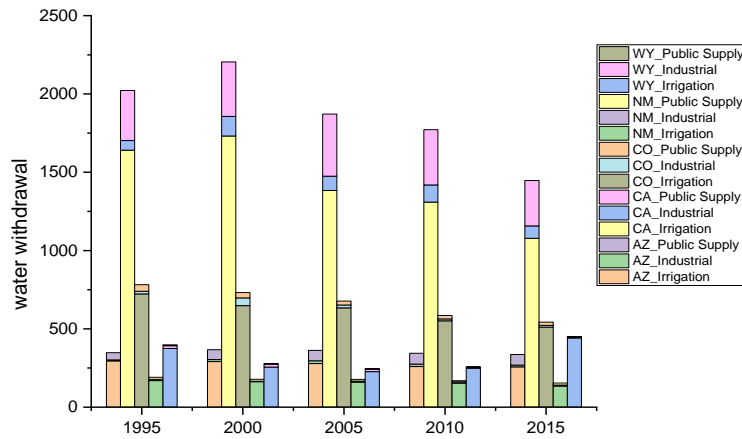


Figure 2. Water Use in the Five States in Three Areas.

Based on the gross product by the industry for specific states given by the U.S. Bureau of Reclamation, we filtered and summed to get the gross product for agriculture and industry from 2015-2022, such as table 2 [5].

Table 2. Data of Output and Water Withdrawal in 2015.

	Arizona	California	Colorado	New Mexico	Wyoming
Output/(millions of current dollars)					
Agricultural	4026.2	63967.1	5135	2444.5	1339.4
Industrial	243944.3	2329854.7	275625.2	74559.9	55396.7
Water Withdrawal/km					
Irrigation	257.16	1077.88	511.03	134.74	442.21
Industrial	11.30	79.72	170	6.53	5.96
Public Supply	67.85	289.36	20.44	12.66	2.90

2.2. Method

To give the operating scenarios for the dams, we chose to use a linear programming model to determine the appropriate amount of water to be taken from the two lakes to respond to a fixed set of water supply and demand conditions for the five states. The programming model is first needed constraints.

Constraints on minimum water level (MinM/MinP: minimum value of power generation constraint, ecological constraint):

$$S_M \geq \text{MinM} \tag{1}$$

$$S_P \geq \text{Minp} \tag{2}$$

Constraints on flood control (MaxM/MaxP: maximum value of reservoir storage allowed for flood control):

$$S_M \leq \text{MaxM} \tag{3}$$

$$S_P \leq \text{MaxP} \tag{4}$$

Constraint on power generation flow (Estimated based on the total energy output quota of hydropower generation according to the U.S. Energy Administration):

$$\rho g V_{EM} (M + H_M) + \rho g V_{EP} (P + H_P) \geq \sum B_i * 0.0255, \tag{5}$$

$i = AZ, CA, WY, NM, CO$

It is assumed that the generating unit can fully convert the water potential energy into the required electrical energy.

Constraint on water use:

$$V_{WM} + V_{WP} \geq \sum A_i, \quad i = AZ, CA, WY, NM, CO \quad (6)$$

A more accurate value of reservoir capacity is obtained by fitting a curve of water level to reservoir capacity. Constraints on reservoir capacity:

$$V_{WM} + V_{EM} - V_{PM} + S_M \geq V_M \quad (7)$$

$$V_{WP} + V_{EP} + V_{PM} + S_P \geq V_P \quad (8)$$

Non-zero:

$$V_{WM}, V_{WP}, V_{EM}, V_{EP}, V_{PM}, V_{PM}, S_M, S_P \geq 0 \quad (9)$$

The final objective function in the planning model will reflect the level of satisfaction with the water allocation scheme. The satisfaction value will give a good indication of the reasonableness of the operating scheme.

F represents the ratio of water and electricity supply to demand, responding to the degree of satisfaction, the higher the ratio of water and electricity supply to demand, the higher the satisfaction. The following equation is built from the relationship between supply and demand.

$$F = \frac{\rho g V_{EM}(M + H_M) + \rho g V_{EP}(P + H_P)}{\sum B_i} + \frac{V_{WM} + V_{WP}}{\sum A_i}, \quad (10)$$

$i = AZ, CA, WY, NM, CO$

Up to here, we can get the plan for the water we take from each lake with the determined water level.

