Modeling and Analysis of the Entire Process of Building Seismic Loss Based on Probability Theory

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Abstract. The objective of this paper is to explore the relationship between earthquakes and building losses and to attempt to develop an ideal model to analyze post-disaster building losses in Sichuan Province, China for economic analysis, seismic analysis, and risk prediction for post-disaster reconstruction. Based on existing theories, this paper follows the assumptions and theories of Poisson distribution in time and exponential distribution in intensity level for random earthquakes, and validates the analysis using recent years of earthquake data from Hokkaido, Japan, and recent years of local earthquake data from Sichuan respectively; After calculating the probability of occurrence of earthquakes within Sichuan, the international earthquake classification scale and Chinese national norms are combined to determine the damage caused to buildings After calculating the probability of occurrence of earthquakes in Sichuan, the minimum seismic rating and probability of occurrence were determined by combining the international seismic classification scale and the Chinese national code; Finally, the final regional loss model was fitted and applied to an earthquake in a remote area of Sichuan.

Keywords: *Earthquake; Building loss; Probability and Statistics; Risk assessment; Probability distribution.*

1. Introduction

Located at the junction of the Indian Ocean plate and the Eurasian plate, Sichuan Province, China, has frequent seismic activities. The famous Wenchuan earthquake (seismic intensity greater than 8) occurred in the Sichuan region. Seismic activity has affected the lives of residents and caused considerable economic losses. For buildings, frequent earthquakes often shorten the service life of buildings and increase potential safety risks such as collapse. Since 1933, there have been more than 32 earthquakes of magnitude greater than five in Sichuan. According to the classification of the seismic intensity scale, earthquakes of magnitude five or more will cause varying degrees of damage to buildings, and the damage of buildings is directly related to the safety and security of residents. Economic construction is related, it is necessary to establish an earthquake-building loss estimation model in Sichuan.

At present, the 1-year scale earthquake prediction model, which is directly based on the magnitude-frequency G-R relationship, is one of the few accepted laws in seismology and is the basis for statistical probability prediction of earthquakes [2]. According to the previous research, in Sichuan Province, affected by local economic development, architectural styles, local building materials, and other factors, its spatial distribution ratio and seismic performance have obvious regional characteristics, which can be divided into three major regions: high-altitude western mountainous areas, the central and northern economically developed urban areas, and other densely populated areas. For different building types and seismic fortification levels, earthquakes of different magnitudes will damage houses differently [1]. Earthquakes of magnitude more than five will cause different types of damage to the building, which can be included in the calculation.

Building loss in seismic activities depends on the magnitude of earthquakes and resistance of building structures. However, it is not very realistic for researchers to predict the ability to build resistance in potential seismic disasters since all building parameters are unknown or unclear during seismic prediction. There are some methods for earthquake damage prediction of building groups considering seismic fortification levels. At present, the domestic research on the seismic analysis of dense building groups is quite mature, especially after the Tangshan earthquake and Wenchuan earthquake [2]. Big data analysis and experiments have proved the reliability of these models.

At present, the research on the Sichuan earthquake is generally divided. For example, the earthquake loss of a single building seismic model is different from a group building model. The former focuses on universality and can be directly used in various regions, but the group model is often a specific condition is required, such as a certain city, or a certain mountain range, and a specific building type, such as a high-code concrete building group and an undefended brick-concrete building group. It is worth noting that timber frame buildings have better seismic performance than concrete buildings [5]. But timber construction is not widely available in China. The parameters and results calculated by them are very different.

The loss estimation model is based on previous studies and summarizes the architectural characteristics of Sichuan and determines the basic conditions and parameters of building losses under seismic activity.

This article presents a methodological approach to the study of earthquake building loss modeling, presenting the methodology and results in terms of earthquake distribution models, classification of earthquake magnitudes according to the degree of building damage, and the expected losses of buildings in earthquakes, respectively.

