

Construction of Power Grid Stability Analysis System Based on Stochastic Process Theory

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Abstract. In recent years, the proportion of new energy power generation in the power supply has increased yearly. However, the random volatility generated by new energy generation poses a new challenge to the stability of the power system. In terms of stability analysis, the traditional methods based on the deterministic system can not fully describe the influence of random fluctuation. In this case, this paper introduces the power grid stability analysis method considering random influence. Based on the original model, this paper introduces the random fluctuation simulated by the Gaussian process, establishes the power grid model described by nonlinear stochastic differential equations, and gives the numerical solution format of the model with the help of the Euler method. Secondly, compared with the traditional method of quantifying the size of the escaping state of the system, this method considers the influence of the random variable on the stability of the system at the same time. Therefore, this paper establishes the stability analysis system of the power grid and further considers the influence of power grid topology and system parameters on system stability. By analyzing the Brass paradox, it is found that it is more reasonable to quantify the stability of the system with exit time. Finally, this paper uses the stability analysis system to study the stability of the high voltage power grid in Shandong Province and gives suggestions to improve the stability.

Keywords: Nonlinear coupled stochastic model, stability analysis system, topology structure, Shandong power grid.

1. Research background

1.1. Project Background and Significance

The power grid is one of the most important infrastructures in today's society. The normal operation of the power grid is a necessary condition for the regular operation of society. With the continuous development of the economy, the traditional energy power generation gradually highlights its disadvantages due to its non-renewable and the seriousness of pollution to the ecological environment, when the development and use of new energy have become an unstoppable trend. In 2020, China proposed at the UN General Assembly and the Climate Ambition Summit to achieve a carbon peak by 2030. Increasing the proportion of wind power, solar power generation, and other new energy sources in the energy consumption structure is essential to achieving carbon peak and carbon neutrality. Over the past decade, the integration of renewable energy sources has increased rapidly, such as wind and solar power, due to their sustainability.

Meanwhile, new energy generation also brings some problems. As is known to know, this renewable energy generation depends on the weather that cannot be controlled or even accurately predicted, which is random and intermittent. In the traditional energy power system, the uncertainty usually only comes from the consumer side, while the uncertainty of the new energy generation comes from the load and power generation sides. Compared with the traditional energy generation, the new energy generation brings more volatility and randomness, so it will be more challenging to manage. These fluctuations reduce the quality of the power supply and reduce the stability of the power system. Therefore, in the context of increasing the proportion of new energy generation in the energy consumption structure, it is necessary to study how to overcome the impact of new energy volatility on the whole national grid to improve the stability of the power system, and this has been the focus of scholars at home and abroad.

1.2. Traditional power grid mathematical model and synchronization characteristics

The following equations can describe the power system of the power grid:

$$\begin{cases} \dot{\delta}_i = \omega_i \\ M_i \dot{\omega}_i = P_i - D_i \omega_i - \sum_{j=1}^n l_{ij} \sin(\delta_i - \delta_j) \end{cases} \quad (1)$$

Stability analysis of the power system uses the concept of steady-state, also called the synchronous state, considering the power system (1). If there is a synchronous state, it is satisfied:

$$\begin{cases} \omega_i(t) = \omega_{syn} \\ \delta_i(t) = \omega_{syn} t + \delta_i^* \end{cases} \quad (2)$$

Where δ_i^* is determined by the energy flows

$$P_i - D_i \omega_{syn} - \sum_{j=1}^n l_{ij} \sin \delta_{ij}^* = 0 \quad (3)$$

With $\delta_{ij}^* = \delta_i^* - \delta_j^*$, and the synchronization frequency satisfies:

$$\omega_{syn} = \frac{\sum_{i=1}^n P_i}{\sum_{i=1}^n D_i} \quad (4)$$

Without losing the generality of the investigation of the frequency synchronization, we assume in this paper that the power supply and demand are balanced $\sum_{i=1}^n P_i = 0$, which leads $\omega_{syn} = 0$.

