

Error compensation analysis of time-grid angular displacement sensor

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Abstract. The time-grid angular displacement sensor is an advanced angular displacement measurement device widely used in various precision instruments, industrial control systems, and robotics, with high accuracy, fast response, strong anti-interference ability, and long life. In this paper, the sensing mechanism of the embedded time-grid angular displacement sensor is used to analyze the compensation effects of Harmonic error compensation, HHT-ELM Error Compensation, PSO-BP neural network error compensation, and Multi-step compensation effects of Harmonic error compensation, HHT-ELM Error Compensation, PSO-BP neural network error compensation, and Multi-step calibration method are analyzed and compared to find that the Multi-step calibration method has the best compensation effect but the compensation system is complex and suitable for places with high accuracy requirements. compensation, HHT-ELM error compensation, and Harmonic error compensation. and as the compensation effect decreases the compensation method also tends to be simpler.

Keywords: Embedded Time-Grid Angular Displacement Sensor, Angular Displacement Sensing Mechanism, Error Traceability, Error Compensation, Neural Network.

1. Introduction

1.1. Background

With the development of industrial technology and the information technology industry, the importance of precision measurement technology and instrumentation, represented by sensors, is becoming increasingly prominent in machine building and the entire industrial system. Measurement technology is a key technology for cognition and transformation of the world, widely used in the field of measurement and testing and machine building, and its level directly affects the development of the machinery manufacturing industry.

The core idea of a time-grid angular displacement sensor is "inter-temporal and spatial conversion", which is based on the principle of a time-grid sensor, measuring the rotation angle by measuring the time difference between a fixed number of grids and the timing pulses at different positions, thus avoiding the measurement error caused by the limitation of inscription accuracy of traditional grid displacement sensor. And it has the characteristics of high accuracy, high stability, long life, high adaptability, high sensitivity, etc.

The history of time-grid angular displacement sensors can be traced back to the early 1970s. The earliest time-grid sensor was invented by John Kitching and Don Monroe of Stanford University in 1973. This sensor is used to measure the displacement by using the interference signal generated by the laser beam passing through two gratings, with high accuracy and high resolution. In the early 1980s, an American electrical engineer, John Shaw designed the first time-grid angular displacement sensor, which could measure the angular displacement and rotational speed of high-speed rotating mechanical parts with high resolution and sensitivity.

With the development of technology, the time-grid angular displacement sensor has gradually been widely used. In the medical field, the time-grid angle displacement sensor is used to measure the rotation angle of human joints to help doctors diagnose and treat joint diseases; in the industrial field, the time-grid angle displacement sensor is used to measure the rotation angle of the blades of wind turbines, the rotation angle of the crankshaft of automobile engines, etc.; in aerospace, the time-grid angle displacement sensor can be used to measure the attitude and rotation speed of aircraft to ensure

the safety and stability of aircraft; in precision manufacturing, the time-grid angle displacement sensor can be used to measure the attitude and rotation speed of aircraft to ensure the safety and stability of aircraft. In precision manufacturing, the time-grid angular displacement sensor can measure the small deformation of mechanical parts, to improve the accuracy and stability of products; in semiconductor manufacturing, the time-grid angular displacement sensor can be used to measure the rotation speed and angle of silicon wafers, to improve the accuracy and quality of wafer manufacturing.

With the deepening of the productization of time-grid sensors, the market has put forward higher requirements on the measurement accuracy, stability, and resolution of the sensors. To further improve the measurement performance of the time grid, an in-depth analysis of the mechanism of the measurement error of the magnetically modulated time grid is needed to lay the theoretical foundation for optimizing the sensor structure and studying the error compensation method.

1.2. Recent Progress

Han Yu proposed an error compensation method based on the Hilbert-Huang transform-limit learning machine for the problem of large original errors of embedded angular displacement sensors and carried out experimental research on sensor error compensation. And for the problem of accuracy degradation during the use of an embedded time-grid angular displacement sensor, the self-calibration method of sensor error based on the single measurement head error phase offset method is proposed. The error self-calibration model is verified, and the results show that the error of the sensor is reduced to 6.0" after self-calibration[1].

Qing-Song Zhou addresses the short-period error calibration problem of the embedded time-grid angular displacement sensor and proposes an over-limit learning machine (ELM) calibration method and a comparative study with the Fourier approximation method is carried out. The Fourier approximation approach, the adaptive mean square difference threshold method, the ELM, and the particle swarm optimization (PSO) method are coupled to increase the sensor's overall measurement accuracy. The step-by-step error calibration model is proposed to compensate for the systematic, gross, and random errors of measurement. The calibration model is proposed to compensate for the systematic error, coarse error, and random error, and finally achieve the high accuracy calibration of the sensor. The experimental results of the error calibration show that the short-period error of the sensor reaches 4.0" after calibration by the over-limit learning machine, which is higher than that of the Fourier The calibration results show that the short-period error of the sensor reaches 4.0" after calibration by the overrun learning machine, which is more effective than the Fourier approximation method for the short-period error calibration. The sensor measures with a final precision of 0.9" following a multi-step calibration process, and this method is important for the high-accuracy measurement of embedded time grid sensors. This method has important application value to realize the high accuracy measurement of embedded time-grid sensors[2].

To realize the error separation of the time-grid displacement sensor, and then correct the error to improve the measurement accuracy of the time-grid displacement sensor, Zhong-Hua Gao adopts the principle of multi-signal superposition to separate the errors of the stator and rotor of the time-grid displacement sensor during processing, and eliminate most of the long-period errors, and conduct harmonic analysis on the separated error components to establish an error correction model. The error correction model is used to correct the line slot indexing errors of the stator and rotor of the time-grid displacement transducer, and the accuracy of the field-type time-grid displacement transducer is significantly improved to better than $\pm 2''$. It is proved that this method of error separation is very effective in eliminating the circumferential indexing error, and the error correction model established is effective in correcting the sensor error[3].