2. Methodology

The earthquake prediction model (for magnitude and frequency) and building loss model are involved in the total loss estimation model. To maximize the data and construct a more accurate model, Hokkaido in Japan and Sichuan in China were chosen as our data collection points and building loss collection point respectively. Hokkaido had numerous earthquakes in the past decade but less house damage, which is conducive to the establishment of the earthquake prediction model. Sichuan province had fewer seismic activities while Sichuan province had high representativeness in building damage, which is suitable to establish the loss estimation model.

a) The figure below illustrates the whole procedure of the loss estimation model. Before data collection and analysis, here are four assumptions to obtain the model: firstly, the probability of an earthquake occurring anywhere in the potential hypocenter area is ought to be the same; Secondly, the number of earthquakes decreases with the increase of magnitude; Thirdly, site seismic parameters are a function of magnitude and distance; Finally, assuming the occurrence of earthquake satisfies Poisson distribution, and time of earthquake incidence is independent and random. According to Figure.1, there are several steps in this model: Earthquake distribution model: when the duration is t, the number of occurrences of earthquake $m(a, a+t)$ follows Poisson distribution whose parameter isλt. The distribution function is shown as below:

$$F_i\big(\xi \leq m(a, a+t)\big) = \sum_{k=0}^{m(a,a+t)} \frac{\lambda^{kt}}{k!} e^{-\lambda t} \tag{1}$$

b) Verification of the earthquake distribution model: to test the accuracy of the distribution model, the Sichuan province seismic data from 2008 to 2011 whose magnitude is over 4.0 are collected in this section. We could figure out the probability distribution function (PDF) of the earthquake distribution case:

$$P = \left|1 - 2\sum_{k=0}^{m(a,a+t)} \frac{\lambda^{kt}}{k!} e^{-\lambda t}\right| \tag{2}$$

After that, the parameter λt could be decided by the number of earthquakes and the time duration. The probability came out from the PDF function with the given parameter. Then, we collected the aggregate of magnitudes among minimum magnitude and maximum data, finding the maximum difference between this set of data, which is regarded as R. The magnitude range of the potential earthquake is ought to locate between the existing maximum magnitude and the maximum magnitude plus the difference R. An earthquake happened in 2012, in Sichuan province helped us to check the accuracy of the earthquake distribution model and the results were acceptable. Moreover, to indicate the earthquake distribution follows the exponential distribution, the seismic data from 2017-2019 in Hokkaido, Japan were chosen to calculate the parameter and RMSE via Python.

c) Building seismic classification: As reported to the Modified Mercalli Scale (MMS) to describe the building performance during seismic activity. When the earthquake magnitude over V the building will get damaged and cause the finical loss in general condition. Hence, apart from earthquake distribution and occurrence, we must filter out earthquakes with an earthquake intensity greater than 5 from the annual forecast data as our goal. However, the criteria for building damage vary from region to region. For example, the Japanese Meteorological Agency stipulates that a degree of damage can be caused to non-reinforced concrete buildings if the intensity is greater than four, and it is important to analyze and classify them according to local circumstances during the study.

d) Building loss model: the total price of a building during its service life can be expressed as follows:

$$L = C_C + C_t + C_m \tag{3}$$

When L is the loss of the total damage, C_c is the initial construction cost, C_t is the testing cost, C_m and is the maintenance cost. Building property includes residential land, public management, and public service land, different types of buildings differ from the inspection cost. Table.2 illustrates the differences among these constructions. For building maintenance, the coefficient is 2% or 2.5% if the building is equipped with elevators. This is a simplified model. Damage to a building structure is often not caused by earthquakes alone, for example, landslides [10], liquefaction of foundations, and other phenomena that occur during earthquakes can cause damage to the structure and result in losses, which are calculated as part of the maintenance costs.