For resolving the competing interests of water availability for general (agricultural, industrial, residential) usage and electricity production, we chose NSGAI [6] to obtain the best method to maximize the competitive benefits by continuously optimizing the objective function with constraints on indicators such as water storage, power generation flow, etc.

First of all, our decisions on supply and demand are measured on a one-day cycle.

Based on the supply-demand balance, there is the following equation:

$$G_i = In_i + Ir_i + Re_i, \quad i = AZ, CA, WY, NM, CO \quad (11)$$

$$V_p = V_{p0} + S - G_i - VE1 \quad (12)$$

$$V_M = V_{M0} - G_i - VE2 \quad (13)$$

Based on the actual situation, we converted the conditions into mathematical form to obtain the following constraints needed for the model.

Constraints on minimum water level (MinM/MinP: minimum value of power generation constraint, ecological constraint):

$$V_M \geq \text{MinM} \quad (14)$$

$$V_p \geq \text{MinP} \quad (15)$$

Constraints on flood control (MaxM/MaxP: the maximum value of reservoir storage allowed for flood control):

$$V_M \leq \text{MaxM} \quad (16)$$

$$V_p \leq \text{MaxP} \quad (17)$$

Constraints on power generation flow:

$$QGN_m \leq VEm \leq QGM_m \quad (18)$$

$$QGN_p \leq VEp \leq QGM_p \quad (19)$$

Constraint on water supply:

$$G_i \geq U_i, \quad i = AZ, CA, WY, NM, CO, ME \quad (20)$$

Constraint on Mexico's rights:

$$V_{Em} \geq V_{min} \tag{21}$$

Constraints on supply generation:

$$\sum G_i + VEP + S \leq V_{P0} - MinP, i = WY, NM, CO \tag{22}$$

$$\sum G_i + VEM \leq V_{M0} - MinM, i = AZ, CA \tag{23}$$

General Constraint:

$$X \geq 0 \tag{24}$$

We also need to establish the objective function of the model and then find the optimal solution by the NSGAI to obtain the best allocation scheme.

Industrial [7]:

$$W_{Ini} = a \times Ini, i = AZ, CA, WY, NM, CO \tag{25}$$

Agriculture [8]:

$$W_{Iri} = b \times Iri, i = AZ, CA, WY, NM, CO \tag{26}$$

Residential:

$$W_{Rei} = c \times Rei, i = AZ, CA, WY, NM, CO \tag{27}$$

By dealing with the relationship between the maximum power generation and the maximum flood discharge, we can get hydroelectric power generation.

Electricity:

$$W_E = d \times (VE1 + VE2) \tag{28}$$

Satisfaction function (Minimization of regional water shortage):

$$Satisfaction = \frac{\sum(G_i - U_i)}{\sum U_i}, i = AZ, CA, WY, NM, CO \tag{29}$$

