

# Allocation and Scheduling of the Colorado River System

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**Abstract.** Generally, dams serve as reservoirs for water storage. The Colorado River Basin is currently in a state of continuous drought, and if the five states of AZ, CA, WY, NM and CO maintain the agreements made hundreds of years ago, there will not be enough water to supply in the near future. We propose a multi-objective programming algorithm under inequality constraints, under the condition that the sum of the competition coefficients is minimized, we solve the proportion of water resources that should be cut for general water usage and hydropower production at different water scarcity rates. We develop some policies to deal with water scarcity. We use the univariate analysis method to rationally analyze and reconstruct our model according to three different scenarios, and finally predict different situations, like additional water supply is needed or whether there will be more water flowing to the Gulf of California.

**Keywords:** Differential equations, kinetic equations, entropy method, competition coefficients, multi-objective programming algorithms, univariate analysis.

## 1. Introduction

As one of the major inventions in human history, dams have played an important role in managing the allocation of water resources for hundreds of years. Reservoirs built behind dams can store water for various purposes, providing areas for recreation and preventing downstream flooding. At the same time, after the dam opens the valve, the high-speed flow of water can drive the turbine to rotate, providing electricity power for humans. The initial need for a reservoir was realized in 1936 with the completion of Hoover Dam in Black Canyon, storing 32 million acre-feet in the mammoth reservoir of Lake Mead[1]. 30 years later, a 710-foot high dam called Green Canyon Dam, built by the U.S. Bureau of Reclamation, created Lake Powell, which holds 27 million acre-feet of water[2].

Unfortunately, in the event of dry weather, the water level in the reservoir tends to become low, and water supply and power generation will not be available. Five states and Mexico around the Colorado River System are facing such a problem[3-4]. Therefore, it is of great significance to develop a reasonable water allocation plan to ensure the basic water and power generation needs of the five states and Mexico around the Colorado River system[5-7].

In our paper, we build three models to solve the problems:

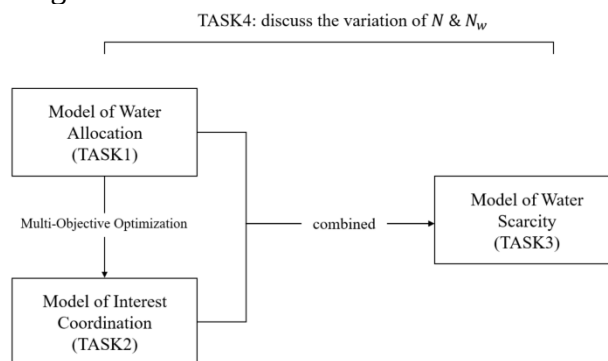
- To solve the water allocation problem under fixed supply and demand conditions, we developed a water allocation and scheduling model for the Colorado River system using differential equations and kinetic equations.

- To address the competing benefits of general usage and electricity production, we collected data on water use from five states and calculated each weight using the entropy weighting method based on Task1's model. Then we modeled the benefit distribution using a multi-objective optimization approach.

- To discuss the water scarcity situation, we designed different steps of water scarcity rates and integrated the models of Task1 and Task2 with the inequality constraint approach to design the model for the water scarcity condition.

- In order to apply the model to the case of demographic, industrial and agricultural changes, a multi-factor analysis was conducted to discuss the changes and optimization of the above model.

Our work is as shown in Figure 1.



**Figure 1.** Our Work

## 2. Assumptions and Justifications

To simplify the problem, we make the following basic assumptions, each of which is properly justified.

- **Assumption 1:** In the process of building the model, the data we collect is authentic and credible.

**Justification:** The geographic location, dams and other information we collect are all sourced from the official or government data on websites such as USGS, which can be considered real data.

- **Assumption 2:** Assuming that when there is a shortage of water in the reservoir, all additional replenishment comes from rainfall, and the entire volume of rainfall will flow into the reservoir.

**Justification:** Through the analysis of meteorological websites and geographic locations, almost all the additional water replenishment in Lake Mead and Lake Powell come from rainfall, and simplifying the form of rainfall is more conducive to building the model for this problem.

- **Assumption 3:** The inflow rate of the reservoir is constant, but the outflow rate is determined by the water level and the dam.

**Justification:** The reservoir's intake water mainly comes from the Colorado River, so its intake rate will not change much over a long period of time, while the different water levels will lead to different discharge rates, and the dam can also control its discharge rate by opening and closing valves.

- **Assumption 4:** Evaporation of water is always present and can be approximated as a constant.

**Justification:** Since the evaporation of water is only highly variable when there is a large difference in temperature, and the Colorado River system is mainly in the northern temperate zone where the temperature difference is small, the amount of evaporation can be approximated as a constant.