Tianheng Zhang proposed an error compensation and parameter identification method based on sinusoidal function and fast Fourier transform (FFT) to improve the measurement accuracy and calibration efficiency of the time grating angular displacement sensor. The experimental results show that the error of the time grating angular displacement sensor is reduced by 38.4 times after the

compensation. The application of the new method can greatly improve the measurement accuracy and calibration efficiency of the time-grating angular displacement sensor[4].

Min Liang proposed a software error compensation method based on the PSO-BP neural network model to further improve the measurement accuracy of the sensor. The experimental results show that the PSO-BP neural network-based time grating sensor error compensation method can achieve a better compensation effect under the three set speeds, and the peak-to-peak value of the whole-period error is reduced by 97% after compensation, and the accuracy is within 0.0137° , and the method has better compensation effect compared with the harmonic correction method[5].

1.3. This Paper

This article starts from the mechanism of error generation of embedded sensors, analyzes four error compensation methods: harmonic error compensation method, HHT-ELM error compensation method, PSO-BP neural network error compensation method, and multi-step calibration method, compares the compensation effect of these four error compensation methods, and analyzes what kind of error each is suitable for compensation and the applicable environment.

Chapter 2 introduces the working principle of the time-grid angular displacement sensor, and then introduces the theories of the five error compensation methods. Chapter 3 introduces the process and compensation effect of each of the three error compensation methods, and then compares the three methods. Chapter 4 presents the comparison results and the scope of the application.

The application range of error compensation technology for time-grid angular displacement transducers involves many fields such as robotics, aerospace, manufacturing, automotive manufacturing, etc. It can provide high accuracy, high stability, and high reliability measurement and control for industrial applications.

2. Principle and error calibration method

2.1. Principle of the time-grid angular displacement sensor

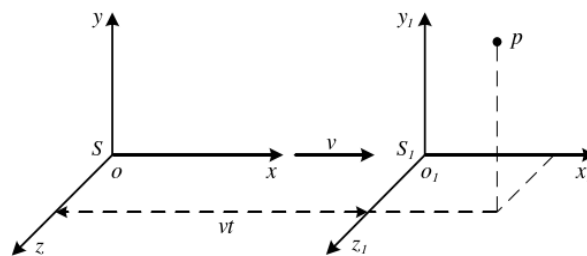


Figure 1. Spatial coordinate system transformation[1]

Let the coordinates of point p in the figure in the coordinate systems S and S1 be (x, y, z) and (x_1, y_1, z_1) , respectively, so that the interrelationship between point p in the two coordinate systems is as follows:

$$\begin{cases} x = x_1 + vt \\ y = y_1 \\ z = z_1 \\ t = t_1 \end{cases} \quad \text{or} \quad \begin{cases} x_1 = x - vt \\ y_1 = y \\ z_1 = z \\ t_1 = t \end{cases} \quad (1)$$

Constructing a dual coordinate system in time and space makes it possible to measure space by time. By converting the requirement of the uniform velocity of the measurement object into the requirement of the uniform velocity of the relative motion between the two coordinate systems, it is possible to The time measurement of variable speed motion is possible.

Tectonic traveling waves have the advantages of similar spatial and temporal periodic propagation with natural traveling waves and simultaneously possess the qualities of a controllable vibration source, the ability to remove outside interference, cheap cost, etc. Therefore, artificially constructing

tectonic traveling waves similar to natural traveling waves in the time grid is a key way to realize the conversion of measurement reference space-time coordinates.

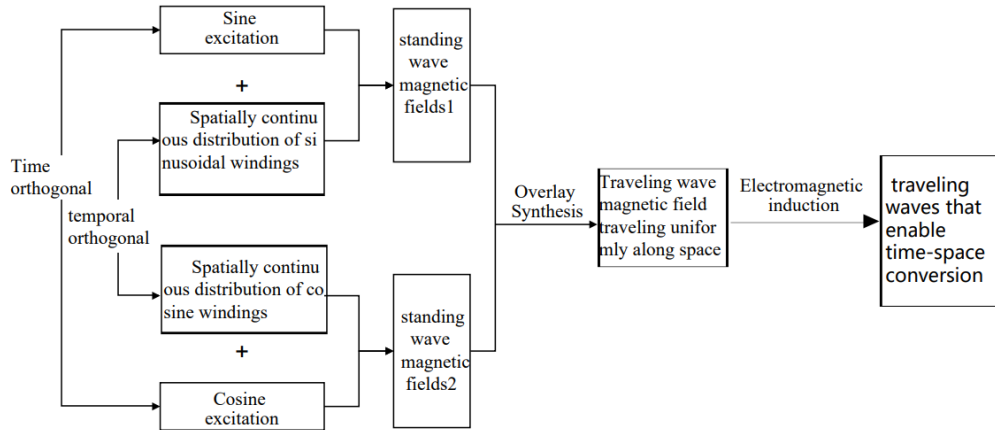


Figure 2. Time-grid traveling wave formation method [2]

Phase analysis of the sensed traveling wave signal can solve the spatial angular displacement information based on the relationship between phase and angular displacement.

