

The Application of Bayesian Theorem

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Abstract. There are huge amount of mathematics model can be used to make decisions and make predictions, one of a useful model is the Bayesian model. It is built based on Bayesian theorem. In this article, the advantages and disadvantages of this theorem are discussed. Then some main applications of Bayesian theorem will be introduced and illustrated with some specific cases. Specifically, applications of Bayesian theorem in fields such as disease diagnosis, making risk decisions and data diagnosis are found by us. Bayesian theorem is worth to be used in disease diagnosis based on the positive results which calculated by scholars and making risk decisions since it increases the confidence of investment for investors. To be conclude, Bayesian theorem is a effective model to be used in making decisions and predictions, and this model definitely contributes a lot to the develop of the whole society. The study of the application for Bayesian theorem in different settings contributes to the development of relevant theories.

Keywords: Bayesian theorem, Disease diagnosis, Risk decisions, application.

1. Introduction

China is experiencing a sustainable economic growth nowadays. These huge companies have to make strategic economic decisions frequently to make sure it can run smoothly and generate profits. In order to make useful and helpful decisions, many models can be used such as Bayesian theorem, decision tree and linear regression. Bayesian theorem is named after English statistician, Bayes, who discovered the formula in 1763 [1]. Bayesian theorem is applied in many different fields including disease diagnosis and making risk decisions. Yu and Nijaiti found that Bayesian theorem is useful to make risk decisions. After applying a specific case, they concluded that if the expected value of return is greater than firm's investment, then it is worth to invest and vice versa [2]. Furthermore, Zhang also investigated application of Bayesian theorem in making decisions [3]. Xia, Zhang and Yan did research on this filed as well [4, 5]. In addition, Gao found that Bayesian theorem is useful in disease diagnosis. She gave a specific example to show application of this theorem and concluded that Bayesian theorem can help to calculate data and make a chart, and this chart is of great clinical significance and it is helpful for the diagnosis of disease [6]. Tang and Zhang found that Bayes theorem can be applied in disease diagnosis as well [7,8].

This model is investigated by many other people. Wang did some research and showed that Bayesian theorem owns some limitations [9]. Additionally, a scholar named Heide also did some researches on this theorem and contended that Bayesian theorem is not useful under some occasions [10]. However, researchers such as Wang argued that there are some limitations when people use Bayesianism to solve theoretical and practical problems. He argued that Bayesian approach cannot combine conflicting beliefs that are based on different implicit conditions. Additionally, a scholar mentioned that the standard Bayesian framework does not offer a way how to incorporate new hypothesis. Heide also mentioned that standard Bayesian inference can fail to concentrate in this best element in the model.

Despite the fact that some scholars argued that Bayesian theorem has some limitations, it is still widely used to solve some problems and make decisions, meanwhile it is an effective and helpful method to solve problems and make decisions.

In this paper, definition of Bayesian theorem is introduced and in order to see the effectiveness of making decisions or making predictions by using Bayesian theorem. Advantages and disadvantages of Bayesian theorem is provided in this article. Applications of this theorem in different fields such as disease diagnosis, making risk decisions and data diagnosis are also introduced in detail.

2. bayesian theorem

2.1. Bayesian Theorem

Bayesian theorem is used in various disciplines, with medicine and pharmacology as the most notable examples and it is commonly employed in different fields of finance. Some of the applications include but are not limited to, modeling the risk of lending money to borrowers or forecasting the probability of the success of an investment.

If event A has already happened, this event might be included by a set of causative events, then under the condition that the event A has already happened, the probability that a specific reason occurs is $P(B_i|A)$. It equals to the probability of this reason happens $P(B_i)$ when the event A has not occurred times the probability of this event under this reason $P(A|B_i)$. Then this multiple is divided by the total probability that the event A happens. Thus:

$$P(B_i|A) = \frac{P(B_i)P(A|B_i)}{\sum_{i=1}^n P(B_i)P(A|B_i)} \quad (1)$$

Even though Bayesian theorem is widely used in many fields such as making strategic economic decisions and making predictions. This kind of model still has some limitations. To begin with, it is worth to point out that this model is super sensitive to the pattern and the form of the input data. Before using the Bayesian theorem to make predictions of strategic economics decisions, people need to get the prior probability, while the Bayesian theorem do not own a general approach to represent and deal with the uncertainty within the background knowledge and the prior probability function. So, Bayesian theorem would be less likely to work under the condition that there are some uncertainties. A scholar who is called Wang agreed with this piece of statement and he contended that Bayesianism has limitations in representing and processing uncertainty, which is not new to all. From different considerations, many people reached the same conclusion, that is, using a probability distribution function alone to represent uncertainty is not enough, because it fails to show the ignorance, or uncertainty about the function itself.

2.2. Bayes Theorem's Application in Making Risk Decisions

Many scholars prove that Bayesian theorem can help firms to make decisions. There is a specific case showing in Yu and Nijaiti's article [2].

The decision makers of an enterprise plan to increase investment to increase investment to improve rate of producing products with high quality, and it is expected to invest 900,000 yuan to improve all equipment. After the improvement of the equipment, the quality rate can be increased to 90%, and the management thinks it can be increased to 70%. Based on experience, decision makers believe that the former is 40% and the latter is 60%. According to the calculation, if the fruit quality rate rises to 90%, it can increase the income of 200,000; if the quality rate is increased to 70%, the profit will be increased by 100,000. In order to avoid unnecessary risks, decision makers improve a set of equipment and trial-produced 5 products, the results are all high-quality products. How should decisions be made?

They assume rates of producing superior goods are θ_3 and θ_4 where $\theta_3 = 90\%$, $\theta_4 = 70\%$, and set θ_3 , θ_4 to represent the superior goods. According the case in front, $P(\theta_3) = 0.4$ and $P(\theta_4) = 0.6$, these are the decision make' perspectives towards those two opinions based on their experience, which is the prior probability. Then, they calculate the condition probabilities which are $P(A_2|\theta_3)$ and $P(A_2|\theta_4)$ by using function of binomial distribution to calculate that: [8]

$$P(X = k) = \sum_{k=1}^n \binom{n}{k} p^k (1-p)^{n-k} \quad (2)$$

$$P(A_2|\theta_3) = 0.90^5 = 0.590 \quad (3)$$

$$P(A_2|\theta_4) = 0.70^5 = 0.168 \quad (4)$$

Then, we can use Bayesian theorem to calculate the posterior probability of θ_3 and θ_4 :

$$\begin{aligned} P(\theta_3|A_2) &= \frac{P(\theta_3)P(A_2|\theta_3)}{P(\theta_3)P(A_2|\theta_3)+P(\theta_4)P(A_2|\theta_4)} \\ &= \frac{0.40 \times 0.590}{0.40 \times 0.590 + 0.60 \times 0.168} = 0.70 \end{aligned} \quad (5)$$

$$\begin{aligned} P(\theta_4|A_2) &= \frac{P(\theta_4)P(A_2|\theta_4)}{P(\theta_3)P(A_2|\theta_3)+P(\theta_4)P(A_2|\theta_4)} \\ &= \frac{0.60 \times 0.168}{0.40 \times 0.590 + 0.60 \times 0.168} = 0.30 \end{aligned} \quad (6)$$

It is clear that decision makers' credibility on the two opinions changes to 0.70 and 0.30 respectively. Next, the prior probability and the posterior probability are used to calculate the expected value of return and make final decisions with the results.

Expected returned value before experiment is:

$$\begin{aligned} &E(\text{before the experiment}) \\ &= (0.40 \times 20 + 0.60 \times 10) \times 6 = 84.0 \end{aligned} \quad (7)$$

Expected returned value after experiment is:

$$\begin{aligned} &E(\text{after the experiment}) \\ &= (0.70 \times 20 + 0.30 \times 10) \times 6 = 102.0 \end{aligned} \quad (8)$$

Apparently, after the calculation, we can know that:

$E(\text{after the experiment}) > 90.0 > E(\text{before the experiment})$. This shows that firms should not invest on the production; the expected value of return is greater than the value of investment when they use the posterior probability to calculate the expected value, showing that firms should invest on the production.