1.3. Traditional stability analysis method

The stability domain is often used to describe the stability of the dynamic system. The power grid system can be regarded as a nonlinear autonomous dynamic system and can be described by the following set of differential equations

$$\dot{x} = f(x), x \in M \quad (5)$$

The equilibrium point of a dynamic system is defined as the zero point of the vector, the solution of the following equation:

$$f(x) = 0 \quad (6)$$

In real problems, the large perturbations suffered in power systems are usually accompanied by local disequilibrium. The state variable (θ_i, ω_i) changes from the initial synchronous state $(0, 0)$ to the disturbed state $(\theta_i, \omega_i)_{t_1}$ during the disturbing process. If it is within its stable domain B , the generator can return to its synchronous state again. For a single-node power grid model, we defined:

$$S(B) = \int \chi_B(\theta, \omega) \rho(\theta, \omega) d\theta d\omega \quad (7)$$

Where:

$$\chi_B(\theta, \omega) = \begin{cases} 1, & \text{if } (\theta, \omega) \in B \\ 0, & \text{else} \end{cases} \quad (8)$$

The probability value is when the system returns its synchronous state with a probability density ρ after a large random perturbation. When $S = 0$, we call this synchronization state unstable. For the general multi-node power grid model, we can also make a similar definition, which we will not repeat here.

2. Construction and stability analysis based on stochastic theory

2.1. Nonlinear coupled stochastic power grid model

In practice, due to various disturbances, the state of the power system does not stay in the synchronous state but fluctuates around the synchronous state. In order to study the fluctuation characteristics of the system, we simulate the random interference of renewable energy power generation to the power grid stability with Gaussian noise, and the system is characterized by the following second-order coupled stochastic differential equation:

$$\begin{aligned} d\delta_i &= \omega_i dt \\ M_i d\omega_i &= (P_i - D_i \omega_i - \sum_{j=1}^n l_{ij} \sin(\delta_i - \delta_j)) dt + b_i dw(t) \end{aligned} \tag{9}$$

Where $w(t)$ is a Gaussian noise and b_i denotes the intensity of the disturbance.

2.2. Exit time and stability analysis system

2.2.1 Definition of the exit time

Due to the unpredictable non-deterministic impact of introducing new energy power generation technology on the power system, we always hope to give an early warning of power grid instability with higher confidence. Therefore, after considering the stability domain of the coupled oscillator model solution, we set a smaller safety range:

$$\begin{cases} \Gamma = [-\varepsilon, \varepsilon] \\ \Theta = \{\delta \in \mathfrak{R}^n \mid |\delta_i - \delta_j| \leq \frac{\pi}{2}, \forall (i, j) \in \varepsilon\} \end{cases} \tag{10}$$

In the numerical simulation process, the exit time T is obtained by traversal the value of the earliest node deviated from the range. It can be the index value to evaluate the stability of the power grid.

2.2.2. Numerical solution method for the stochastic model

This paper uses Euler-Maruyama methods to solve the second-order coupled stochastic differential equation proposed by (9). Then we can obtain a numerical approximation scheme for the solution of this model:

$$\begin{cases} \delta_{i+1} = \delta_i + \omega_i \Delta t \\ \omega_{i+1} = \omega_i + \frac{1}{M_i} (f(\omega_i, \delta_i, \delta_j) \Delta t + b_i \Delta W_i) \\ f(\omega_i, \delta_i, \delta_j) = P_i - D_i \omega_i - \sum_{j=1}^n l_{ij} \sin(\delta_i - \delta_j) \\ \Delta W_i = z_i \sqrt{\Delta t}, z_i \sim N(0,1) \end{cases} \tag{11}$$

