Attitude control of quadrotor UAV based on fuzzy PID control under small disturbance

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Abstract. This paper proposes a attitude control method based on fuzzy control. Firstly, the basic model of quadrotor UAV is constructed, then the fuzzy PID controller based on fuzzy control strategy is introduced, and finally the simulation comparison experiments under PID control and fuzzy PID control are conducted. The experimental results show that the parameters of each link in the PID control method are easy to set, but the overshoot amount is large and the adjustment time is long; The simulation model of fuzzy PID control is relatively complex, and it is difficult to reach the predetermined expectation and difficult to accurately control. However, the output of overshoot, short regulation time and the waveform peak are small. In terms of anti-interference, fuzzy PID is more adaptive than PID control.

Keywords: Four-rotor UAV, PID Control, Fuzzy Control, Small Perturbations.

1. Introduction

The application of quadrotor UAV is more and more widely used, and has gradually become a research hotspot [1-3]. The core control technology of UAV is attitude control, and now the PID control algorithm is widely used. The algorithm scales, integrates and differential input deviation, and uses the superposition results to control the actuator, thus realizing the subtle control [4] that cannot be felt by human senses.

However, the attitude of UAV controlled by classic PID is greatly affected by external interference, and the fixed parameters are not easy to adjust. To overcome these shortcomings, we introduce the method [5] of fuzzy PID control. PID control and fuzzy control [6] are two control modes with distinct features and some connections. We combined the two methods. Our basic idea is to introduce fuzzy controller on the basis of classical PID control, so that the two can cooperate with each other and give full play to the regulation experience of fuzzy rule library to regulate the non-linear system online, so that the quadrotor UAV has a better control effect under the condition of different input variables.

2. The basic model of the quadrotor UAV

2.1. Basic model framework

The objective of creating a control model for a quadrotor aircraft is to examine how external forces and torques affect its position and attitude. The model is comprised of two parts: a kinetic model and a kinematic model. Figure 1 illustrates the process of constructing the model.

Figure 1. Control model of the quadrotor aircraft
In this paper, two coordinate systems will be used, namely, inertial coordinate system (ground coordinate system) and non-inertial coordinate system (body coordinate system) will be used. Both coordinate systems follow the right-handed rule. The coordinate system was established as shown in Figure 2.

![Figure 2. The coordinate system of the model](image)

Through derivation, we can obtain a rotation matrix of two coordinate systems, as shown in formula 1.

\[
R_b^e = \begin{bmatrix}
\cos \theta \cos \psi & \cos \psi \sin \theta \sin \varphi - \sin \psi \cos \varphi & \cos \psi \sin \theta \cos \varphi + \sin \psi \sin \varphi \\
\cos \theta \sin \psi & \sin \psi \sin \theta \sin \varphi + \cos \psi \cos \varphi & \sin \psi \sin \theta \cos \varphi - \cos \psi \sin \varphi \\
-\sin \theta & \sin \varphi \cos \theta & \cos \varphi \cos \theta
\end{bmatrix}
\]  

(1)

2.2. Dynamics model of the quadrotor UAV

The input of the kinetic model is external force and external moment, and the output is velocity and angular velocity. Suppose the vehicle is subject to gravity and propeller pull.

2.2.1 Position dynamics model

According to Newton's Second Law:

\[
\dot{v}^e = g^e + R_b^e \frac{T}{m}
\]  

(2)

Expand into a matrix form, where \(g\) is the gravitational acceleration and \(T\) is the total lift generated by the four propellers:

\[
\begin{bmatrix}
\dot{v}_x \\
\dot{v}_y \\
\dot{v}_z
\end{bmatrix} = \begin{bmatrix}
g & 0 \\
0 & \frac{1}{m}
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
-\tau
\end{bmatrix}
\]  

(3)

2.2.2 Postural dynamics model

Through Euler equation, the gyro moment and total moment of inertia are derived and can be obtained:

\[
\begin{bmatrix}
l_{xx} & l_{yx} & l_{yz} \\
l_{yx} & l_{yy} & l_{yz} \\
l_{yz} & l_{yz} & l_{zz}
\end{bmatrix} \begin{bmatrix}
\dot{p} \\
\dot{q} \\
\dot{r}
\end{bmatrix} + \begin{bmatrix}
p & q & r \\
q & i & j \\
r & j & k
\end{bmatrix} = \begin{bmatrix}
l_p(\omega_1 - \omega_2 + \omega_3 - \omega_4) + \tau_x \\
l_p(-\omega_1 + \omega_2 - \omega_3 + \omega_4) + \tau_y \\
l_p(\omega_1 + \omega_2 + \omega_3 + \omega_4) + \tau_z
\end{bmatrix}
\]  