This case showed that it is meaningful to use Bayesian theorem to make decisions, and this can increase the confidence of investment for investors. In the main while, since this model can effectively help firms to make better and more appreciate decisions, it can generate external benefits to the whole society since the quality of products rises significantly after investing on improving capitals for production.

2.3. Application of Bayes Theorem in Disease Diagnosis

Bayesian theorem can be used to predict the probability of a patient having or not having a disease, and this can contribute to the accuracy of results of diagnosis. A scholar named Gao explained application of Bayesian theorem in this field in detail [6].

The probability of positive results in diagnosis tests is (develop a disease):

$$P(D+/T+) = P(D+)P(T+/D+)/P(D+)P(T+/D+) + [1 - P(D+)] [1 - P(T-/D-)] \quad (9)$$

The probability of negative results in diagnosis tests is (no disease) [4]:

$$P(D-/T-) = [1 - P(T-/D-)]P(T-/D-)/[1 - P(D+)]P(T-/D-) + P(D+)[1 - P(T+/D+)] \quad (10)$$

Where $P(T+/D+)$ is the probability of disease among positive results; $P(D+)$ is the frequency of disease in the subject population; $P(T+/D+)$ is the probability of positive test results of patients; $P(T-/D-)$ is the probability of negative results for patients without disease.

There is a specific case to show how Bayesian theorem is applied in disease diagnosis. Thinking about a question, what is the probability of coronary heart disease in a 36-year-old man with atypical angina? The calculation steps are as follows: According to literature reports, among 23996 male patients aged 30 to 39, 57 of them had coronary heart disease and 2897 males did not have coronary heart disease; among non-male patients aged 30 to 39, 1506 of them had coronary heart disease and 19536 of these patients did not have coronary heart disease, the results can be obtained:

$$P(T+/D+) = 57/1563 = 0.036 \quad (11)$$

$$P(T-/D-) = 19536/22433 = 0.871 \quad (12)$$

According to the research article, we can know that the incidence of coronary heart disease in patients with atypical angina pectoris was 49.9%, which means that $P(D+) = 0.499$. We can substitute it in to the formula:

$$P(D+/T+) = 0.499 \times 0.036 / 0.499 \times 0.036 + (1 - 0.499)(1 - 0.871) = 21.8\% \quad (13)$$

Besides, this model can be used to calculate $P(D+/T+)$ of each subgroup with different age, gender and symptom. According to the degree of ST segment decent of the electrocardiogram, it can be divided to 6 subgroups with $P(T+/D+)$ and $P(T-/D-)$. The probability of coronary heart disease in each subgroup can be calculated according to the age, symptoms and the degree of ST segment descent of the electrocardiogram. Then we can make a chart. This chart is of great clinical significance and it is helpful for the diagnosis of disease.

2.4. Application of Bayes Theorem in Data Diagnosis

Assuming they $0.001 \leq P(A|B)$, then we have:

$$-20 \leq 10[1 + \lg P(A|B)] \leq 10 \quad (14)$$

Since the integer part of $10[1 + \lg P(A|B)]$ is kept, the value of $P(A|B)$ can be gained. Hence the value of $10[1 + \lg P(A|B)]$ can be gained (the result is an integer and the author named it as scoring). According to equation (14), the standard scoring table can be gained (Table 1).

Table 1. Standard scoring table

$P(A B)$ (%)	scoring	$P(A B)$ (%)	scoring	$P(A B)$ (%)	scoring
(0,0.11]	-20	(1.12,1.41]	-9	(14.12,17.78]	2
(0.11,0.14]	-19	(1.41,1.77]	-8	(17.78,22.38]	3
(0.14,0.17]	-18	(1.77,2.23]	-7	(22.38,28.18]	4
(0.17,0.22]	-17	(2.23,2.81]	-6	(28.18,35.48]	5
(0.22,0.28]	-16	(2.81,3.54]	-5	(35.48,44.66]	6
(0.28,0.35]	-15	(3.54,4.46]	-4	(44.66,56.23]	7
(0.35,0.44]	-14	(4.46,5.62]	-3	(56.23,70.79]	8
(0.44,0.56]	-13	(5.62,7.07]	-2	(70.79,89.12]	9
(0.56,0.70]	-12	(7.07,8.91]	-1	(89.12,100.00]	10
(0.70,0.89]	-11	(8.91,11.22]	0		
(0.89,1.12]	-10	(11.22,14.12]	1		