3. Analysis of the influencing factors of power grid stability

3.1. Impact of network topology on stability

In the nonlinear random grid model proposed in this paper, the exit time T is used as the grid stability assessment criterion to study the influence of increasing cycle on grid stability in the simplified grid topology, as shown in Figure1(a,b). In this model, the power generation and consumer ends are simplified, including the inertia parameters $M=1$, the damping coefficient $D=1$, and the disturbance strength $b=0.01$. In topology a and b, the red point represents the generator, the generation power is $P=4$, the blue point represents the consumer end, and the power is $P= -3$:

After adding new connections, both topological structures can reach a stable state, the frequency converges to 0, as shown in Figure1(c), and the system exit time increases after increasing cycles, as shown in Figure1(d), which indicates the system is more stable, which is consistent with the results described by deterministic grid systems. It also proves the results of the non-random model.

3.2. The Brass Paradox

Although increasing the circulation network structure will improve the overall stability of the grid, some specific potential connections will reduce the total grid capacity, even destroying the grid lock [10]. It also has

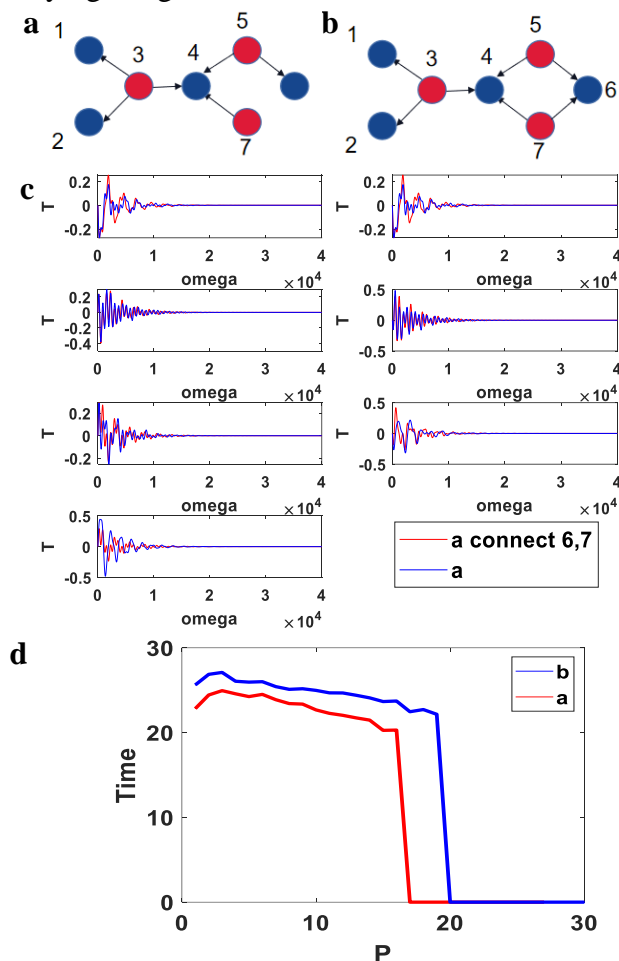


Figure 1 Influence of Topological Structure on Stability. (a, b) topology structure. (c) Frequency convergence diagram. (d) Exit time comparison chart

been observed that the addition of the link undermines the overall synchronization stability of the system in the network, as shown in Figure2(a, b). It is the Brass paradox in the oscillator network.

In this network, four power generators $P_i = +P$ are connected to four power users $P_i = -P$ by transmission lines with a transmission capacity of L . If $L > L_c = P$, the original network structure is stable, the system will quickly relax to this phase-locked state, as shown in Figure2(c). If a connection is added to the original topology to form the second oscillatory network structure shown in Figure2(b), the oscillator does not lock the phase, as shown in Figure2(d).

When additional transmission lines are added, the flow is redistributed, and the upper line transmits more electricity than the lower line, resulting in the overload of the connection line, as shown in Figure2(e), which makes the oscillator system lose its steady state.