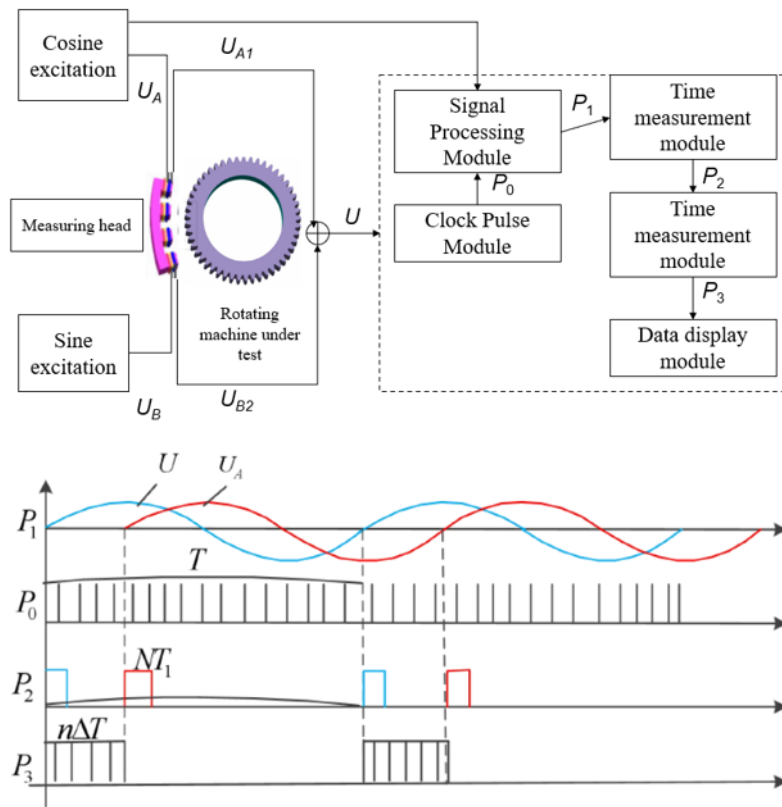


Figure 3. Block diagram of measurement principle of embedded time-grid angular displacement sensor [2]

2.2. Harmonic error compensation method based on fast Fourier transform Error compensation method

The measurement error curve of the embedded time-grid angular displacement sensor is a periodic continuous function, which satisfies the Dirichlet Conditions. Therefore, based on the Fourier principle, the measurement error curve is decomposed into a linear superposition of infinite error harmonics, and the compensation of the original error can be realized by calculating the frequency, amplitude, and phase of each harmonic error.

2.3. Hilbert-Huang Transform

The Hilbert-Huang Transform (HHT) is a non-stationary signal analysis method that consists of two parts: the Hilbert transform and the empirical mode decomposition (EMD). The Hilbert transform can decompose a real function into an orthogonal function pair, which includes both global and local information of the original function, the EMD, on the other hand, can decompose a nonlinear and nonsmooth signal into a superposition of several eigenmode functions (IMFs), each representing a different frequency and amplitude component of the signal. Through the combination of these two steps, the local frequency and energy distribution of the signal can be obtained, thus enabling the analysis and processing of nonlinear and non-stationary signals.

2.4. Adaptive Mean-Squared Thresholding method

Using the Bessel formula to calculate the mean square error value of the entire error sequence, and then setting a threshold based on analysis to determine the range of coarse errors. When the mean square error of the error sequence is greater than the set threshold, sort the error data from large to small, determine whether the endpoint data is a coarse error, and locate and replace it until the mean square error value is less than the defined threshold.

2.5. Extreme Learning Machine

The feedforward neural network can be considered a linear system, and the output weights can be calculated by a simple generalized inverse operation of the output matrix H of the hidden layer to achieve the approximation of any continuous system. Compared with BP neural network, this method is faster to learn and more accurate. The learning speed is faster, the accuracy is higher, and the generalization performance is better.

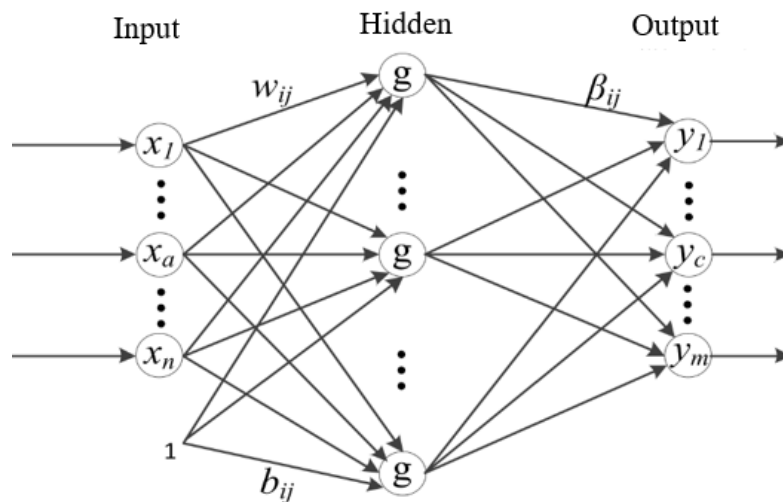


Figure 4. ELM network structure diagram [3]

2.6. Optimization via Particle Swarm

An intelligent optimization program is known as the particle swarm optimization algorithm was created by modeling how birds forage in the wild, in which individual particles in a swarm share information to complete the search for the optimal value. Each particle has a memory function that allows it to remember its optimal position while searching for the optimal value as the local optimal value of the particle. It can also use this memory function to determine the optimal position of the particle based on information sharing among the particles. Each particle has an adaptation function related to the target value to determine the adaptation value to judge the current position of the particle. Each particle in a D -dimensional space has a D -dimensional position vector and velocity vector. These vectors are used to compute the adaptation value of the current location and velocity about the goal value and to identify the ideal solution for the entire particle swarm.

3. Comparison of error compensation process and results

3.1. Harmonic Error Compensation and HHT-ELM Error Compensation

Han Yu's team first conducted an error test experiment on the sensor, and the result showed that the sensor's initial inaccuracy for the entire circumferential range is 117.9", which is a large original error, we hope to solve the problem of large original error in the next data processing.

