An Optimized Model for the Approximate Iterative Adjustment of the "FAST" Active Reflector

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Abstract. This paper analyzes the relevant data and finds that the angle formed by the upper endpoint, the lower endpoint, and the main cable node of 2216 groups of actuators is between 179.95° and 180°. Therefore, it is considered that the main cable nodes move mainly in the radial direction, and only a small amount of tangential displacement occurs. Then, mathematical models are established for the radial and tangential displacements, and after several iterations, the coordinates before and after the displacement of the main cable node are obtained. Since the adjustment factors of the reflective panel need to be considered in combination, the reflective panel is easy to adjust. The shortest total telescopic distance of the actuator is the optimization goal, and the optimization model is established with the adjustment factors and other constraints as the constraints.

Keywords: active reflector; iterative adjustment; optimization model.

1. Introduction

FAST is the largest single-aperture radio telescope globally, and it belongs to the national "Eleventh Five-Year" major scientific engineering project. The flexible integral triangular cable net structure is adopted, and the actuators are laid on the ground to pull down the cables so that the shape of the active reflection surface changes. This project has made outstanding contributions to the development of the field of astronomical observation. The initial state of the active reflector is a spherical surface with a diameter of 500 meters. It is adjusted to a rotating paraboloid with a diameter of 300 meters and a line of symmetry between the observed celestial body and the feed bin during operation. The ratio of the distance between the feed bin and the reflective sphere and the radius of the initial reflective sphere, the focal-to-diameter ratio is 0.466, and the feed bin can only accept valid signals within 1km. The actuator is connected to the main cable node through the pull-down cable, and its telescopic range is -0.6m-0.6m. The azimuth of the celestial body S is represented by the azimuth angle α and the elevation angle β. This article attempts to analyze when α=0°, β=90°, combined with the adjustment factors of the reflective panel, to determine the ideal paraboloid.

2. Model building and analysis

2.1. Preprocessing: Determine the position coordinates of the main cable node on the reference sphere

The position of the main cable node given by the data is not on the reference sphere. First, the position of the main cable node is contracted to the reference sphere. According to the method of iteratively correcting the displacement of the main cable node in 5.3, the main cable node is first contracted to the reference sphere in the radial direction. Then, the influence of the movement of the adjacent main cable node on the tangential displacement is considered, and then the movement trajectory of the main cable node is iteratively adjusted.

Let the coordinates of the i-th main index node be (x_i, y_i, z_i), and the coordinates of the center of the reference sphere are (0, 0, 0),

The straight-line Equation of the i-th main cable node along the radial direction is:

$$\frac{x-x_i}{x_i}=\frac{y-y_i}{y_i}=\frac{z-z_i}{z_i} \tag{1}$$

The Equation of the base sphere is:

$$x^2+y^2+z^2=90000 \tag{2}$$

Simultaneous formulas 1 and 2, the straight line along the radial direction of the main cable node and the spherical reference equation

$$\begin{cases}\dfrac{x-x_i}{x}=\dfrac{y-y_i}{y}=\dfrac{z-z_i}{z}\\ x^2+y^2+z^2=90000\end{cases} \tag{3}$$

$\overrightarrow{BB'}$ on $\overrightarrow{AA'}$ the direction $\vec{m}$:

$$\vec{m}=\frac{\overrightarrow{BB'}\cdot\overrightarrow{AA'}}{\left|\overrightarrow{AA'}\right|^2}\times\overrightarrow{AA'} \tag{4}$$

$\overrightarrow{BB'}$ on $\overrightarrow{AA'}$ the normal plane $\vec{b}$:

$$\vec{b}=\overrightarrow{\mathrm{BB'}}-\vec{m} \tag{5}$$

In the same way, from equations 4 and 5, the projection on the normal plane can $\overrightarrow{\mathrm{CC'}}$ be $\overrightarrow{AA'}$ obtained $\vec{c}$:

$$\vec{c}=\overrightarrow{\mathrm{CC'}}-\frac{\overrightarrow{\mathrm{CC'}}\cdot\overrightarrow{AA'}}{\left|\overrightarrow{AA'}\right|^2}\times\overrightarrow{AA'} \tag{6}$$