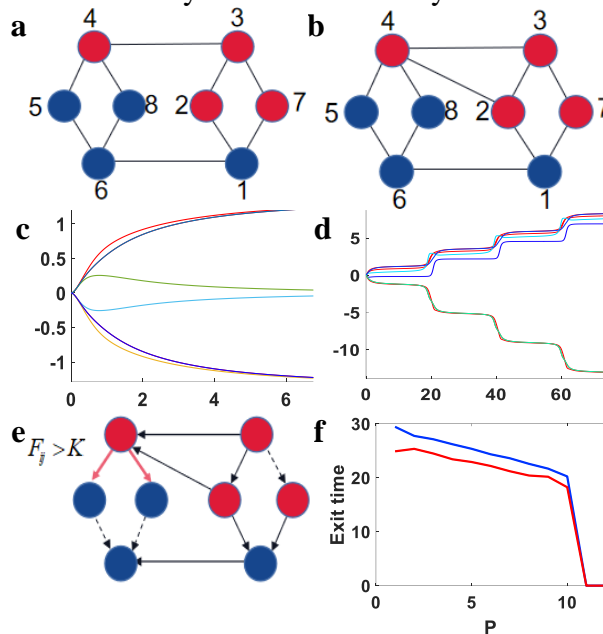


Figure 2 The Blass paradox structure diagram. (a, b) Structure diagram of oscillator network. (c, d) Phase difference diagram. (e)Flow Overload Diagram. (f)The change chart of exit time.

We use the exit time index to complete system stability analysis in the stochastic model. We adjust the initial value of P to 2 and then increase the value of P until it cannot reach the phase-locked state.

As shown in Figure2(f), the exit time of the added topology at different power P is studied. It should be noted that the group model should calculate the exit time of the model under the premise of reaching a stable state. This reduced model has inertia parameter $M=1$ and the perturbation strength $b=0.01$.

4. Stability Analysis of high-voltage power grid in Shandong Province

In the previous part, we analyzed the stability analysis system in the specific grid topology and parameter setting, showing the evaluation and prediction function of the system in theoretical analysis. For some special networks, the system can not only prove and reproduce some recent research results, pointing out that in the construction of a power system, the optimization problems of the existing power system.

Below, we use the theory to analyze the actual large power grid and give the numerical results of the power grid simulation. On this basis, the system will make a comprehensive evaluation of the stability of the power grid and provide a theoretically effective power grid optimization strategy.

As shown in Figure3(a), the square represents the power generation node, and the dot indicates the substation connected by 500kv transmission lines. In fact, this is just a simplified representation of the provincial grid, and each node also represents a small local grid, which means that we can conduct a stability analysis at that level.

We first judge the stability of power grid topology by using the proposed extensive power grid stability analysis system. Export the system parameters from the given actual power grid data

we first set the perturbation strength b to zero and observe whether the system state variables converge under the deterministic condition. We simulate the sample path with the Monte Carlo method and set the step length as $h=0.0003$, and the iteration time is 300s. The results are as Figure3(b, c).

According to the image, the frequency ω_i of each node rapidly converges to 0 after a certain disturbance for a short period. Then, the sample path of the phase observation shows that the system also enters the phase-locked state after a period of disturbance. It shows that the power system has a stable equilibrium point.

Based on this study, we continue the stability analysis of the power grid under stochastic perturbations, which is described by the equation (9). The random disturbance intensity is set $b=0.007$, and the iteration parameters are consistent with the previous one. The stability analysis algorithm calculates the escape time of the system state variables and the phase difference, and the escape time is $T=137.2161s$. The escape happens when the system state variable is almost at the end of the iterative row. While considering the mean and variance of the phase difference, it always remains in the relatively safe range. It is shown that the Shandong electric power system's topology design and operation parameter setting are in a relatively reasonable state.