Han Yu performed the traditional harmonic error compensation method:

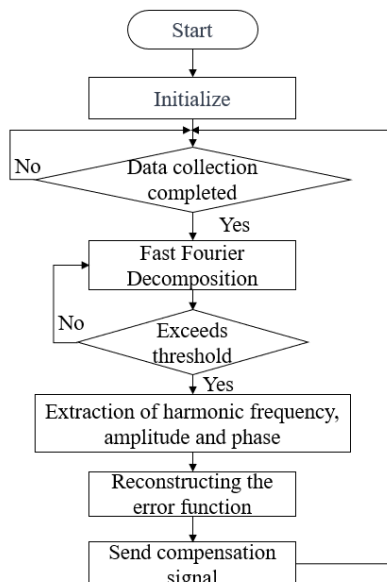


Figure 5. Harmonic error compensation experiment flow chart [1]

MATLAB numerical analysis software's rapid Fourier transform of the sensor's initial error sequence indicates that: the original error of the embedded time-grid angular displacement sensor is mainly the low and medium frequency harmonic error within 800 times, which satisfies the system error characteristics, so the frequency, phase, and amplitude of the main harmonic error are selected to reconstruct the original error, and the reconstruction effect is analyzed with both the short- and long-term errors of the sensor.

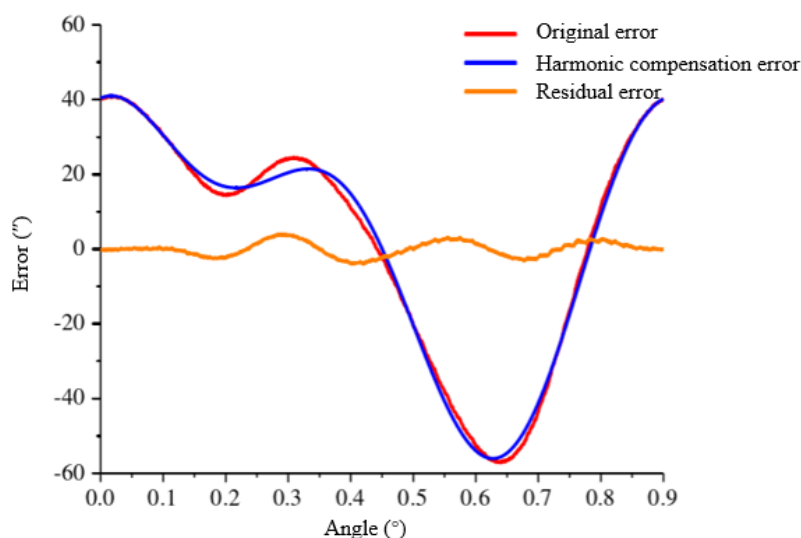


Figure 6. Short-period error reconstruction effect diagram [1]

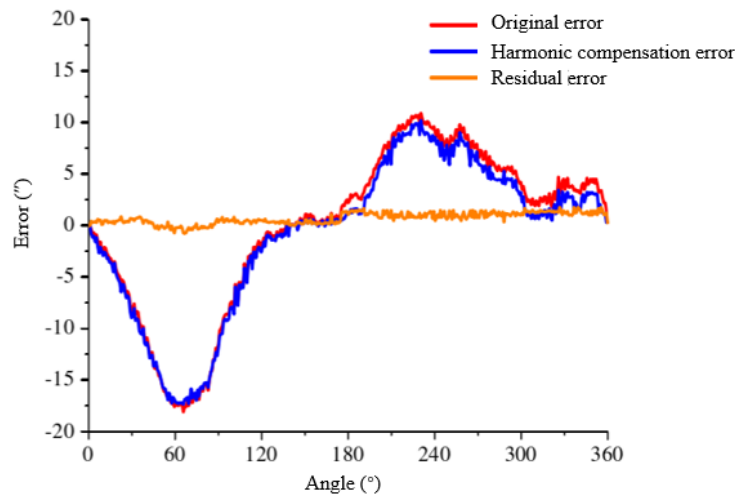


Figure 7. Long-period error reconstruction effect [1]

The error obtained from the reconstruction of the harmonic error compensation model is essentially the same as the overall dynamics of the original error. After compensation, the short-period error of the sensor decreased from 98.0" to 8.6" and the long-period error decreased from 29.0" to 2.5". The final measurement error of the sensor was obtained by linearly subtracting the original error series from the error series reconstructed by the harmonic error compensation model.

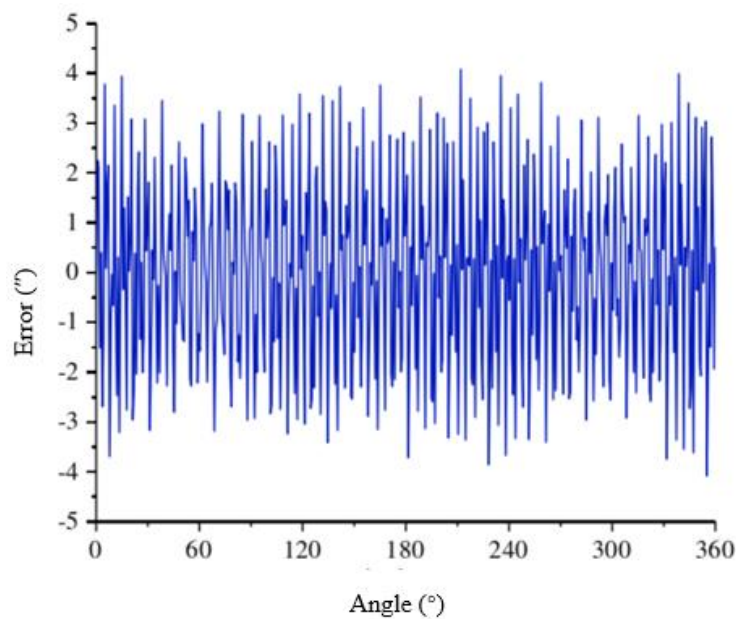


Figure 8. Sensor final measurement error [1]

Figure 8 shows that after the harmonic error compensation, the sensor's measuring inaccuracy is significantly decreased across the entire circumferential range, and the measurement error's peak-to-peak value is decreased from 117.9" to 9.0", which can effectively reduce the system error of the sensor.

