# Research on automatic parking based on A* search algorithm

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**Abstract.** Automatic parking is one of the most landing scenarios in autonomous driving technology. Automatic parking refers to the automatic parking process of cars in parking lots. In this paper, we consider three parking models under vertical, horizontal and bending conditions. We first establish physical models according to different conditions, introducing A* search algorithm is d as the optimal path selection algorithm for practical cases. We also set up a constrained optimization problem for the situation of car occupancy in the garage. By solving this constraint, we can efficiently calculate the optimal parking path to avoid the situation of car occupancy.

**Keywords:** Automatic parking, A*search algorithm, constrained optimization.

## 1. Introduction

Automatic parking is one of the most landing scenarios in autonomous driving technology. Automatic parking refers to the automatic parking process of cars in parking lots. In big cities with limited parking space, it is a practical function, which reduces the difficulty for drivers to drive into narrow Spaces. This paper takes unmanned passenger vehicles as an example to realize the function of automatic parking in the parking lot. The core problem of automatic parking process is how to identify the optimal target parking space in the parking lot when the unmanned vehicle drives to the designated location (such as the entrance), and how to quickly arrive and park safely according to the target parking space.

## 2. Three scenarios of unmanned vehicle parking track

### 2.1. The case of vertical warehousing

We established the track model of parking Spaces entering into the garage vertically, which is a vertical 90 degree turn in the process of horizontal driving. Therefore, we designed the following model based on the optimal turning range:

Define a downward vertical convolution curve [6]:

$$\kappa(s) = \alpha s \tag{1}$$

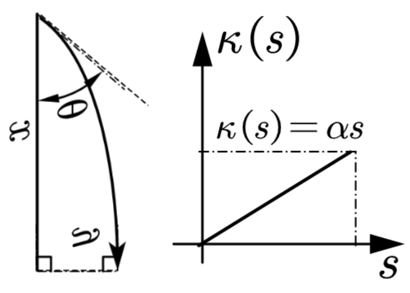


**Figure 1.** Optimal range of curvature

$$\theta = \frac{1}{2}\kappa s = \frac{1}{2}\alpha s^2 \tag{2}$$

Now we consider the speed modeling in the process of turning into storage. It is a general model and can be calculated only by changing relevant parameters in the subsequent analysis.

After establishing the trajectory planning model of the unmanned vehicle turning around, a vehicle velocity planning model is required to be established in order to plan the speed, acceleration and other information of the unmanned vehicle at each trajectory point to realize the pre-planned path [7].

A vehicle velocity planning model is proposed in [8] which can plan the shortest time speed series of vehicle movement on a specific trajectory given the maximum acceleration, initial position and speed, and target position and speed as follows:

$$\varphi = \sqrt{\frac{1}{a_{\max}}} \tag{3}$$

$$\beta = \varphi a_{\max} \tag{4}$$

$$\tau = \beta t \tag{5}$$

$$z_1 = \varphi \beta s \tag{6}$$

$$z_2 = \varphi \frac{ds}{dt} n \tag{7}$$

Introducing control variables $u \in [-1,1]$. We have:

$$f_t / F_{\max} = u\sqrt{1-\left(z_2^2 / R\left(z_1\right)\right)^2} \tag{8}$$

$$m\frac{d^2 s}{dt^2} = \frac{m\left(\frac{ds}{dt}\right)^2}{R(s)} = f_1 \tag{9}$$

$$\dot{z}_1 = z_2, \tag{10}$$

$$\dot{z}_2 = u\sqrt{1-\left(z_2^2 / R\left(z_1\right)\right)^2} \tag{11}$$

$$H = 1 + \lambda_1 z_2 + \lambda_2 u\sqrt{1-\left(z_2^2 / R\left(z_1\right)\right)^2} \tag{12}$$

$$\dot{\lambda}_1 = -\frac{\partial H}{\partial z_1} = -\lambda_2 u \frac{z_2^4}{\sqrt{1-\left(z_2^2 / R\left(z_1\right)\right)^2}} \frac{R}{R^3} \tag{13}$$

$$\dot{\lambda}_2 = -\frac{\partial H}{\partial z_2} = \lambda_1 + 2\lambda_2 u \frac{z_2^3}{R^2\sqrt{1-\left(z_2^2 / R\left(z_1\right)\right)^2}} \tag{14}$$

$$u^* = -\operatorname{sgn}\lambda_2 = \frac{\partial R}{\partial z_1} \tag{15}$$

Case 1 provides a simple scenario of the vehicle turning around vertically and entering the storage, with no obstacles on the road. The horizontal right lane was selected as the target lane, the right endpoint of the obstacle was selected as the target point of turning trajectory planning, and the problem was solved according to the trajectory optimization model. The resulting trajectories are line, arc, line, arc, line. The specific simplified trajectory results are shown below.