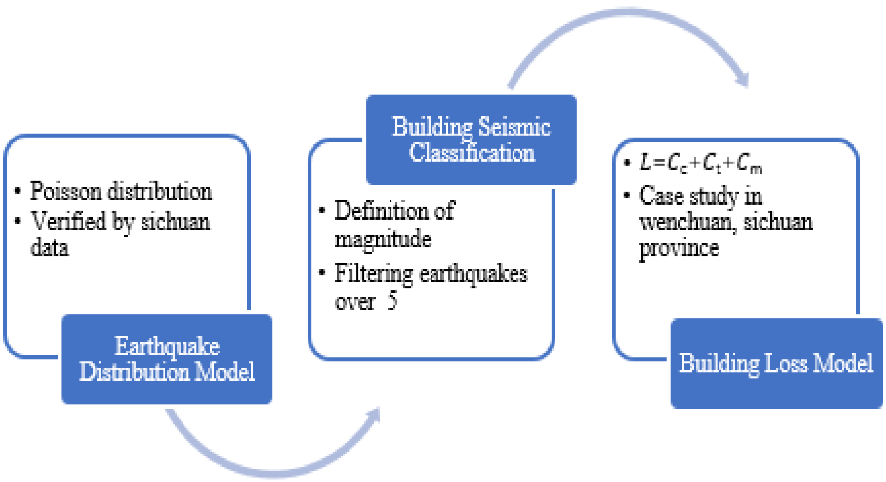

Figure 1. Building loss estimation procedure.

3. Data analysis

For a study, the selection of data is very important. All our subsequent conjectures and verifications are based on the displayed data, not fabricated out of thin air. All theories are based on reality.

To better carry out our research, we chose two high-frequency seismic areas, one is Sichuan Province of China, and the other is Japan. As an area in the same seismic zone, earthquakes are particularly common in these two areas.

A) For whether the earthquake obeys Poisson distribution, we choose Sichuan as the research target. Because earthquakes occur frequently in cities, counties, and districts of Sichuan Province, and the magnitude is different, it is suitable for simulating and discussing different scenarios.

We randomly selected Guangyuan, Sichuan Province as the reference object, and retrieved the earthquake data from 2008 to 2011 from China Earthquake Administration. After sorting, a total of 15 large and small earthquakes occurred and they are shown in Figure.2.

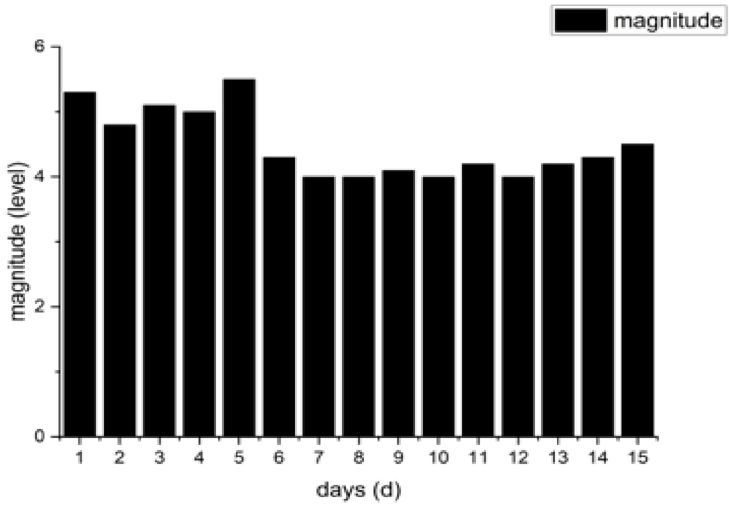

Figure 2. Magnitude map in Guangyuan.

B) Because when the earthquake magnitude is greater than 4, the building will have the risk of collapse. Therefore, after proving the Poisson distribution, we decided to calculate the probability of magnitude greater than 4 through the study of Japanese data, and compare and analyze the prediction conclusions obtained from Poisson distribution. Therefore, due to the comprehensive distribution of earthquake magnitude in Japan, we have collected earthquake data from many places in Japan.