In the next Han-Yu designed the HHT-ELM error compensation experiment, the original sensor error sequence was first subjected to HHT transformation to reconstruct most of the systematic error components in the original error, and then the ELM model was used to reconstruct the residual error to obtain the final measurement error of the sensor.

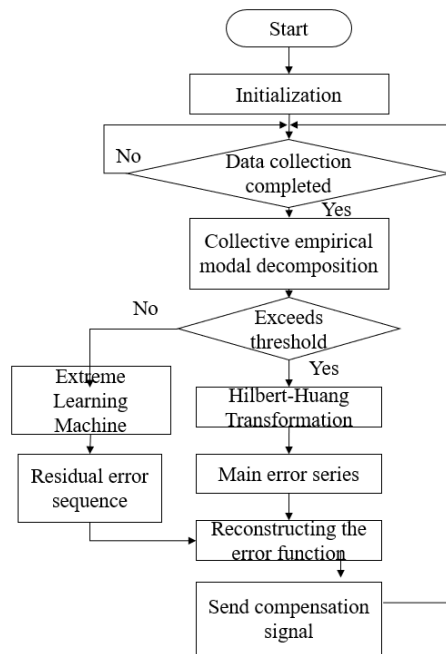


Figure 9. HHT-ELM error compensation experiment flow chart [1]

The instantaneous frequency and instantaneous amplitude of the main IMF components are brought into equation (4.9) to reconstruct the original error and analyze the reconstruction effect with both the short- and long-term errors of the sensor.

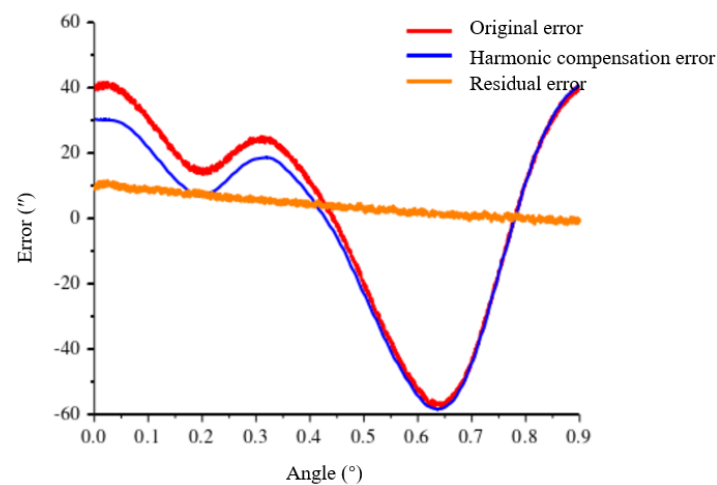


Figure 10. HHT model short-period error reconstruction effect [1]

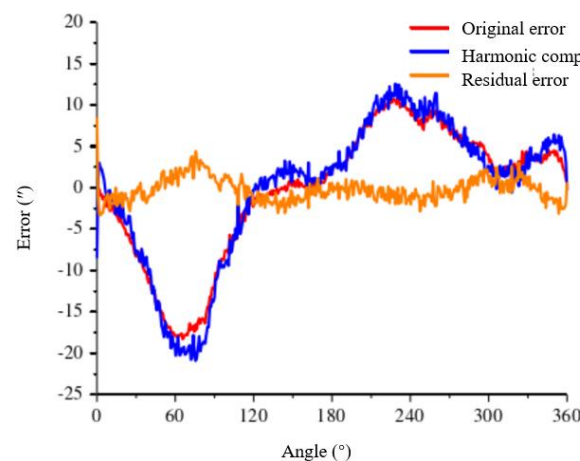


Figure 11. HHT model long-period error reconstruction effect [1]

From Figures 10 and 11, the reconstructed error series is the same as the original error series in terms of overall dynamics, which proves the feasibility of the HHT model. After the HHT error compensation, the peak value of the short-period error of the sensor decreases from 98.2" to 13.6", and the peak value of the long-period error decreases from 29.0" to 11.8".

To improve the compensation efficiency, the residual error is compensated based on the ELM algorithm, and the compensation effect is shown in Figure 12. At the same time, the residual error is linearly subtracted from the error reconstructed by the ELM model, and the final error of the sensor is obtained, as shown in Fig. 13.