Further observation of the grid architecture shows that the existing 24 nodes are only connected by 28 transmission lines, which inevitably makes some nodes not in the transmission cycle. Moreover, the above theoretical analysis shows the grid's stability during operation. We can reverse prove it in the numerical model by the optimization based on the theory. The specific treatment method is: A transmission line is added between the two nodes of Xin'an Station and Yuncheng Station in Hebei Province (green line in Figure3(a)). We calculate the escape time T and other evaluation metrics for the state variables in the improved transmission network, the escape time $T=148.2313s$. The numerical results confirm our conjecture that the improved grid structure has better stability, which gives a strategy of grid optimization to some extent.

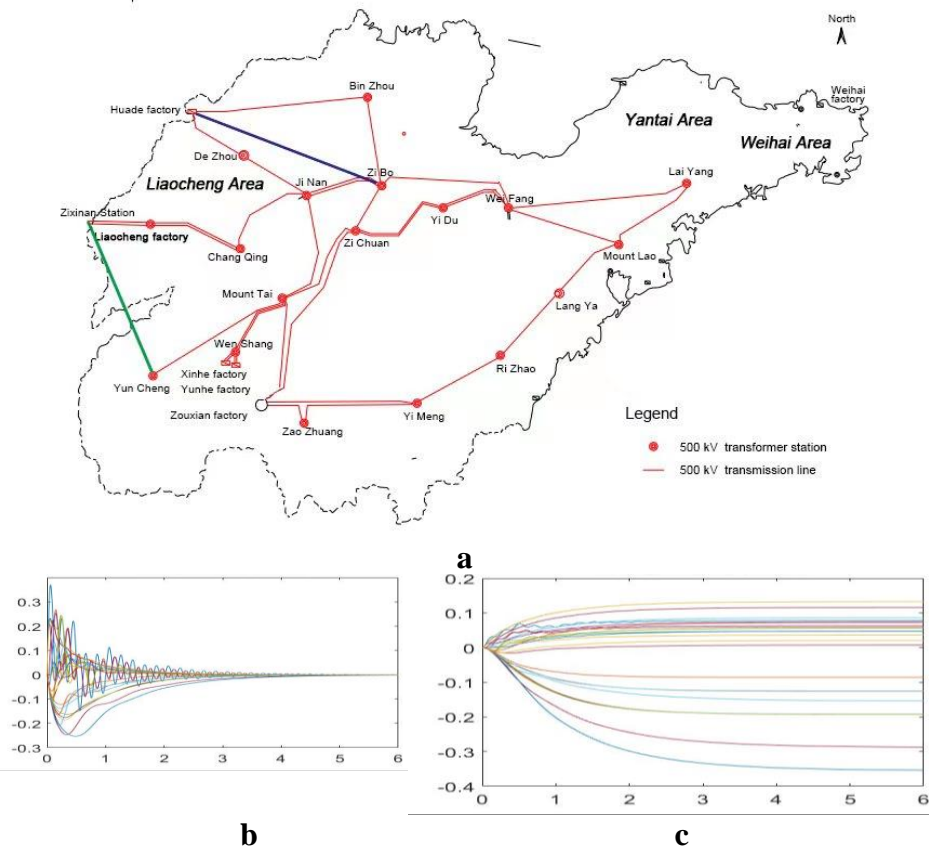


Figure 3 Shandong power grid. (a)Topological Structure Diagram of Main Nodes in Shandong Power Grid. (b)Frequency convergence diagram. (c)Phase difference diagram.

In the actual operation of the grid, managers always want to improve the stability of the grid by adding lines, and the numerical simulation in the last part also justified this idea. However, in some cases, increasing the line makes the grid much less stable, and the Braess paradox will appear, which is reproduced in the following simulations. We added a transmission line between Zibo and Huade factory (blue line in Figure3(a)) and then conducted a numerical simulation to obtain the numerical results. The exit time is $T=104s$, and the sample mean and variance of the phase difference are shown in the attachment. We found that the escape time is about 30s before the processing premise, and the mean of the phase difference of the added lines also fluctuates considerably compared with the variance, which brings great risks to the safe operation of the power grid. It is shown that numerical simulations of the actual power grid once again demonstrate the existence of the Braess paradox.

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