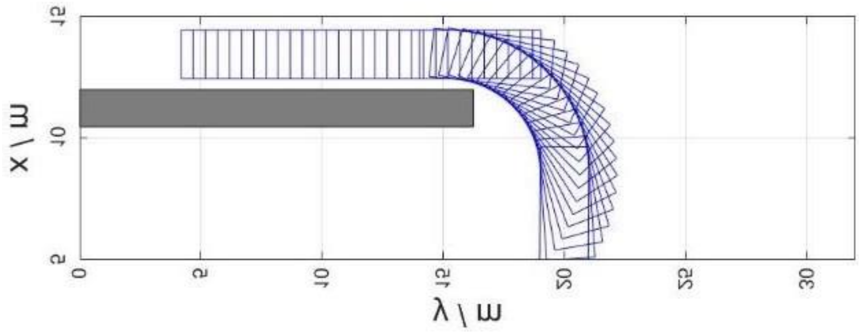

**Figure 2**. Modeling of parking situation 1

The trajectory obtained according to Figure 2 is in good line with the optimization objective, and the unnecessary line pressing is reduced while the traffic efficiency is as high as possible. The track first passes through an arc with the maximum required curvature. Due to the limitation of scene, if the middle lane is selected as the target lane, lane line will be crushed. Therefore, choosing lane 3 as the target lane can ensure high traffic efficiency while ensuring less line pressure.

According to the speed and trajectory, the time of each point on the trajectory is obtained, and some data are shown in Table 1:

**Table 1.** Case 1 trajectory data graph

| X | Y | Speed | Time | Speed difference | Acceleration | azimuth |
|---|---|---|---|---|---|---|
| 6.28 | 1.5708 | 2.44949 | 0.204124 | 0.71744 | 3.514724 | 13.44 |
| 6.78 | 1.5708 | 3 | 0.166667 | 0.55051 | 3.30306 | 13.44 |
| 7.28 | 1.5708 | 3.4641 | 0.144338 | 0.4641 | 3.215378 | 13.44 |
| 7.78 | 1.5708 | 3.87298 | 0.1291 | 0.40888 | 3.167168 | 13.44 |
| … | … | … | … | … | … | … |
| 6.8264 | -1.5708 | 4.47214 | 0.111803 | -0.52786 | -4.72133 | 2.76857 |
| 6.3264 | -1.5708 | 3.87298 | 0.1291 | -0.59916 | -4.64107 | 2.76857 |
| 5.8264 | -1.5708 | 3.16228 | 0.158114 | -0.7107 | -4.49486 | 2.76857 |
| 5.3264 | -1.5708 | 2.23607 | 0.223607 | -0.92621 | -4.14214 | 2.76857 |