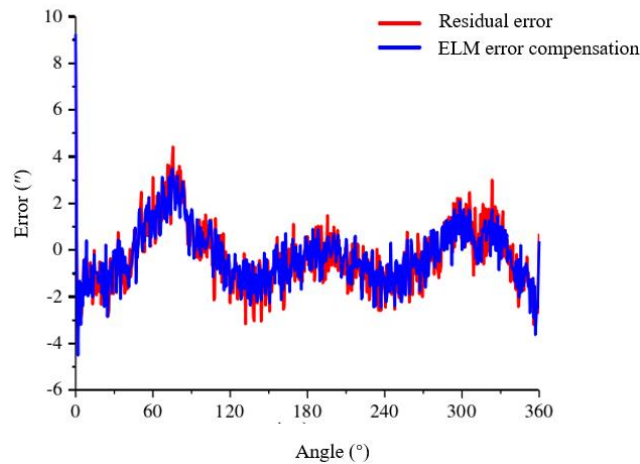


Figure 12. HHT model short-period error reconstruction effect [1]

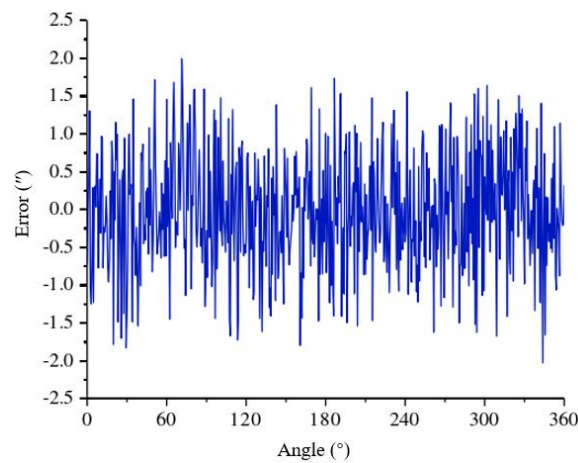


Figure 13. HHT model short-period error reconstruction effect [1]

From Fig. 12, it can be seen that the error sequence reconstructed by the ELM algorithm has a good overlap with the residual error sequence and can compensate for most of the residual error. From Figure 13, the peak value of the final measurement error of the sensor decreases from 117.9" to 4.0" after the error compensation. The experimental results show that the method can be used for error compensation of the sensor.

The experimental results show that the original error peak value of the embedded time-grid angular displacement sensor is 117.9"; After harmonic error correction, the sensor's ultimate error peak value is 9.0"; after HHT-ELM error correction, it is 4.0". By comparing and analyzing the reconstruction effects in multiple spatial periods, it is found that the HHT-ELM error compensation model is significantly better than the harmonic error compensation model.

3.2. Error Compensation Method for PSO-BP Neural Network Model

Min Liang software compensation technique using a particle swarm algorithm-optimized BP neural network is proposed using neural network technology.

The learning rule uses the gradient descent method since the BP neural network is globally optimal, which has the drawbacks of sluggish convergence speed and ease of falling into local optimality. To address the shortcomings of the BP neural network, Min Liang employs the PSO approach to optimize it, taking advantage of the PSO algorithm's quick convergence and robust global search capabilities to enhance the BP neural network model. The specific plan is to train the BP neural network after replacing its initial weights and thresholds with the results of the PSO algorithm.

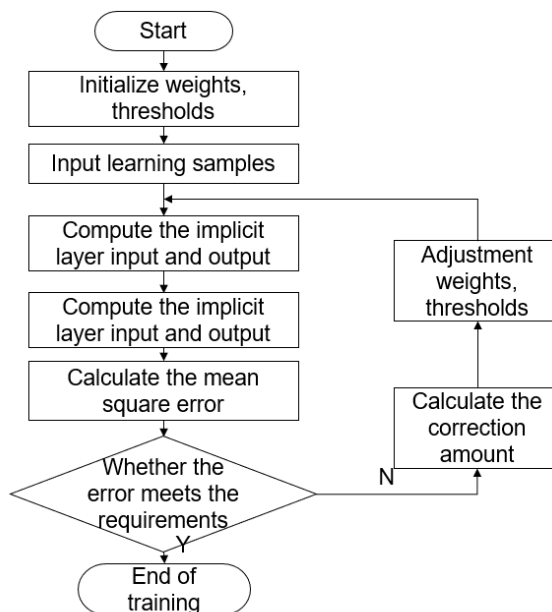


Figure 14. Training flow chart of BP neural network [5]

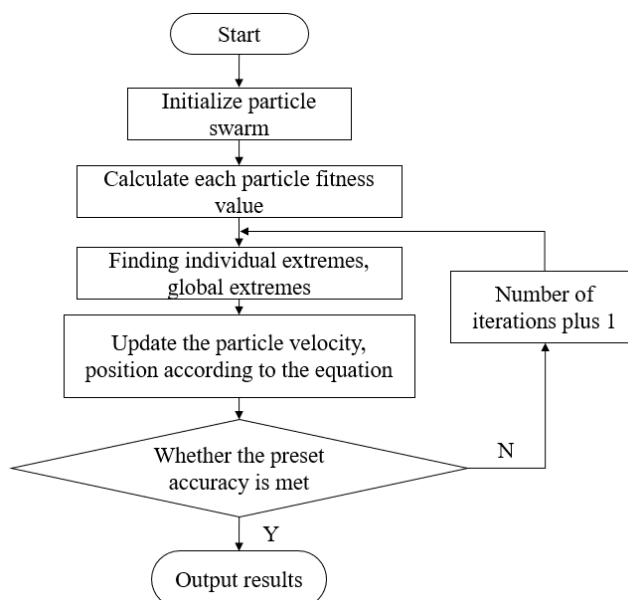


Figure 15. Particle swarm algorithm implementation flow chart [5]

The BP neural network model is created first, and then the PSO-BP neural network model is created by optimizing the BP neural network with the PSO algorithm.

BP neural network and PSO-BP neural network models are created using the MATLAB neural network toolbox and the training neural network and PSO algorithm parameters. The BP neural network and PSO-BP neural network models are trained and tested using sensor simulation data, and simulation performance graphs for both models are obtained (see Figures 16 and 17).