(4)
\( \tau \) represents the propeller's torque on the body axis, which includes rolling torque \( (\tau_x) \) around the \( O_bX_b \) axis, pitch moment \( (\tau_y) \) around the \( O_bY_b \) axis, and yaw moment \( (\tau_z) \) around the \( O_bZ_b \) axis. The three components of \( \omega_b \) in the organism axis are represented by \( p, q, \) and \( r \). Sorted out:

\[
\begin{align*}
\dot{p} &= \frac{1}{I_{xz}} \left[ \tau_x + qr (I_{yy} - I_{zx}) - I_z q (-\omega_1 + \omega_2 - \omega_3 + \omega_4) \right] \\
\dot{q} &= \frac{1}{I_{yy}} \left[ \tau_y + pr (I_{zz} - I_{xy}) + I_z p (-\omega_1 + \omega_2 - \omega_3 + \omega_4) \right] \\
\dot{r} &= \frac{1}{I_{zz}} \left[ \tau_z + pq (I_{xx} - I_{yz}) \right]
\end{align*}
\]

(5)

2.3. Kinematic model of the quadrotor UAV

The inputs of the kinematic model are velocity and angular velocity, and the outputs are position and pose. Equation for the velocity and the position:

\[
\begin{align*}
\dot{p}^e &= v^e \\
p^e &= [x \quad y \quad z]^T \\
[\dot{x} \quad \dot{y} \quad \dot{z}]^T &= [v_x \quad v_y \quad v_z]^T
\end{align*}
\]

(6)

The body's velocity can be expressed as the rate of change of its attitude angle:

\[
\begin{align*}
\dot{\Theta} &= W \cdot \omega_b \\
\dot{\Theta} &= [\phi \quad \dot{\theta} \quad \psi]^T \\
\omega_b &= [p \quad q \quad r]^T \\
W &= \begin{bmatrix} 1 & \tan{\theta}\sin{\phi} & \tan{\theta}\cos{\phi} \\
0 & \cos{\phi} & -\sin{\phi} \\
0 & \frac{\sin{\phi}}{\cos{\theta}} & \frac{\cos{\phi}}{\cos{\theta}} \end{bmatrix}
\end{align*}
\]

(7)

Sorting out available:

\[
\begin{align*}
\begin{bmatrix} \dot{\phi} \\
\dot{\theta} \\
\dot{\psi} \end{bmatrix} &= \begin{bmatrix} 1 & \tan{\theta}\sin{\phi} & \tan{\theta}\cos{\phi} \\
0 & \cos{\phi} & -\sin{\phi} \\
0 & \frac{\sin{\phi}}{\cos{\theta}} & \frac{\cos{\phi}}{\cos{\theta}} \end{bmatrix} \begin{bmatrix} p \\
q \\
r \end{bmatrix}
\end{align*}
\]

(8)

2.4. Control model of the quadrotor UAV

The dynamic models and kinematic models of the quadrotor aircraft are combined. The arrangement is as follows:

\[
\begin{align*}
\dot{p}^e &= v^e \\
\dot{v}^e &= g^e - \frac{1}{m} R_b^e T^e \\
\dot{\Theta} &= W \cdot \omega_b \\
J \dot{\omega}^b &= -\omega^b \times J \omega^b + G_a + \tau
\end{align*}
\]

(9)

3. Design of fuzzy PID controller

3.1. PID control principle

PID control is to form a control deviation based on the given value and the actual output value [7]. PID control law:

\[
u(t) = k_p \left[ e(t) + \frac{1}{T_1} \int_0^t e(t) \, dt + \frac{T_D}{T_1} \frac{de(t)}{dt} \right]
\]

(10)
The parameters in this equation are: $k_p$, which is the scale coefficient, $T_i$, which represents the integral time constant, and $T_D$, which represents the differential time constant.