## 3. Symbols and Definitions as shown in Table 1

**Table 1.** Symbols and Definitions

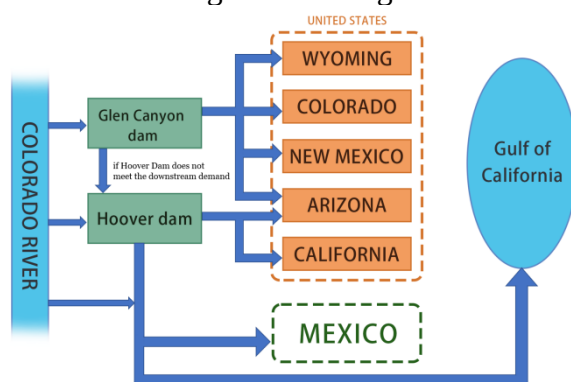
| Symbols | Definitions                                                             |
|---------|-------------------------------------------------------------------------|
| H       | Colorado River runoff                                                   |
| N       | Total demand in the five major states and Mexico                        |
| $P_0$   | Minimum water level in Lake Powell capable of meeting specified demands |
| $M_0$   | Minimum water level in Lake Mead capable of meeting specified demands   |
| $S_P$   | Bottom area of Lake Powell                                              |
| $S_M$   | Bottom area of Lake Mead                                                |
| $R_P$   | Evaporation rate of Lake Powell                                         |
| $R_M$   | Evaporation rate of Lake Mead                                           |
| J       | The amount of water flowing into the Gulf of California                 |
| R       | Loss of water transported from Glen Canyon Dam to HooverDam             |
| L       | Transport distance between Lake Powell and Lake Mead                    |

|          |                                                                                      |
|----------|--------------------------------------------------------------------------------------|
| $v_i$    | Reservoir inflow/outflow rates                                                       |
| $N_i$    | Water demand by state                                                                |
| $w_{ij}$ | Water weighting factors used by states for different aspects                         |
| $e$      | Water required for electricity generation as a percentage of total fixed demand      |
| $r$      | Water resources required for general water use as a percentage of total fixed demand |

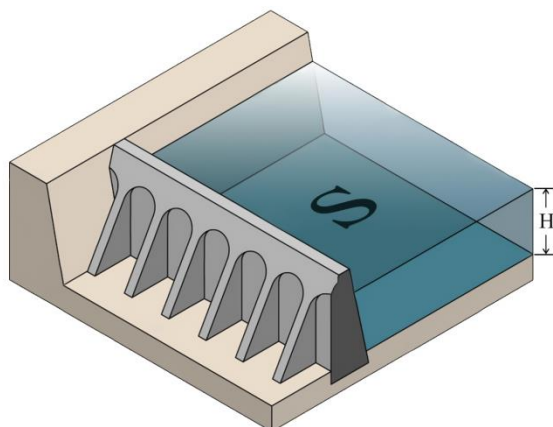
#### 4. Model Preparations

Before developing a water allocation plan, we first analyzed the geography around the two dams and the water supply targets for Lake Powell and Lake Mead. By reviewing the relevant information and analyzing the geographical location, we roughly determined that Glen Canyon Dam is in the upstream of the Colorado River, while Hoover Dam is in the downstream area. Meanwhile, the upstream part of the area is less likely to experience water shortage, but the downstream area often experiences widespread drought, so we can assume that when the downstream part of the water shortage, the water from Glen Canyon Dam will directly supply Lake Mead.

WY, CO, and NM are mainly located in the upper part of the Colorado River, so the water and electricity of these three states mainly come from Glen Canyon Dam; AZ is partly located in the upper area and partly located in the lower area, so both Glen Canyon Dam and Hoover Dam will provide water and electricity; CA is located in the lower area, and the water and electricity mainly come from Hoover Dam; finally, it also needs to provide water to Mexico, and finally the excess water will drain into the Gulf of California. As shown in Figure 2 and Figure 3.



**Figure 2.** Colorado River Water Supply System



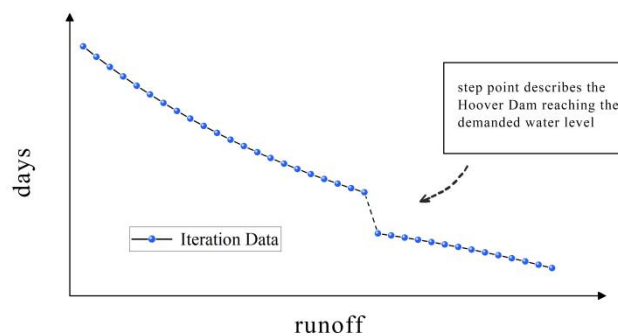
**Figure 3.** Diagram of Dam

The study[5] explored the design of the physical model of the dam, which used the *Seep/w* module in Geo-studio to build the finite element model of the dam. Therefore, we believe that the Glen Channel Dam and Hoover Dam can be simplified to a cubic model, and the water capacity of the dams can be roughly obtained by calculating the product of the bottom area of their reservoirs and

the height of the water table. This would make our model as simplified and generalizable as possible without loss of generality.