Bayesian theorem can be applied in data diagnosis. If event A represents that a patient has a disease, and two independent groups of events $A_1, A_2, \dots, A_n$ represent the symptom, symptom $A_i (i =$

$1, 2, \dots, n$). Then we take S_i type of states $A_{i1}, A_{i2}, \dots, A_{is}$ and $A_{i1}, A_{i2}, \dots, A_{is}$ is a two mutually incompatible complete preparation event group. The disease is divided into M categories. Hence event group $B_1, B_2, \dots, B_m$ is a division of the population $U = \{\text{certain diseases}\}$, Bayes theorem tells that:

$$P(B_i|A) = \frac{P(A|B_i)P(B_i)}{\sum_{j=1}^m P(A|B_j)P(B_j)} \quad (i = 1, 2, \dots, m) \quad (15)$$

Statistical probability of $P(B_i)$ and $P(A|B_i)$ can be known through the accumulation of data from a large number of trials (patients). According to the optimal subordination principle, the conditional probability $P(B_i|A)$ ($i = 1, 2, \dots, m$) can be obtained. The corresponding B_i is the result of data diagnosis when maximum value of i is used. According to (14), it is obvious that the denominators of m posterior probabilities are the same, so m numerator terms are obtained:

$$P(A|B_i)P(B_i) \quad (i = 1, 2, \dots, m) \quad (16)$$

When the maximum value of i is employed in the formula, then the result of diagnosis B_i can be gained. Let $A = A_1 A_2 \dots A_n$, then:

$$\begin{aligned} P(A|B_i) &= P(A_1 A_2 \dots A_n | B_i) = \prod_{j=1}^n P(A_j | B_i), \\ 10[n + \lg P(A|B_i)] &= \sum_{j=1}^n 10[n + \lg P(A_j | B_i)] \quad (i = 1, 2, \dots, m) \end{aligned} \quad (17)$$

For a certain patient, symptom $A_i = A_{ik}$, where k is one from a group of number $1, 2, \dots, S_i$. For a fixed i ($i=1, 2, \dots, m$), the value of $P(A_i|B_i)$, for each additional item on the right of equation (17), refers to the scoring standard table to get the corresponding score, and the total score on the right side of equation (17) can be obtained by summing up all items together. And this will be taken as:

$$R(A|B_i) = R(A_1 A_2 \dots A_n | B_i) \quad (i = 1, 2, \dots, m) \quad (18)$$

Then we can multiple R_i with its corresponding $P(B_i)$ and take it as L_i . Hence:

$$L_i = R_i P(B_i) \quad (i = 1, 2, \dots, m) \quad (19)$$

Apparently, let the i in equation (16) be the maximum number so that it is equivalent to the result which can be gained by using equation (18) when L_i is the greatest. Therefore, to obtain the maximum serial number i of L , the disease diagnosis result B can be obtained. In this way, L can be obtained by calculating the sum of score R , if:

$$L_i = \max\{L_1, L_2, \dots, L_m\} \quad (20)$$

Then the results of numerical diagnosis of disease is pseudo B_i . The author named this numerical diagnosis method as scoring method [11].

3. Conclusion

To be conclude, a bunch of applications for Bayesian theorem in different fields such as disease diagnosis and making risk decisions are showed in this article. Benefits and limitations of it are also discussed and compared in this article. According to Gao's article, Bayesian theorem is beneficial to be applied in disease diagnosis since the data calculated by this model can be used to make a chart, and this chart is informative and can contribute a lot to disease diagnosis. This theorem is also worth to be applied in making risk decisions. At the meanwhile, it can help firms to make a more appropriate

decision. Thus, the whole society can develop more smoothly and sustainably. Finding the application of Bayesian theorem can let humans realize the effectiveness and significance of this type of model. If it can be widely used, then the whole society is more likely to be benefited.

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