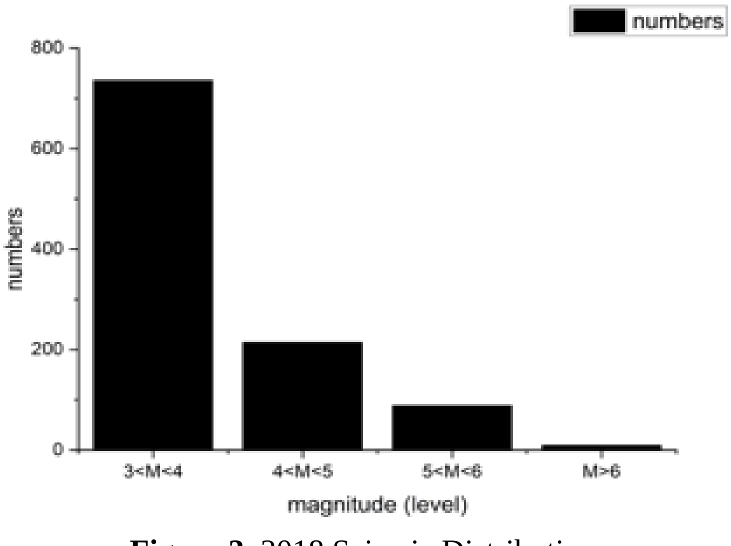

Figure 3. 2018 Seismic Distribution.

c) The May 12 Wenchuan earthquake caused a direct economic loss of 845.14 billion yuan. Among them, the loss of buildings and infrastructure is very large, accounting for 70% of the total loss. The loss of civilian housing and urban residents' housing is the largest, accounting for 27.4% of the total loss. The loss of infrastructure, including roads, bridges, and other urban infrastructure, was the second, accounting for 21.9%, and the loss of other non-residential houses, including schools and hospitals, was the third, accounting for 20.4%. And some general data is shown in Table.3.

From this, we can see that the economic loss caused by building collapse accounts for a very high proportion of the earthquake, and the loss is very serious. We established a mathematical model and verified the rationality and applicability of the model through an earthquake loss.

On August 13, 2010, after the Wenchuan earthquake, another earthquake occurred in Wenchuan. 36 civilian houses were destroyed and 356 plank houses for the resettlement of earthquake victims were destroyed.

Figure 4. Damage site.

4. Result

In fact, we all know the damage to buildings caused by earthquakes, but before studying economic losses, we must find out the magnitude and probability of earthquakes. In this regard, we cited and compared the data of Sichuan and Japan for research.

4.1. Distribution Model

Before that, some scientists have studied that the distribution of earthquakes is likely to meet the Poisson distribution. In order to make our research more accurate, we summarized some seismic data in the two regions and verify whether their properties meet the Poisson distribution [9], so as to use this conclusion later in this research. The expectation and variance of Poisson distribution are λ. Parameters of Poisson distribution λ is the average incidence of random events per unit time (or unit area). Poisson distribution is suitable to describe the number of random events in unit time.

The occurrence of earthquakes also belongs to random events. For this, we randomly selected one from the large earthquakes in Sichuan in recent years for data analysis. The data are shown in Table 1 below. From the data, we can get the fact that aftershocks occur every day after the earthquake in a period of time. Professor Yu of Eastern Liaoning University has studied a series of earthquakes in Liaoning and obtained the properties of stationarity and ineffectiveness of earthquake occurrence, so as to infer that the seismic random event flow approximately obeys Poisson distribution

4.1.1. Stationarity

Suppose m (a, b] represents the number of seismic events in a basic unit in the time interval (a, b]. From the data we collect, we find that the longer the interval (a, a+t] is, the more earthquakes. So we have the relation:

$$p[\xi = m(a, a+t)] = p[\xi = m(0,t)] \tag{4}$$

4.1.2. Mutual anisotropy

From Fujian Earthquake Agency, we know that after the Wenchuan Earthquake, earthquakes of the same magnitude did not occur again, but many low-level aftershocks occurred. In time t, the probability of two or more earthquakes is $p[\xi > 1]$. It means that when t→0, $p[\xi > 1] = o(t)$ (infinitesimal of higher-order).