## 5. Task 1: Water allocation model under fixed demand

The geographic analysis shows that the water supply for the five states comes mainly from the Colorado River system, and when the dry season comes, the reservoirs in these areas are constantly decreasing. In order to meet the fixed water supply demand of the five states, it is very important to rationalize the water supply scheduling of Glen Canyon Dam and Hoover Dam. As shown in Figure 4.



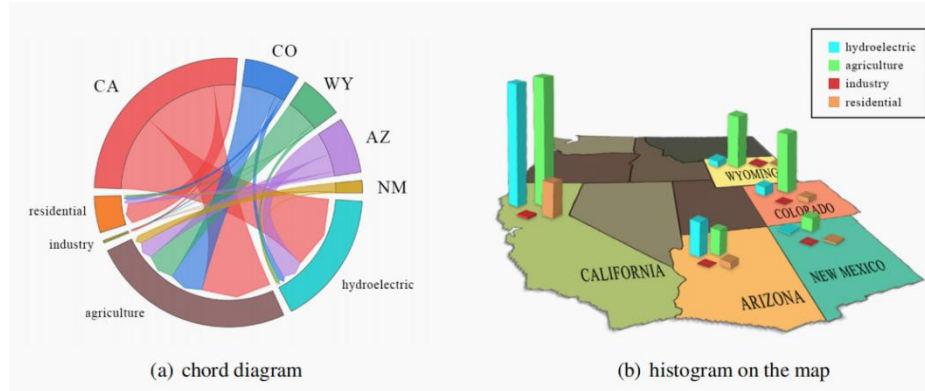
**Figure 4.** Competition Coefficient Curve

The trend of its solution shows that when the Colorado River run off decreases, the time required generally shows a monotonic increase, which is in line with our expectation. Moreover, there is a step point in the figure, which indicates that the water level of Lake Mead downstream has reached the minimum water limit at this time and needs to be replenished by Lake Powell upstream, so the steep increase in time is also in accordance with our expectation.

However, when  $H \leq MS_M(1 + R_M) + PS_P(1 + R_P)$ , we believe that the river run off is not sufficient to meet the fixed demands of the states while maintaining the water levels of the two reservoirs constant. Thus, at this time, the decision maker needs to determine by judging the water level of the two reservoirs to increase the opening of the gate of that reservoir to meet the demand of the states: when  $P > P_0, M > M_0$ , increase the rate of Hoover Dam; when  $P > P_0, M < M_0$ , the rate of Glen Channel Dam needs to be increased to meet the demand of the states, while replenishing the water level of Lake Mead to  $M_0$ , and at the same time through the kinetic equations to calculate The time required for the transport process is related to the run off volume of the river.

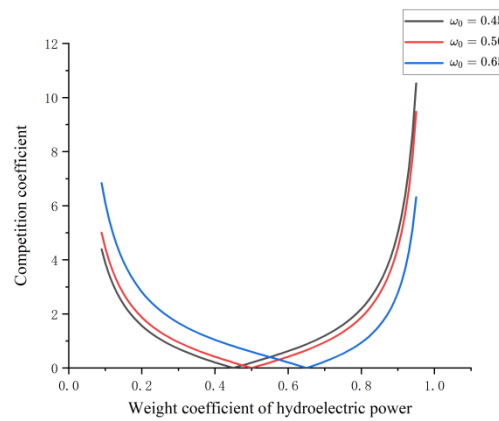
## 6. Task 2: Competitive interest coordination model

The second section is to address the competing interests among agriculture, industry, housing and electricity. First we analyze the water use of the five states and their allocation ratios and determine the weight size of each area by the entropy weighting method, and finally address their competing interests by multi-objective planning. By reviewing the relevant information, we roughly map out the proportional allocation of water use in the four areas in the five states. As shown in Figure 5.



**Figure 5.** Allocation of Water Use by State

It can be seen from the competition coefficient formula that, when a state in the general use of water and electricity water resources weighting coefficient and the standard weighting coefficient is equal, then for the competition coefficient is 0, indicating that at this time the state in general use and electricity production water allocation to achieve the best, and when the weighting coefficient is not consistent with the standard weighting coefficient, we simulate  $w_{i10} = 0.45, 0.5, 0.65$ . As shown in Figure 6.



**Figure 6.** Competition Coefficient Curve

It is obvious from the figure that when the difference between the weight coefficient and the standard weight coefficient is larger, the competition coefficient is larger, which meets our design requirements.