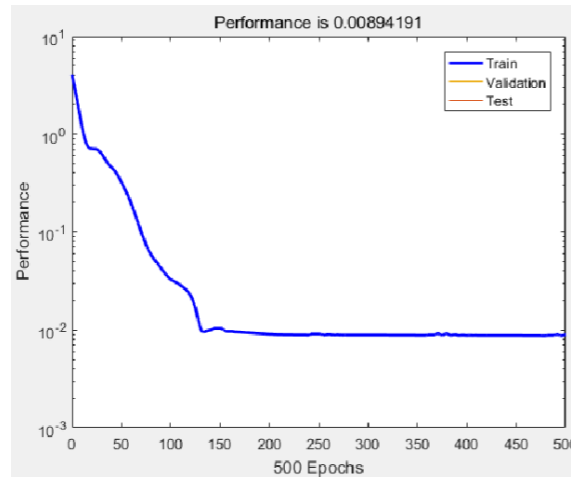


Figure 16. BP neural network model [5]

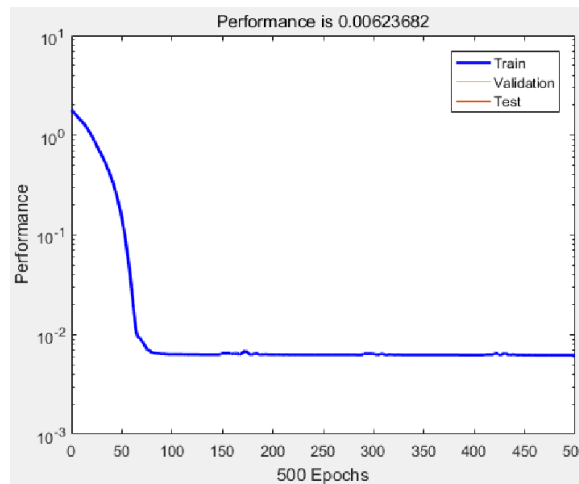


Figure 17. PSO-BP neural network model [5]

The BP neural network model has 186 iterations and the best calibration error is 0.0089, whereas the PSO-BP neural network model optimized by the PSO algorithm has 71 iterations and the best calibration error is 0.0062. When comparing the two models, the BP neural network model fluctuates more at the start of training, whereas the PSO-BP neural network model's performance is smoother and the simulation converges faster. The PSO-BP neural network model's performance curve is smoother, the simulation converges faster, and the number of iterations and best calibration error are lower, demonstrating that the PSO algorithm's optimization of the BP neural network is effective. As a result, the PSO-BP neural network model is chosen as the time-grid sensor's error compensation model.

The measurement errors collected by the time grid sensor at 5 rpm, 10 rpm, and 20 rpm were tested and analyzed by software compensation effect and rotational speed to test the effectiveness of the established error compensation model's compensation and to study the effect of the error compensation model on the error compensation effect of the sensor at different rotational speeds.

The error compensation model of the Neural network PSO-BP was used to compensate for the measurement error in the full circumference (360°) range, and the peak value of the compensated full circumference error was 0.0122° at 5 rpm of the time grid sensor. Compared with the original error peak value of 0.6° before compensation, the compensated measurement error peak value is reduced by 98%; the time grid sensor is at 10 rpm, and the peak value of the compensated error is 0.0131° at 10 rpm of the sensor. Compared with the original error peak value of 0.6° before compensation, the peak value of the compensated measurement error was reduced by 97.8%. The peak value of the error after compensation is 0.0137° at 20 rpm of the sensor. Compared to the original error peak value of 0.6° before compensation, the peak value of the compensated measurement error is reduced by 97.7%.

When the neural network model using PSO optimization error compensation model is used to compensate the time-grid sensor, the error compensation effect is affected by the sensor speed, but the effect is not significant at the three speeds mentioned above, and the peak value of the full-period error after compensation is within 0.0137° . The peak-to-peak error value is reduced by 97%, which proves the feasibility and effectiveness of the neural network-based error compensation method for the time-grid sensor.

3.3. Multi-step Calibration Method

To better calibrate the measurement errors and improve the calibration accuracy, a multi-step calibration method is proposed from the error traceability analysis of the embedded time-grid displacement sensor, combining Fourier approximation theory, adaptive mean square difference threshold method, overrun learning machine algorithm and particle swarm optimization algorithm.

The error model of the embedded time-grid angular displacement sensor is further subdivided into systematic error and nonlinear error, in which the nonlinear error is mainly coarse error and random error, and different calibration methods are used to calibrate these three errors in steps, to achieve higher measurement accuracy.

First, the systematic errors are calibrated by Fourier approximation; then the coarse errors are located and rejected by the adaptive mean square error threshold method; finally, the random errors are calibrated by the particle swarm optimization algorithm.

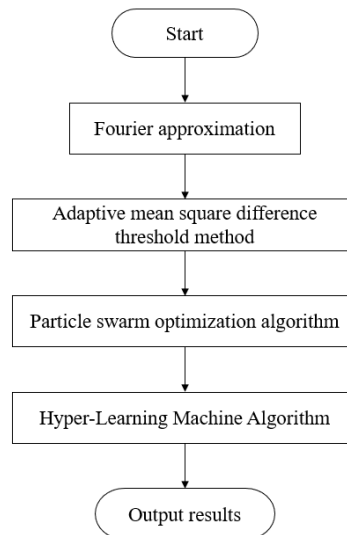


Figure 18. Flow chart of the multi-step calibration method

Based on the Fourier approximation theory, the harmonics with a greater influence on the whole period error are selected to complete the reconstruction of the periodic component of the sensor system error, to calibrate the periodic element of the system error in the total measurement error of the sensor. When the number of periodic terms is $n=300$, the calibration effect is shown in Figure 19 and Figure 20.