4.1.3. Independent

From Figure 1, we knew the number of earthquakes in the same area at the time (a, b] does not affect the number of earthquakes in time (c. d] and that means they are independent so we can get:

$$\begin{aligned} & p[(\xi = m(a,b)) \cap (\xi = m(c,d)] \\ & = p[\xi = m(a,b)]p[(\xi = m(c,d)] \end{aligned} \tag{5}$$

When these three conditions are satisfied at the same time, we can confirm that the occurrence of earthquakes follows the Poisson distribution.

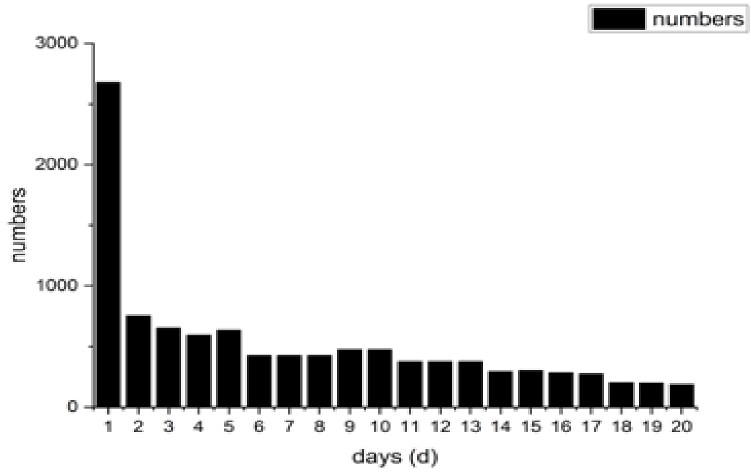

Figure 5. Aftershock statistics.

Since the occurrence of earthquakes obeys Poisson distribution, we can use this condition to predict the earthquake situation of the next year. In order to confirm the validity, we selected Guangyuan City that is a region in Sichuan to predict the magnitude of the earthquake which may occur in the next year. From 2008/8/6 to 2011/11/1, a total of 15 earthquakes occurred in this region. The specific magnitude is shown in Figure 3.

Let ξ follows Poisson distribution (ξ is the number of earthquakes) and substitute the data into the formula (2), and then we can get:

$$F(\xi \le m(2011{,}2012)) \approx 0.125 \tag{6}$$

So the probability of occurrence is 0.751, A= {5.4, 4.8, 5.1, 5.0, 5.5.......} and A is the aggregate of magnitudes. Max=5.5, Min=4, R= Max-Min =1.5.

From the theory of earthquakes: Magnitude ϵ [Max, M+R], we can predict the Range of the next earthquake is in [5.5, 7.0], and the probability P=0.751.

After the calculation, we compared it with the real situation and found that there was indeed a big earthquake with a magnitude of 5.7 in the second year in that region. So we can use this model to predict the magnitude of the earthquake which may occur in next year in this region. However, one defect of this model is that it cannot accurately predict the exact time of the earthquake. This defect is also one of the research directions of scientists, but so far it cannot be accurately predicted.

4.2. Building seismic classification

The internationally used Richter scale generally divides earthquakes into magnitude 9, and the energy released by a magnitude 6 earthquake is equivalent to the energy of the atomic bomb dropped by the United States on Hiroshima, Japan. For each magnitude difference, the energy difference is about 1000 times. According to the data, we have collected, when the magnitude is greater than 4, it is likely to cause building damage. Therefore, in order to get the loss caused by building damage, we need to calculate the probability of earthquakes greater than magnitude 4 per year.

Table 1. Earthquake magnitude classification.

Magnitude	M<3	3<M<4	4<M<6	M>6
Result	Generally imperceptible	We can feel it, but it won't cause damage	it is likely to cause building damage	Serious damage

We select some earthquakes that happened in Hokkaido. With the verification of Poisson distribution, the exponential distribution could be chosen as our target model. After the assumption of methodology, we found that the distribution of earthquake magnitudes in the same year is very similar to the exponential distribution. Python was introduced to draw the earthquake magnitude distribution image in recent years.

With the calculation, we get the parameter λ and the root mean square error (RMSE) from 2017-2019 shown in Table.2.