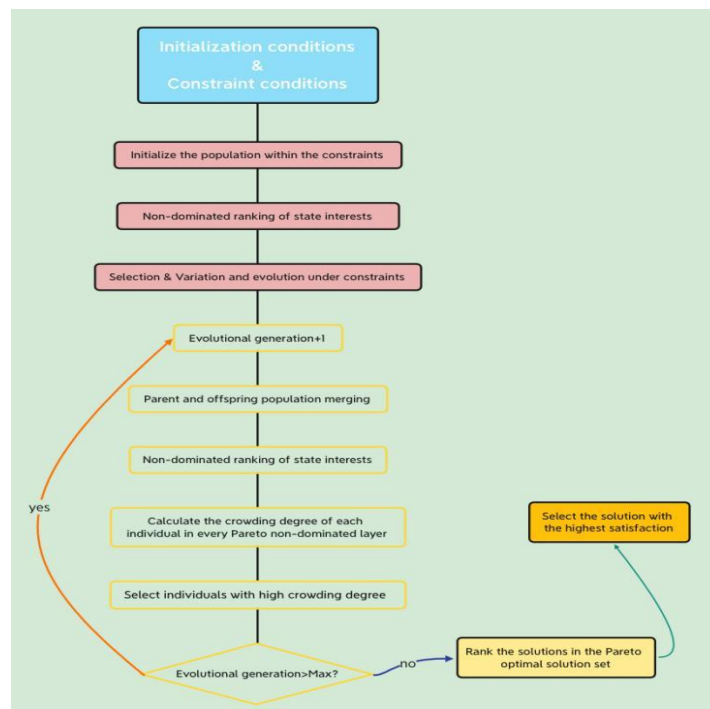


Figure 3. Algorithm Flow Chart of our model.

Algorithm Flow Chart of our model

The flow chart of the algorithm for our model is shown in Figure 3.

How long it takes to re-run the model mainly depends on the satisfaction of the optimal solution.

Based on the trend of water volume change and satisfaction, we set the satisfaction level to be greater than 10% when the optimal solution from the original data no longer matches the available water volume. In this case, the model needs to be run to find a better solution.

Substituting $(1 \pm \alpha \%)$ of the total water volume into our model, when the satisfaction function increases or decreases by 10%, we can solve for $\alpha = 0.05$.

The period for re-running is $\frac{\text{Water volume}}{V}$ days with additional replenishment of V water per day.

Finally, substituting the latest data, the following results can be calculated. This verifies the feasibility of our model.

Table 3. Results of water allocation (km3).

Industry-use in AZ	Industry-use in CA	Industry-use in WY	Industry-use in NM	Industry-use in CO
0.001967852	0.017646565	0.001808718	0.001963386	0.002662765
Agriculture-use in AZ	Agriculture-use in CA	Agriculture-use in WY	Agriculture-use in NM	Agriculture-use in CO
0.080995436	0.58421797	0.00362348	0.020321767	0.021394225
Residential-use in AZ	Residential-use in CA	Residential-use in WY	Residential-use in NM	Residential-use in CO
0.022745742	0.129509821	0.00052945	0.002149509	0.009299634
Waterflows from Lake Powell to Lake Mead	Waterflows out from Lake Mead			
0.006602664	0.00753196			

By using this model, we will obtain the water allocation scheme for the target optimal case to address the competing interests of available water.

3. Results and Discussion

3.1. Dam Operations

Based on the mathematical model we constructed, the following recommendations are given to state natural resources negotiators to make a reasonable and sustainable allocation of the Colorado River's water resources.

This section focuses on new operation scenarios for Glen Canyon Dam and Hoover Dam under current drought conditions to achieve a balanced sharing of limited water resources for water and power generation. The general premise is that dam operating programs need to respond to a fixed set of water supply and demand conditions. We give the following suggestions based on the modeling results according to calculation results based on brought the latest data into the model.