### 3.2. Fuzzy PID control principle

Compared with the traditional controller, the fuzzy controller replaces the traditional controller $C$ with the fuzzy controller $FC$. The essence of fuzzy PID controller is to establish the relationship between error $e$ and error rate $ec$ as input and $k_p$, $k_i$ and $k_d$ as output, and adjust parameters under different error and error rate online according to fuzzy control principle. The basic structure of fuzzy PID controller is shown in Figure 3 below.

![Figure 3. Structure of fuzzy PID controller](image)

### 3.3. Input and output blurring

This paper constructs a fuzzy controller based on Matlab/Simulink. First, open the fuzzy module, and select the Mamdani type fuzzy inference editor to establish a two-output three-output variable. At the same time, the range of membership function of each variable is taken and the membership function of each variable is established. Let the fuzzy universe of input $e$ and $ec$ be [-6,6], and the fuzzy subset be {NB, NM, NS, NZ, ZO, PS, PM, PB}. Where, NB represents a large negative error, NM represents a large negative error, NS represents a small negative error, ZO represents an error of 0, PS represents a small positive error, PM represents a large positive error, and PB represents a large positive error; The fuzzy domains of output $k_p$, $k_i$ and $k_d$ are [-0.3,0.3], [-0.06,0.06], [-3,3], respectively. The required stable output can be obtained by adjusting the relevant parameters of $e$ and $ec$.

### 3.4. Fuzzy rule establishment

Fuzzy rules are a knowledge base of fuzzy rules based on the theoretical knowledge and practical operation experience of experts and staff. The fuzzy condition language is used to describe the control logic and fuzzy relationship, and then determine the fuzzy relationship between the three output parameters of PID and the two inputs, so that the four-rotor UAV has better control effect under different input variables [8].

According to the PID parameter adjustment method, the fuzzy control rules are established according to the relationship between the parameters and the control effect in the actual control and adjustment process. According to different combinations of input $e$ and $ec$, different PID parameter control rules are corresponding. There are 7 language fuzzy variables for each of the two inputs, and there are 49 fuzzy rules in total. See Table 1, 2 and 3 below:
Table 1. $k_p$ fuzzy rule reasoning table

<table>
<thead>
<tr>
<th>$k_p$</th>
<th>$e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>PB PB PB PM PM PS ZO ZO</td>
</tr>
<tr>
<td>NM</td>
<td>PB PB PM PM PM PS ZO ZO</td>
</tr>
<tr>
<td>NS</td>
<td>PM PM ZO NS NM NB NB</td>
</tr>
<tr>
<td>ZO</td>
<td>PB PB PM ZO NM NB NB</td>
</tr>
<tr>
<td>PS</td>
<td>PM PM ZO NS NM NB NB</td>
</tr>
<tr>
<td>PM</td>
<td>ZO ZO NS NM NM NB NB</td>
</tr>
<tr>
<td>PB</td>
<td>ZO ZO NS NM NM NB NB</td>
</tr>
</tbody>
</table>

Table 2. $k_i$ fuzzy rule reasoning table

<table>
<thead>
<tr>
<th>$k_i$</th>
<th>$e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>NB NB NM NM NS NS ZO ZO</td>
</tr>
<tr>
<td>NM</td>
<td>NB NB NM NM NS NS ZO ZO</td>
</tr>
<tr>
<td>NS</td>
<td>NB NM NS NS ZO PS PS</td>
</tr>
<tr>
<td>ZO</td>
<td>NM NM NS ZO PS PM PM</td>
</tr>
<tr>
<td>PS</td>
<td>NM NS ZO PS PS PM PB</td>
</tr>
<tr>
<td>PM</td>
<td>ZO ZO PS PS PM PB PB</td>
</tr>
<tr>
<td>PB</td>
<td>ZO ZO PS PM PM PB PB</td>
</tr>
</tbody>
</table>