Then, the displacement of the main cable node A after an adjustment is made $\overrightarrow{AA''}$:

$$\overrightarrow{AA''}=\overrightarrow{AA'}+\vec{b}+\vec{c} \tag{7}$$

The iterative method is used to solve the displacement of the main cable node after multiple adjustments:

$$\overrightarrow{AA''}^{(k+1)}=\overrightarrow{AA'}^{(k)}+\vec{b}^{(k)}+\vec{c}^{(k)} \tag{8}$$

Take a small integer ε, and use $aa_i^{(k)}$ to $aa_i^{(k+1)}$ represent the component vector $\overrightarrow{AA'}^{(k)}$ of $\overrightarrow{AA''}^{(k+1)}$ the i-th row, that is, the displacement of the i-th main cable node after adjusting k times. The termination criterion of the iterative process is:

$$\left|aa_i^{(k+1)}-aa_i^{(k)}\right|\le\varepsilon \tag{9}$$

2.2 Model establishment

1. Establishing an optimal model to solve the ideal parabolic Equation

Consider first the two-dimensional parabolic Equation. The position of the focal point of the parabola equation is known. The position of the feed bin and the parabola equation can be obtained only by considering the focal length of the parabola. Considering the adjustment factor of the reflective panel, in order to ensure that the radial expansion and contraction range of the actuator is within -0.6~+0.6 meters, the expansion and contraction of each actuator should be as small as possible. The ideal paraboloid should be as close to the spherical reference surface possible.

Therefore, the objective optimization model is established with the focal length as the independent variable. Then, the optimization model to reduce the telescopic amount of each actuator as much as possible is established.

The i-th main cable node coordinates are set on the reference sphere (x_i, y_i, z_i), and the corresponding actuators are adjusted to the coordinates (xi', y_i', z_i') on the paraboloid of the reference circle. The coordinates of the lower endpoint of the actuator on the surface (x_b, y_b, z_b), the coordinates of the upper endpoint of the actuator on the reference circle (x_p, y_p, z_p), the coordinates of the lower endpoint of the actuator on the parabola It is coordinated (x_b', y_b', z_b'), the coordinates of the endpoints on the parabolic actuator (x_p', y_p', z_p'), the focal length of the parabola p/2. The stretch amount is d_i. The intersection point along the node radial direction and the reference sphere with a radius of 300m is the main cable node position corresponding to the actuator before the adjustment. The intersection point along the node radial direction and the parabola is the leading cable node position corresponding to the adjusted actuator.

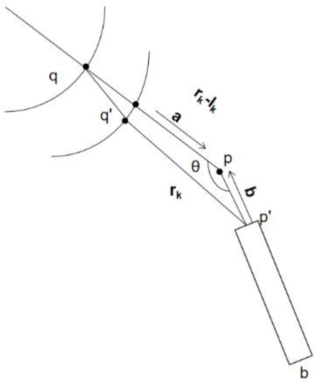

Figure 1. A simplified diagram of the actual main cable node expansion and contraction

Since the actuator can only move along the radial direction of the actuator, the three points p, p', and b are collinear.

$$\frac{x_p - x_p'}{x_p - x_b'} = \frac{y_p - y_p'}{y_p - y_b} = \frac{z_p - z_p'}{z_p - z_b} \tag{10}$$

Since the pull-down cable cannot be stretched,

$$\sqrt{\left(x_q - x_p\right)^2 + (y_q - y_p)^2 + (z_q - z_p)^2} = \sqrt{\left(x_q' - x_p'\right)^2 + (y_q' - y_p)^2 + (z_q' - z_p)^2} \tag{11}$$

Simultaneously (10) and (11) can solve the coordinates of p' point.
Therefore, the displacement of the actuator is

$$d_i = \sqrt{\left(x_q' - x_p'\right)^2 + (y_q' - y_p)^2 + (z_q' - z_p)^2} \tag{12}$$