Lake Powell initial volume: P0

Lake Mead initial volume: M0

Lake Powell storage capacity: P

Lake Mead storage capacity: M

Water used in Lake Powell for power generation: P_Electricity

Water used in Lake Mead for power generation: M_Electricity

Lake Powell to Lake Mead: P_M

Water withdrawal from Lake Powell: P_Water

Water withdrawal from Lake Mead: M_Water

The Sankey diagram of water allocation is shown in Figure 4

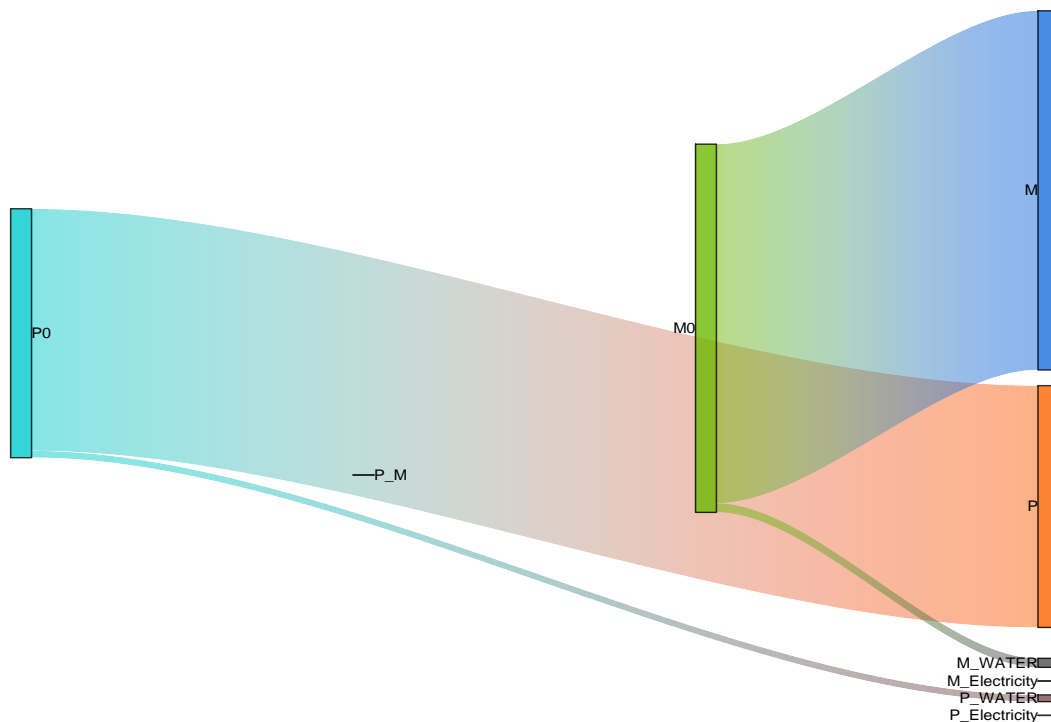


Figure 4. Sankey Diagram of Water Allocation.

At a water level of P (Substitute the latest water level data) at Lake Powell, the amount of water that can be taken is $0.0769582622813485 \text{ km}^3$. Based on the model, it can be seen that the resulting water withdrawal can satisfy the water demand of the three states. In addition, this also satisfies the hydroelectric power generation water line requirement of Glen Canyon Dam. In other words, this amount of water can be used in a way that maximizes the efficiency of the resource and is scientifically sustainable. We believe that it can respond positively to the current water shortage situation.

The general water use (excluding water consumed by hydropower) allocation for the three states is approx:

- New Mexico (NM): 0.024434662 km^3
- Colorado (CO): 0.033356624 km^3
- Wyoming (WY): 0.005961648 km^3
- Water use for hydropower generation: 0.006602664 km^3
- Operations of Hoover Dam

At a water level of M at Lake Mead, the amount of water that can be taken is $0.829551425965461 \text{ km}^3$. Unlike Lake Powell, we need to consider that Mexico's interests will be met by taking water in Lake Mead.

The general water use (excluding water consumed by hydropower) allocation for the two states is approx:

- Arizona (AZ): 0.10570903 km^3
- California (CA): 0.731374356 km^3
- Water use for hydropower generation: 0.00753196 km^3

After water allocations from the above plan are implemented, more than 0.00753196 km^3 of water will be allowed to flow into the Gulf of California from the Colorado River.

Mexico's rights [9] create restricted by a water volume of 0.00753196 km^3 . At present, the rights of Mexico can still be guaranteed to some extent. However, the amount of water withdrawn will be reduced compared to the original agreement.

Since natural conditions are always changing, we predicted the duration of the application cycle of our model after simplifying the complex environmental factors. According to the current data: the average daily precipitation of Lake Powell is about 400mm, and the average daily precipitation of

Lake Mead is about 375mm. Substituting the data, we can get that the re-run frequency is 21 days for Lake Powell and the re-run frequency is 25 days for Lake Mead.

The amount of water withdrawn will indeed be less than the amount of water used by the states in previous years. But this is a necessary water conservation measure in the increasingly drought-prone western United States. Our mathematical solution is modeled from the perspective of optimizing the ultimate overall goal so that the data we produce provides a degree of assurance that the basic water needs of each state.