Table 3. $k_d$ fuzzy rule reasoning table

<table>
<thead>
<tr>
<th>$k_d$</th>
<th>$e$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>PS NS NB NB NB NM PS</td>
</tr>
<tr>
<td>NM</td>
<td>PS NS NB NM NM NM ZO</td>
</tr>
<tr>
<td>NS</td>
<td>ZO NS NM NM NS NS ZO</td>
</tr>
<tr>
<td>ZO</td>
<td>ZO NS NS NS NS NS ZO</td>
</tr>
<tr>
<td>PS</td>
<td>ZO ZO ZO ZO ZO ZO ZO</td>
</tr>
<tr>
<td>PM</td>
<td>PB PS PS PS PS PS PB</td>
</tr>
<tr>
<td>PB</td>
<td>PB PM PM PM PS PS PB</td>
</tr>
</tbody>
</table>

3.5. Fuzzy reasoning and output clarity

According to the fuzzy rule table, fuzzy reasoning is carried out on the fuzzy input to obtain the corresponding fuzzy output, and it is clarified to complete the fuzzy control.

According to the fuzzy rule table, the parameters $k_p$, $k_i$ and $k_d$ are dynamically adjusted as follows:

$k_p = k_p' + \Delta k_p$, $k_i = k_i' + \Delta k_i$, $k_d = k_d' + \Delta k_d$.

$k_p'$, $k_i'$, $k_d'$ are the basic value of positive determination using conventional methods. $\Delta k_p, \Delta k_i, \Delta k_d$ are online correction values obtained according to fuzzy rules, then the fuzzy PID parameters $k_p, k_i, k_d$ are the sum of the two.

4. Simulation and result

Enter fuzzy in the main window of Matlab and press Enter to start the fuzzy inference system editor. Set the fuzzy inference system editor and membership function editor according to the requirements of 3.3 above, as shown in Figure 4 below:
Figure 4. Mamdani fuzzy inference editor with two inputs and three outputs

Establish the membership functions of input and output respectively, as shown in Figure 5 and Figure 6:

Figure 5. Membership function of input quantity

Figure 6. Membership function of output
Establish the fuzzy rule combination between variables [9], as shown in Figure 7 below:

![Rule editor](image)

**Figure 7. Rule editor**

The process of fuzzy reasoning can be observed through the observation window of fuzzy rules [10], and the relationship between input variables and output variables of the whole universe can be observed through the observation window of output surface, as shown in Figure 8 and Figure 9 respectively:

![Observation fuzzy reasoning process](image)

**Figure 8. Observation fuzzy reasoning process**

![Output control surface](image)

**Figure 9. Output control surface**

The simulation model is shown in Figure 10:
After the above fuzzy PID controller is established, it is simulated, and the output image is shown in Figure 11 below, among them, blue is the output signal of fuzzy PID control, and red is the output signal of PID control. The simulation is shown in Figure 11, compared with fuzzy PID control, PID control has greater fluctuations, and the system convergence and stability are poor in the later stage; In the later stage of the system, the fuzzy PID appears stable at the 50th control cycle and reaches the desired output position accurately; PID control is stable in the 200th control cycle, while the system output still has a certain deviation, and the system is in a small oscillation [11].

5. Conclusion

In this paper, an attitude control algorithm of four-rotor UAV based on fuzzy PID is proposed. Through the simulation and comparison of PID control and fuzzy PID control, the following conclusions are drawn:

(1) The parameter setting in PID control method is relatively simple, that is, the parameters of each link are easy to set, but the overshoot is large and the adjustment time is long.
(2) Compared with PID control, the simulation model of fuzzy PID control is more complex, and
the adjustment of various parameters is not easy to meet the predetermined expectations, and it is
difficult to control accurately. However, compared with PID control, the output overshoot is small,
the adjustment time is short, and the waveform peak is small. When the parameters of the controlled
object change, the fuzzy PID control is better than the traditional PID control.

(3) PID control and fuzzy PID control are applicable to different occasions respectively. When the
control system has a clear mathematical model and the parameters of the controlled object are not
easy to change, PID control is suitable for application; Fuzzy PID control is suitable for systems that
are difficult to be accurately described mathematically and the parameters of the controlled object are
often changed.

To sum up, PID has simple structure, good robustness and high reliability; Fuzzy PID is the
combination of fuzzy control and traditional PID control, which combines their control advantages
and provides sufficient conditions for improving control accuracy. At present, the four-rotor UAV
has higher and higher requirements for stability, accuracy and anti-interference ability. The traditional
PID cannot meet the control requirements. The fuzzy PID control formed by combining fuzzy control
and PID control has greater research value and practical significance.

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