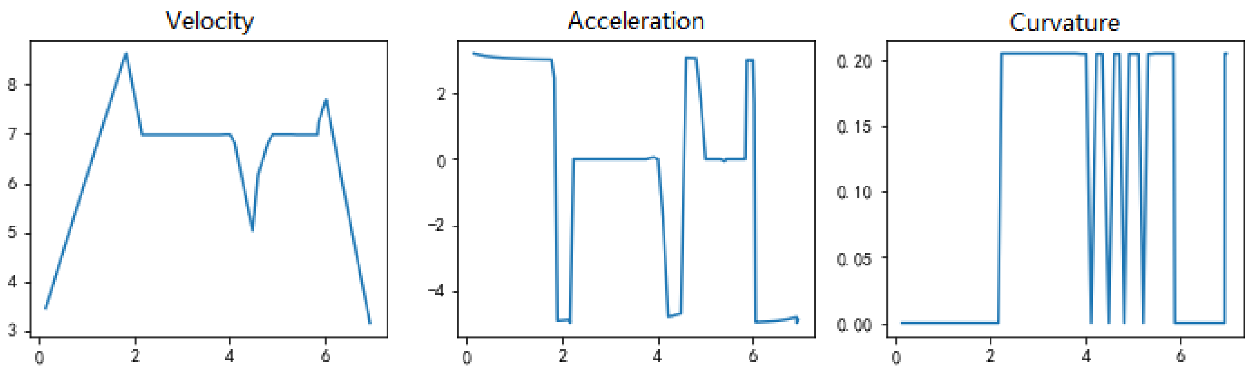


**Figure 3**. Parking situation 1 Motion parameter variation diagram

### 2.2. The case of horizontal warehousing

The scenario provided by situation 2 is the problem of the vehicle turning around at 90 degrees horizontally and entering the warehouse, with no obstacles on the road. The horizontal right lane was selected as the target lane, the right endpoint of the obstacle was selected as the target point of turning trajectory planning, and the problem was solved according to the trajectory optimization model. The resulting trajectories are line, arc, line, arc, line. The specific simplified trajectory results are shown below.

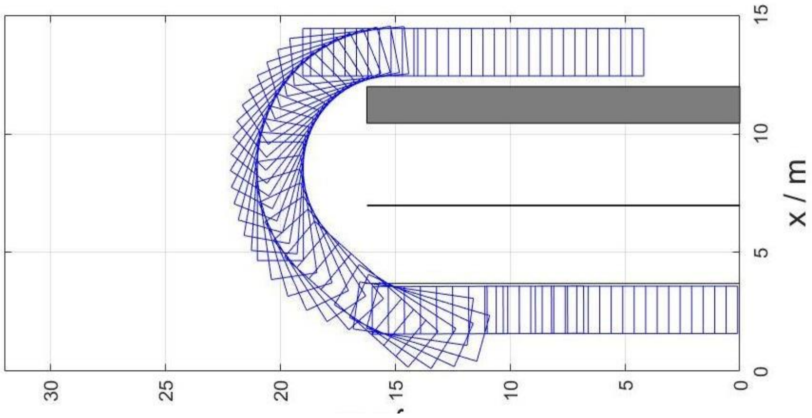


**Figure 4.** Modeling of parking situation 2

According to the results of trajectory optimization, when both G and F exist [8][9], relatively smooth curves can be obtained through trajectory optimization. Put the path points into the velocity planning model, and the obtained items are shown in Table 2:

**Table 2.** Case2 trajectory data graph

| X | Y | Speed | Time | Speed difference | Acceleration | azimuth |
|---|---|---|---|---|---|---|
| 13.44 | 6.28 | 1.73205 | 0.288675 | 1.73205 | 5.99999 | 1.5708 |
| 13.44 | 6.78 | 2.44949 | 0.204124 | 0.71744 | 3.51472 | 1.5708 |
| 13.44 | 7.28 | 3 | 0.166667 | 0.55051 | 3.30306 | 1.5708 |
| 13.44 | 7.78 | 3.4641 | 0.144338 | 0.4641 | 3.21538 | 1.5708 |
| … | … | … | … | … | … | … |
| 3.99129 | 5.48318 | 3.85132 | 0.05193 | -0.25144 | -4.84187 | 4.73245 |
| 3.9953 | 5.28322 | 3.58227 | 0.055831 | -0.26905 | -4.81904 | 4.73245 |
| 3.99902 | 5.09787 | 3.31342 | 0.05595 | -0.26885 | -4.80514 | 4.73245 |
| 3.99929 | 5.08326 | 3.2913 | 0.00444 | -0.02212 | -4.98228 | 4.72946 |

### 2.3. The case of aslant warehousing

The scenario provided by situation 3 is the problem of the vehicle turning around 45 degrees and entering the storage, and there are no obstacles on the road. The horizontal right lane was selected as the target lane, the right endpoint of the obstacle was selected as the target point of turning trajectory planning, and the problem was solved according to the trajectory optimization model. The resulting trajectories are line, arc, line, arc, line.

The path results were imported into the velocity planning model, and the time of each track point was calculated.