Table 2. Calculation Table.

Year	2017	2018	2019
λ	0.991	0.961	0.970
RMSE	0.219	0.201	0.206
R^2	0.814	0.772	0.803

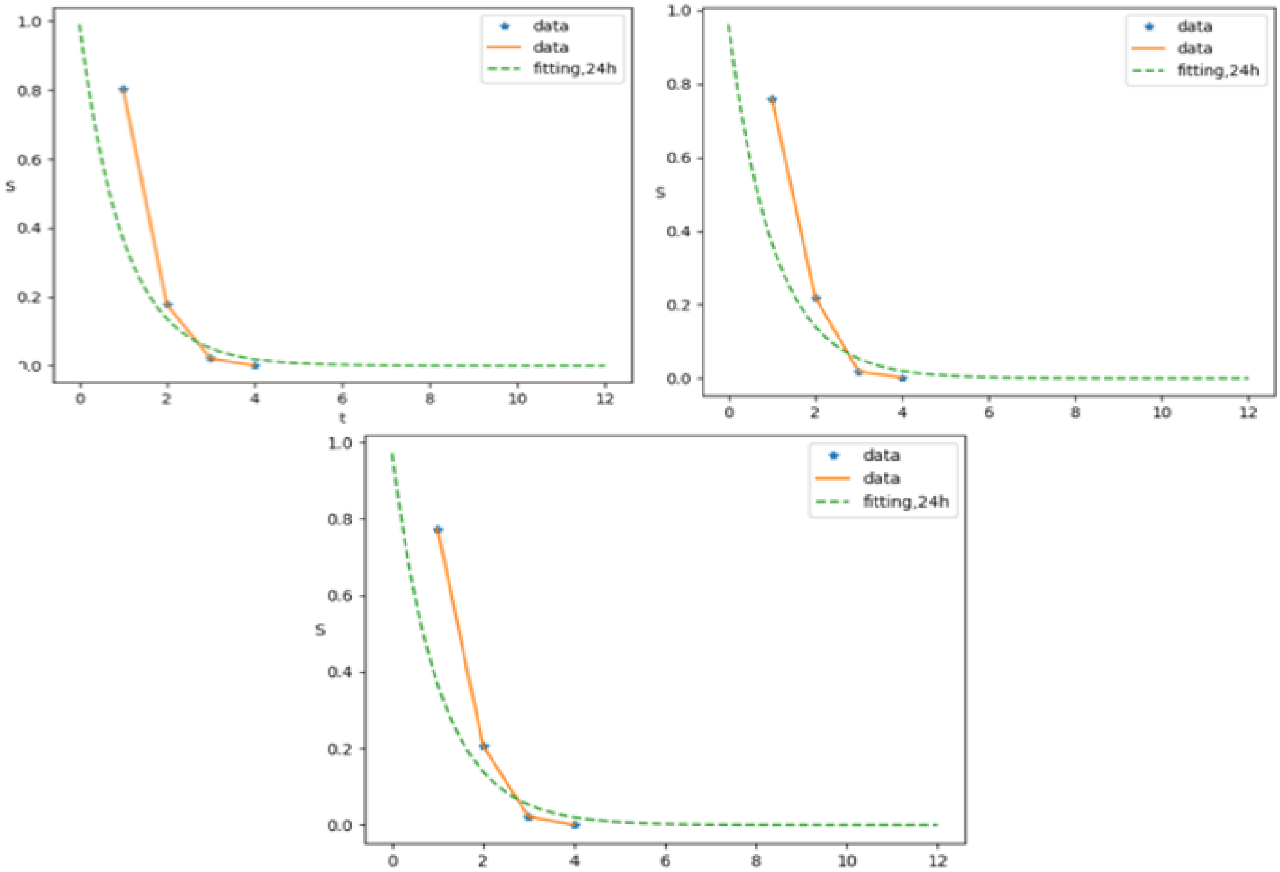

Figure 6. Magnitude distribution from 2017 to 2019.