Therefore, the weight of water resources required for hydropower generation to the total fixed demand water resources is:

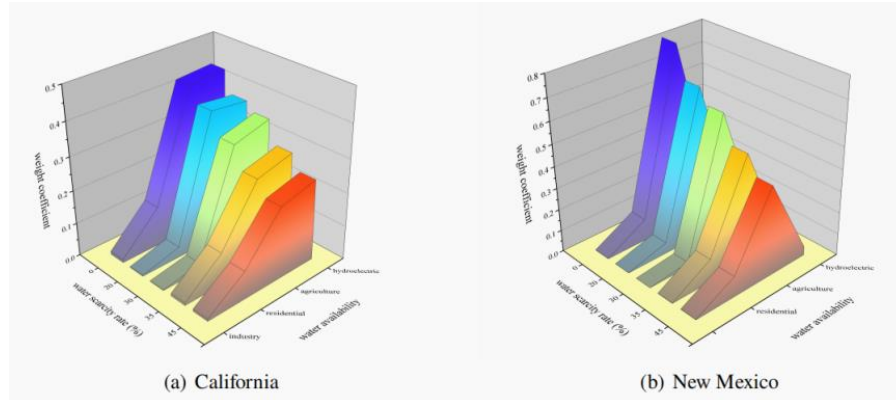
$$e_i = \frac{\sum_i \omega_{i1} N_i}{\sum_i N_i} \quad (1)$$

Then the weight for general water use to total fixed demand water resources is

$$r_i = 1 - e_i \quad (2)$$

## 7. Task 3: Model optimization for different water scarcity scenarios

Consistent with the criteria for addressing competing interests in Task 2, we consider to minimize the sum of the competing coefficients for each state so that we can reach a reasonable allocation of water resources in the event of water shortages. Using California and New Mexico as examples, when the water shortage rates are 20%, 30%, 35%, and 45%, respectively, we calculate the weighting coefficients for the four areas of hydropower, agricultural water, industrial water, and residential water through the equation and visualize the data as Figure 7.



**Figure 7.** The weighting coefficients for the four areas of hydropower, agricultural water, industrial water, and residential water through the equation and visualize the data

The graph indicates that the overall water supplement at different rates of water scarcity. The overall plate of the graph corresponds to the current total water volume. When a particular place is more prominent, it means the corresponding weight coefficient is higher and needs more water supplement. Taking California and New Mexico shown in the graph as examples, the common point is that they both pay more attention to the needs of agricultural irrigation, but CA has a higher demand for electricity production while NM does not.

In the case of continued water scarcity, however, all aspects of development will suffer a greater impact, especially in agriculture where the water weight coefficient is much lower than normal. In the long run, it may make the agricultural harvests of the states decline, which will breed social problems such as famine and is not conducive to the construction of social stability.

## 8. Task 4: Model optimization in different situations

As the population grows gradually, the demand for water resources will increase accordingly, so the water resources provided under the original fixed demand are not enough to maintain the normal operation of the local area, which we can consider as a kind of water shortage status, that is, corresponding to the water shortage model in Task 3. The corresponding water shortage rate at this point is

$$\delta = \frac{N_{in} - N_i}{N_{in}} \quad (3)$$

The reduction in population in the affected area, similar to the above scenario, corresponds to a water surplus, which corresponds to the model in Task 2. We need to update the standard coefficients of our Task 2 model and then use the updated model to optimize the structural distribution of water resources, while more water will be available to the Gulf of California.

Based on the updated standard weighting factors, we apply the model in Task2 to solve for the optimal allocation of water resources for general water use and electricity production at this time.

## 9. Sensitivity Analysis

For the competition coefficient equation in the task2 model

$$C = \frac{1}{\omega(1-\omega)} |\omega - \omega_0| \quad (4)$$

It can be regarded as that when increases by 1%, C increases by 1.5%, so the model is relatively stable and reliable.

## 10. Conclusion

In the first question, we initially simplified the fixed demand of the five states and the water demand of Mexico. We also analyzed various scenarios for different values of and we obtained a more complete water allocation model and used the kinetic equations to calculate the time required for transportation.

In the second question, we used the entropy weighting method to determine the weight coefficients of water of hydroelectric power production, agriculture, industry, and residential to the fixed demand of the states. We also defined the criteria of competing interests, and finally solved the competing interests among each demand using a multi-objective programming algorithm.

In the third question, we optimized the model by analyzing different water shortage scenarios and visualized the changes in weight coefficients for different water scarcity rate. Finally we proposed corresponding policy measures.

In the fourth question, we optimized and reconstructed the model for three different scenarios, and after recalculating the weight coefficients, we proposed corresponding changes as well as countermeasures.

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