**Table 3.** Case3 trajectory data graph

| X | Y | Speed | Time | Speed difference | Acceleration | azimuth |
|---|---|---|---|---|---|---|
| 13.44 | 6.28 | 1.73205 | 0.288675 | 1.73205 | 5.99999 | 1.5708 |
| 13.44 | 6.78 | 2.44949 | 0.204124 | 0.71744 | 3.51472 | 1.5708 |
| 13.44 | 7.28 | 3 | 0.166667 | 0.55051 | 3.30306 | 1.5708 |
| 13.44 | 7.78 | 3.4641 | 0.144338 | 0.4641 | 3.21538 | 1.5708 |
| … | … | … | … | … | … | … |
| 3.99129 | 5.48318 | 3.85132 | 0.05193 | -0.25144 | -4.84187 | 4.73245 |
| 3.9953 | 5.28322 | 3.58227 | 0.055831 | -0.26905 | -4.81904 | 4.73245 |
| 3.99902 | 5.09787 | 3.31342 | 0.05595 | -0.26885 | -4.80514 | 4.73245 |
| 3.99929 | 5.08326 | 3.2913 | 0.00444 | -0.02212 | -4.98228 | 4.72946 |

# 3. Optimization of trajectories

## 3.1. Trajectory optimization model based on A* search

A * search algorithm is A heuristic shortest path search algorithm, not only using the initial state to the current state of node n cost function $g(n)$, using the current state of node n to the target node lowest cost estimation function, searching the shortest path to the destination node, from the initial state by state to the target cost estimates can be represented as $h(n)$.

Using A* search algorithm, the optimal trajectory of unmanned vehicle turn-around can be found in the rasterized scene. However, when the number of cells is too large, A* search algorithm will deal with too many nodes, which will greatly increase the algorithm's time complexity and time. Therefore, an algorithm is needed to make the trajectory converge to A smaller solution space. In this paper, the Reds-Shepp curve model [7] is adopted to further optimize the convergence of the trajectory planned by the A* search algorithm.

## 3.2. Consider the constrained optimization of occupancy in a garage

The occupied parking space in the garage can be regarded as an obstacle problem in automatic driving. In this way, combining with the A* search model in the previous question, we can establish the following optimal parking path selection model with obstacle constraints:

Where v is the speed of the unmanned vehicle and t is the time. When the parking space is occupied by a car, the unmanned vehicle will judge and transmit the data to the background of the on-board system, and the background will call the obstacle algorithm to replan the path.

$$\min t = \frac{v_1}{a_1} + \frac{5-v_1}{a_2} + t_3 + t_4$$

$$\text{s.t.} \begin{cases} 0 \le a_1 \le 3; \\ -5 \le a_2 \le 0; \\ \dfrac{1}{2a_1}v_1^2 + \dfrac{v_1(5-v_1)}{a_2} + \dfrac{a_2\left[(5-v_1)/a_2\right]^2}{2} = 17.12; \\ v_1 > 5; \\ t_3 = \dfrac{3.56R_k}{5}; \\ 5t_4 + \dfrac{a_1}{2}t_4^2 = 25.70. \end{cases}$$

### 3.3. Optimal path solution

First, we use the with the A * optimal path planning with obstacle constraints model above, the initial position of the best parking path, the following is the schematic diagram of model simulation, he and the route chosen by the actual driving in line with the good, the real constraint model shows us A * with obstacles in case have vehicles occupy the path planning ability.