To some extent, our allocation ensures that state water withdrawals are fair to each other.

In response to the emergency intensification of the drought situation, we need to change the dam operation according to specific environmental conditions. The above proposal is a general condition forecast based on current data.

3.2. Means to Resolve the Competing Interests of Water Availability

We obtained the optimal solution for the allocation of water resources with general water use (industrial, agricultural and residential) and hydroelectric power generation as evaluation indicators. The allocation scheme we give can effectively enhance the competitive benefits. At the current level of available water resources, the pie chart gives a better visualization of the share of water use in the four areas (the other areas are not considered in the model because they are less affected by water use): industry, agriculture, residential, and electricity generation.

The water allocation pie chart is shown in Figure 5.

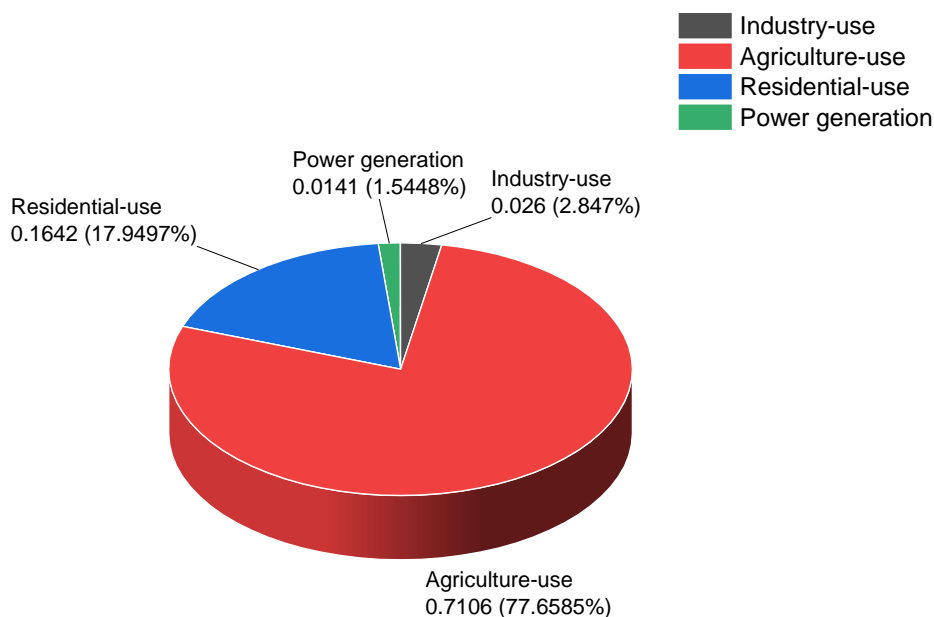


Figure 5. Water Use Allocation Pie Chart.

The decision maker can use our model to find the optimal solution to adjust the water consumption in each area according to the change in water storage to maximize the overall benefit.

3.3. Open Source or Cut Costs

The western United States is suffering from unprecedented drought conditions. In the face of drought, we must prefer to cut costs to allocate the limited water resources more rationally and efficiently. We used our model to measure the scenarios for allocating water to various aspects to achieve the target optimal solution for different water availability. The comparison of the results can guide us to allocate water resources as well as possible to maximize the ultimate combined benefits under the available water supply conditions.

We used the following bar chart for comparison to derive scientifically reliable guidance, such as in figure 6.