According to the focal length of the parabola and the position of the focal point, the paraboloid equation can be determined as:

$$z = \frac{x^2 + y^2}{2p} - 0.534R - \frac{p}{2} \tag{13}$$

It is considered that the expansion and contraction of each actuator should be as small as possible, establish the objective function:

$$\min \sum \left| d_{\mathrm{i}} \right| \tag{14}$$

The constraint condition is that the amount of expansion and contraction is within the range of -0.6m~0.6m

$$\left| d_i \right| \le 0.6 \tag{15}$$

A single-objective optimization model is established to solve the parabolic Equation.

$$\min \sum \left| d_i \right|$$
$$s.t.\begin{cases} \left| d_i \right| \le 0.6 \\ \dfrac{x - x_{\mathrm{i}}}{x_i} = \dfrac{y - y_{\mathrm{i}}}{y_i} = \dfrac{z - z_{\mathrm{i}}}{z_i} \\ x^2 + y^2 + z^2 = 90000 \\ \vec{m} = \dfrac{\overrightarrow{BB'} \cdot \overrightarrow{AA'}}{\left| \overrightarrow{AA'} \right|^2} \times \overrightarrow{AA'} \\ \vec{b} = \overrightarrow{\mathrm{BB'}} - \vec{m} \\ \vec{c} = \overrightarrow{\mathrm{CC'}} - \dfrac{\overrightarrow{\mathrm{CC'}} \cdot \overrightarrow{AA'}}{\left| \overrightarrow{AA'} \right|^2} \times \overrightarrow{AA'} \\ \overrightarrow{AA''}^{(k+1)} = \overrightarrow{AA'}^{(k)} + \vec{b}^{(k)} + \vec{c}^{(k)} \\ \left| aa_i^{(k+1)} - aa_i^{(k)} \right| \le \varepsilon \\ \dfrac{x_p - x_p^{'}}{x_p - x_b^{'}} = \dfrac{y_p - y_p^{'}}{y_p - y_b} = \dfrac{z_p - z_p^{'}}{z_p - z_b} \\ \sqrt{\left(x_q - x_p \right)^2 + (y_q - y_p)^2 + (z_q - z_p)^2} = \sqrt{\left(x_q^{'} - x_p^{'} \right)^2 + (y_q^{'} - y_p)^2 + (z_q^{'} - z_p)^2} \\ d_k = \sqrt{\left(x_q^{'} - x_p^{'} \right)^2 + (y_q^{'} - y_p)^2 + (z_q^{'} - z_p)^2} \\ z = \dfrac{x^2 + y^2}{2p} - 0.534R - \dfrac{p}{2} \\ k = 1, 2, 3, \ldots \end{cases} \tag{16}$$

2.3 Solution of the model

Step 1: Determine the position coordinates of the main cable node on the reference sphere;

The position of the main cable node given by the data is not on the reference sphere. First, the position of the main cable node is contracted to the reference sphere. The straight-line Equation along the radial direction of the main cable node is combined with the reference spherical surface equation, and the intersection of the straight line equation and the spherical surface is obtained, which is the radial displacement of the first movement. Then, the tangential displacement of the node is obtained by calculating the radial displacement of the adjacent node. The resultant displacement is the actual displacement of the first movement. Take the actual displacement as the coordinates of the main cable node, and perform Step 1 again until the main cable node is basically located on the reference sphere.

Step 2: Determine the position of the main cable node on the paraboloid;

Similar to Step 1, the coordinates of the point where the main cable node is located on the paraboloid can be iteratively solved through the coordinate position of the main cable node on the reference sphere.

Step 3: Determine the sum of the absolute values of the telescopic amounts of each actuator according to the coordinates of the main cable nodes on the spherical reference surface and the parabolic surface;

Step 4: Check whether there is an elongation that does not meet the requirements (-0.6~0.6 meters).