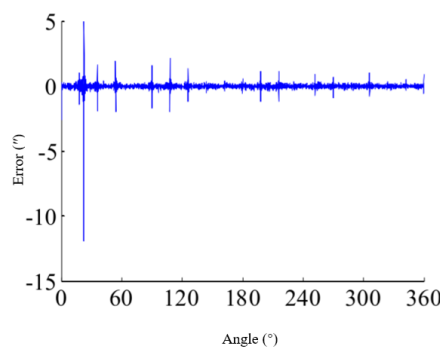


Figure 19. Flow chart of multi-step calibration method [2]

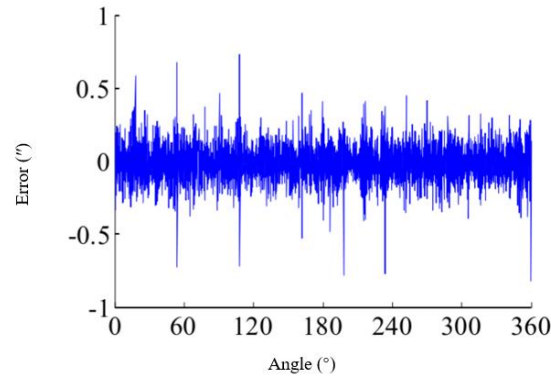


Figure 20. Error curve after coarse error rejection [2]

Figure 19 shows the error curve after Fourier calibration, the error peak value is reduced from 116.7" to 17.6", and the error is reduced by 95.9%. It can be seen that the Fourier approximation calibration method can effectively reduce the periodic component of the measurement error, but there is still a large coarse error and random error.

The error curve after removing the coarse error from the residual components in it is shown in Figure 20, and the error peak value is reduced from 17.6" to 1.7", and the error is reduced by 95.9%.

After removing the systematic error and coarse error by the Fourier approximation method and adaptive mean squared difference threshold method, respectively, the random error still exists in the sensor. The PSO-ELM algorithm was used for error calibration of the random errors in the sensor, and the calibration results are shown in Figure 21.

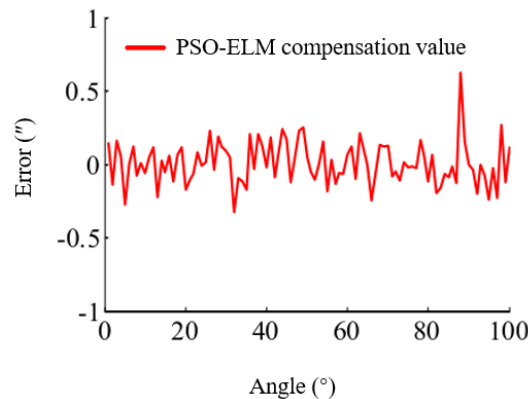


Figure 21. PSO-ELM error calibration [2]

From Fig. 21, it can be seen that after using the PSO-ELM algorithm to calibrate the random error, the peak residual error is reduced from 1.7" to 0.9", and the error is reduced by about 80.8%, which proves the effectiveness of PSO-ELM to calibrate the random error.

The measurement error was reduced from 116.7" to 0.9" after the multi-step calibration method, and the overall error was reduced by 115.8", with an average reduction in the measurement error of about 99.2%.

3.4. Comparison of error compensation methods

Table 1. Comparison Of Error Compensation Effects

Table Head	Table Column Head			
	Harmonic error compensation	HHT-ELM Error Compensation	PSO-BP neural network error compensation	Multi-step calibration method
Error reduction rate	92.4%	96.2%	97%	99.2%

It can be seen that harmonic error compensation processing is not as effective as using neural networks to compensate for errors, because harmonic error compensation can compensate for harmonic disturbances in the system but cannot handle complex nonlinear mapping relationships. Neural networks can be utilized to correct nonlinear flaws, enhancing the system's robustness and control precision. Nevertheless, neural networks need a lot of data to train, which uses up a lot of processing time and resources and can make the system more expensive and complex.

Whether the error is reconstructed with HHT or the parameters of the neural network are optimized with the PSO algorithm, there is little difference between the two approaches for using neural networks for error correction. The multi-step calibration method calibrates the three types of errors, namely systematic error, coarse error, and random error, in different steps using different calibration methods, to achieve the

calibration purpose and higher measurement accuracy. However, due to the complexity of the step-by-step calibration method, it can be applied to the calibration of sensors in measurement environments with high accuracy requirements.

4. Conclusion

Among the four previously mentioned error compensation methods, the multi-step calibration method is superior to the neural network model using the PSO optimization error compensation method is superior to the HHT-ELM error compensation method is superior to the traditional harmonic error compensation method. With the improvement of the compensation effect, the compensation methods become more and more complex.

Systematic errors are suitable for correction using the Fourier approximation method, coarse errors are suitable for rejection using the two-way mean square difference threshold method, and neural networks can handle nonlinear mapping relations and can be used to compensate nonlinear errors, which can be used to correct random errors, thus improving the control accuracy and robustness of the system.

Time-grid angular displacement sensors are involved in high-precision positioning and attitude measurement in robotics, aerospace, and automotive manufacturing. Current research directions include designing higher-resolution time grids and optimizing the optical design of sensor systems and signal processing algorithms to improve the accuracy of positioning and attitude measurements. In addition to this, there are various aspects such as fast acquisition and processing, multimodal measurement, and intelligent applications. With the continuous development and progress of related technologies, the application prospect of time-grid angular displacement sensors in various fields will become broader. The error compensation technology of time-grid angular displacement sensors has important application value in robotics, aerospace, manufacturing, automotive manufacturing, and other fields, which can provide high precision, high stability, and high-reliability measurement and control for industrial applications.

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