After calculation and fitting, we found that for the same place, the probability distribution of earthquake magnitude in different years has very high similarity and obeys the exponential distribution. Our conjecture has been confirmed to some extent.

4.3. Evaluate the Value at Risk

In fact, the Wenchuan earthquake caused huge losses to the economy of the whole province and even the whole country. Especially in terms of economy and casualties, the losses were heavy. In terms of economy, except for the failure of infrastructure and the damage of vehicle pavement, the main economic loss is the economic loss caused by the collapse of buildings. According to the data from China Earthquake Administration, the earthquake caused more than 6.5 million houses to collapse, more than 23 million houses to be damaged.

From the news published in China Daily, we know the earthquake caused more than 6.5 million houses to collapse, more than 23 million houses to be damaged. Such data once again proved the high proportion of building damage to economic losses in the earthquake.

In order to establish the model of economic loss caused by house damage, we should first deeply explore what part of the cost will be consumed by building damage. The construction and use of a building require many different costs, including the initial construction cost, that is, the cost of some materials or structures; Building inspection cost, that is, the cost of safety inspection conducted between each house or building being put into use; Building maintenance cost refers to the cost of repairing the damaged house or problems found in regular maintenance. To verify the effectiveness of formula (3), we calculate the loss caused by building collapse in an example and compare the result to the reality.

To make the model more perfect and detailed, I searched for some information and made the following tables of different costs. The initial construction cost is shown in Table.4 which includes the material and some building structures, and the inspection cost is shown in Table.5. For maintenance cost, there is the coefficient k of maintenance: k=2.5% for the house equipped with an elevator otherwise k=2%. We found the cost in Chengdu is ¥1100/m^2 and we used it for the next example.

Because it happened in 2010, the initial cost may be lower than that in 2021. The cost of a plank house is ¥600/m^2 and the indoor property is ¥30/m^2. The maintenance cost is ¥1100/m^2, and the coefficient is 2%. Because it is located in a remote village, the cost of house detection and decoration is not considered. After substituting these data into the model formula, we know that: Cc=4704000, Ct=235200, Cm=172480

The total economic loss is about ¥5111680 due to the collapse of buildings. Because it is in a remote village, the cost of house detection and decoration is not considered. After substituting these data into the model formula, we estimated that the event caused a total economic loss of the collapse of buildings. In that year, the news reported that the economic losses caused by the collapse of buildings were about 5 million. The difference is estimated to be reflected in the maintenance cost. The maintenance cost in remote villages should be lower than that in cities. The calculated results are close to the actual results, indicating that the evaluation results are reliable.

Table 3. Seismic loss table.

Economic loss	Victims	Injured	Missing
845.14billion yuan	69225	374640	17923

Table 4. Material price list.

Category	Price/Cost	Selection
Reinforced concrete structures	¥1000-1200/m^2	1200
Brick concrete structures (masonry structures)	¥700-800/m^2	800
Ordinary houses	¥700-800/m^2	800

Table 5. Inspection cost list.

Category	Cost
Ordinary houses	M_0=1.33*Area
House without drawing	M=1.3M_0
Ancient Architectural Building	M=2M_0
Floor height >4m	M=1.2M_0
Economical house	M=0.5M_0

Table 6. Magnitude distribution in Hokkaido.

Year	3<M<4	4<M<5	5<M<6	M>6
2017	512	113	13	0
2018	734	212	18	2
2019	503	134	14	0

5. Conclusion

Based on the principles of probability theory, such as Poisson distribution and exponential distribution, this research establishes a mathematical probability model for earthquake events, and obtains the earthquake magnitude and its probability relationship, so as to predict the possible earthquakes in the next year. In this research, two different earthquake distributions in Sichuan Province are successfully verified by using this method. In addition, aiming at the loss caused by building collapse in the earthquake, this paper introduces the evaluation method of earthquake risk probability and considers the value factors related to all aspects of buildings to evaluate the economic loss caused by building collapse. The research results of this paper can provide an effective idea and framework for the risk analysis of building value.

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