Step 5: Traverse p, return to step2, respectively obtain the sum of the absolute value of the corresponding actuator telescopic amount under each focal length value, $\sum \left| d_k \right|$ and compare to get the smallest, $\sum \left| d_k \right|$ and the corresponding paraboloid equation is the ideal paraboloid equation.

The focal length and vertex coordinates of the ideal paraboloid obtained by traversal are 140.2479m and (0, 0, -300.4479), respectively. The Equation of the ideal paraboloid is

$$z = \frac{x^2 + y^2}{560.9916} - 300.4479 \tag{17}$$

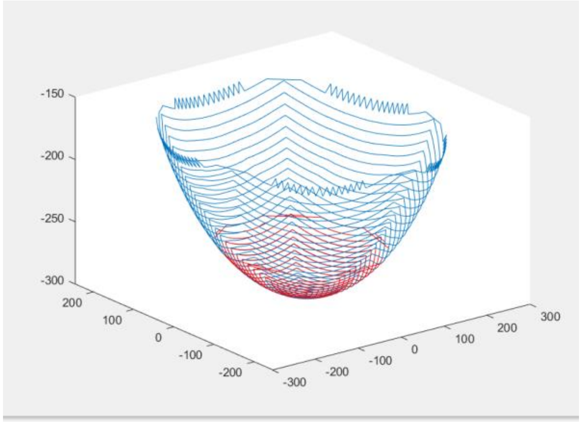

Figure 2 Adjusting the shape of the reflective panel

3. Model conclusion and generalization

In analyzing this problem, when the celestial object to be observed is located directly above the reference sphere, the requirement of "combining the adjustment factor of the reflecting panel" is proposed. It is provided the optimal adjustment strategy for the main cable node under certain conditions. Moreover, based on this, the corresponding ideal paraboloid equation is obtained. The expansion realizes the adjustment of the node position of the main cable, contraction of the actuator, the orientation of the actuator, and the length of the pull-down cable remain unchanged. It can be concluded that there is a certain geometric correspondence between the displacement of the main cable node and the expansion and contraction of the actuator. Therefore, the goal is to minimize the sum of actuator changes without exceeding the actuator expansion and contraction range.

When analyzing the movement trajectory of the main cable node, it is assumed that the main cable node moves roughly in the radial direction, and relevant data can be used to determine whether the direction of the actuator and the direction of the pull-down cable are roughly collinear at this time. Since the cable net is flexible, when the position of the main cable node changes, the adjacent main cable node will undergo adaptive tangential displacement, which will also have a certain impact. On this basis, the tangential displacement of the main cable node is corrected.

The positions of celestial bodies in this article are fixed. However, due to the earth's rotation, the celestial body's position changes every moment, so the shape of the paraboloid changes with time. Therefore, the expansion and contraction amount of the actuator group also changes with time. In reality, due to their gravity, the main cable, pull-down cable, actuator, etc, can generate tension on the main cable node and then change the position of the leading cable node and the shape of the working paraboloid. Moreover, the change in the shape of the working surface will cause a change in the pulling force. Through continuous changes, the whole will reach a balanced and stable state.

References

[1] Luo Pingjie. FAST cable net tensioning process analysis and cable force adjustment [D]. Southwest Jiaotong University, 2016. Shen Shizhao, Fan Feng, Qian Hongliang.

[2] Research on the overall scheme of the FAST active reflector support structure [J]. Journal of Building Structures, 2010, 31(12): 1-8.10.14006/j.jzjgxb.2010.12.001. [3] Qian Hongliang, Fan Feng , Shen Shizhao, Wang Qiming.Analysis of the overall cable network of the supporting structure of the FAST reflector[J].Journal of Harbin Institute of Technology,2005(06):750-752. [3] [4] Guo Yongwei, Wang Qiming. Theoretical modeling and position solution of cable-net mechanism [J]. Chinese Journal of Mechanical Engineering, 2010, 46(01): 18-23.

[3] Li Fuwen, Shen Ruili. Nonlinear static analysis of suspension bridges [J]. Bridge Construction, 1989, (01): 45-52+44.