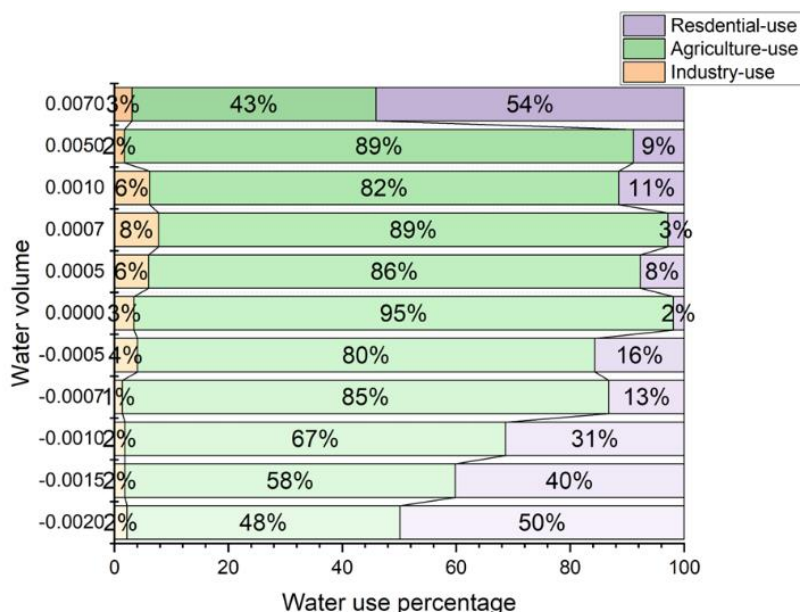


Figure 6. Bar Chart of Water Use Ratio under Different Water Availability.

From the comparison, we can adjust the water consumption in these four areas in response to the reduction in water volume.

In the case of water shortage, the priority is to ensure basic water for people.

Agriculture accounts for the largest share of water use, and it has a greater scope for adjustment.

Industrial water use accounts for a small percentage of water use and will generally decrease with water volume.

Depending on the specific situation, we can substitute water volume and other parameters into the model to obtain the specific optimal water use plan.

Such approaches allow us to maintain a scientific and sustainable integrated development with limited resources. Cities can achieve the best production value while not causing more serious human damage to natural resources.

3.4. Discussion

In case the demands for water and electricity in the communities of interest change, the restraint conditions are not suitable. So we need to modify the demand constraints and the satisfaction function. After that, re-running the model can obtain a new water distribution. We can get the optimal solution in this case. If the general usage shrinks, more water will be used in hydropower to gain additional economic benefits. If the general usage grows, to meet the minimum demands of general usage, we must reduce the hydropower usage.

As the proportion of renewable energy technologies, the importance of hydropower declined. So, water use for hydropower has decreased. To cope with the new situation, we can reduce weight of hydropower in satisfaction function. By re-running the model, we can get the optimal solution. In the case of meeting the demand, the storage in two lakes can be increased, which prospects for sustainable development.

Similar to the first condition, the demands were reduced. Changing the demand constraints and the satisfaction function, we can similarly re-run the model to get the optimal solution. From the result, we conclude that implementing additional water and electricity conservation measures is a good way to help alleviate drought.

The non-dominant comparison method is chosen when comparing allocation options.

The context of this question is a negotiation between the Natural resource officials of five states (Arizona, California, Wyoming, New Mexico, and Colorado), each state's natural resource officials have their own state's interest as the starting point, and they all want to obtain the maximum benefit for their state. Therefore, we adopt the Pareto optimal solution method of multi-objective planning to

simulate the dispute of each state's interests, which can better simulate the negotiation process of Natural resource officials in five states. In this process, the serious imbalance of interests among the states is well avoided.

We construct the 'satisfaction' function to measure the supply and demand of benefit distribution. The larger the value of the function, the more reasonable the supply-demand relationship in the allocation. This simulates the process of macro control. Under a negotiated benefit dispute, an indicator is needed to measure the reasonableness of allocation versus demand. This is a good simulation of this aspect of the negotiation process in which the parties must compromise.

The model can be applied to a variety of situations simply by modifying parameters such as demand by state and current water volume. Instead, it is not necessary to make extensive adjustments to the model to obtain the optimal water allocation scheme for the new conditions. For other capacity classes of reservoirs, the water allocation optimization problem can also be solved by modifying the constraints.

We set a large number of genetic generations when using the genetic algorithm to obtain the optimal solution as much as possible.

We did not find the latest GDP values for each industry in each state, so we used data from 2015 to obtain the industrial water supply efficiency apportionment coefficient and the agricultural water efficiency coefficient. This leads to bias in measuring water use benefits in each state.

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