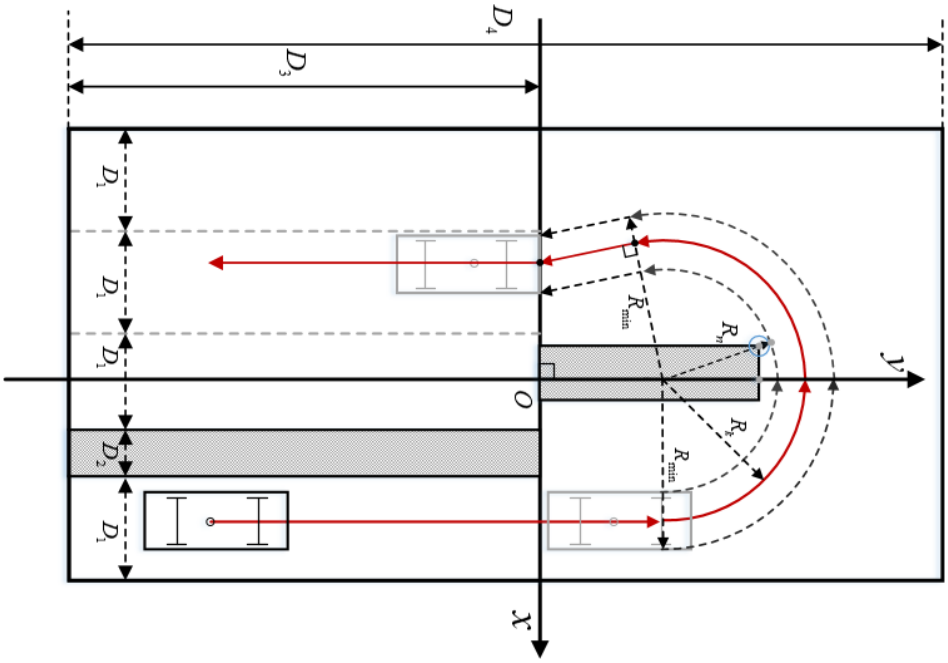

**Figure 5.** Optimal parking path simulation diagram

After obtaining the optimal path planning, we used the model we asked before to calculate the speed and curvature information of each coordinate point in detail, as shown below:

**Table 4.** Optimal trajectory data graph

| X | Y | Speed | Time | Speed difference | Acceleration | azimuth |
|---|---|---|---|---|---|---|
| 13.44 | 6.28 | 1.73205 | 0.288675 | 1.73205 | 5.99999 | 1.5708 |
| 13.44 | 6.78 | 2.44949 | 0.204124 | 0.71744 | 3.51472 | 1.5708 |
| 13.44 | 7.28 | 3 | 0.166667 | 0.55051 | 3.30306 | 1.5708 |
| 13.44 | 7.78 | 3.4641 | 0.144338 | 0.4641 | 3.21538 | 1.5708 |
| … | … | … | … | … | … | … |
| 3.99129 | 5.48318 | 3.85132 | 0.05193 | -0.25144 | -4.84187 | 4.73245 |
| 3.9953 | 5.28322 | 3.58227 | 0.055831 | -0.26905 | -4.81904 | 4.73245 |
| 3.99902 | 5.09787 | 3.31342 | 0.05595 | -0.26885 | -4.80514 | 4.73245 |
| 3.99929 | 5.08326 | 3.2913 | 0.00444 | -0.02212 | -4.98228 | 4.72946 |

According to the data results in the table, it can be seen that the transformation of acceleration, speed, curvature and other indicators is relatively gentle, and the planning results meet good driving experience and traffic efficiency.

### 3.4. Algorithm complexity analysis

The complexity of this algorithm mainly focuses on A* search and solving target constraints, where the complexity of A* search isO(n2*logn), And the complexity of solving the target constraint is $O(T \times (nK + |E|) + nK) = O(T \times (nK + |E|))$ ,Where T is the search time, $K$ is the number of occupied garages, and E is the optimal path length calculated based on A* search. In summary, the complexity of this algorithm is $O(n^2 * \log n + T \times (nK + |E|))$ .It is $O(n^2)$ an algorithm, which is acceptable, and the research on algorithm scalability will be the focus of our subsequent attention.

# 4. Conclusion

## 4.1. Advantage

1) The model can predict the existence of obstacles around, and avoid the obstacles around in advance, so that the vehicle can turn at a uniform speed when turning, and improve the comfort of the passenger;

2) The model adopts interpolation fitting algorithm to carry out path planning for unmanned vehicles, which has the characteristics of simple principal structure and smooth generated path.

3) The model uses a single objective piecewise function to search for optimization. The decision variables are considered comprehensively according to the order of priority, and the goal programming problem can be solved efficiently.

4) In the establishment of the u-turn model of the unmanned vehicle, the u-turn path is divided into the combination of straight path and curve path, which makes it easier to realize the path tracking of the unmanned vehicle.

## 4.2. Disadvantage

5) In the simulation experiment of the proposed path planning algorithm, obstacles are all set in static state, and moving obstacles are not considered;

6) This paper combines the traditional A* algorithm for path planning simulation analysis. Although the simulation results can reach the target point, they also have some defects such as not smooth enough in the planned path.

7) The simple vehicle model does not take into account many actual environmental influences, resulting in large deviations in the path tracking of unmanned vehicles.

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