

Research on azimuth-only passive positioning of UAV formation: A case study of circular formation

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Abstract. In order to maintain the formation, the position of Drones cluster can be adjusted by the method of azimuth-only passive positioning. In this paper, the circular formation is taken as an example. Firstly, when the coordinate information of Drones at the center of the circle and around the circle is known, the coordinates of other Drones are solved according to the Angle signal. By using polar coordinate programming, the Angle between different Drone lines is calculated, and using this information as known information, the nonlinear equations are solved by inverse derivation of their own coordinates. Secondly, in order to determine that at least several Drones need to transmit signals to realize effective positioning of all Drones in the circular formation, this paper concludes that only three Drones at most are needed to realize azimuth-only passive positioning based on the established model.

Keywords: Drones, Azimuth-only passive positioning, Polar coordinates, Geometric model.

1. Introduction

With the improvement of scientific and technological level, UAV has been widely used in industry, military, earthquake relief and other fields. With the unprecedented improvement of the research, design and manufacturing level of UAV, it is playing an increasingly important role in the field of passive positioning [1].

Passive detection technology can receive the signal of radiation source target, find hidden signal parameters, so as to determine the location of the target [2]. Therefore, in order to maintain the formation, the position of UAV cluster can be adjusted by the method of azimuth-only passive positioning. Through passive positioning, the position can be determined by passively receiving the electromagnetic signal of the environment [1].

According to Figure 1 of the circular UAV formation and related information, when the UAV located in the center of the circle and the other two UAVs in the formation are used to transmit signals, and the other UAVs with slightly different positions passively receive signals, how to solve the coordinates of other UAVs according to the Angle signals? Based on the transmitting signals of UAV numbered FY00 and FY01, how many other UAVs are needed to realize the effective positioning of UAVs with azimuth-only passive positioning?

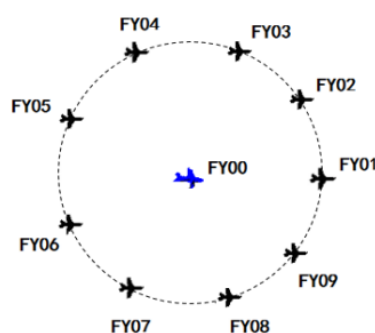


Fig 1. Schematic diagram of circular UAV formation

In order to solve the above problems, Fang Kun proposed the UAV positioning signal separation algorithm based on support vector machine and the UAV positioning error weight compensation algorithm based on random forest [1]. In addition, Xin Zhang et al. used the isolated forest algorithm to assign cooperative targets of UAV formation [5]. What's more, Wu Yunfan et al. proposed a joint location method of arrival Angle location and arrival time difference, derived a passive location algorithm, and provided a quantitative standard for location accuracy. They established a location model for passive receiving UAV [6]. Some scholars have proposed that a weighted least squares localization algorithm based on Angle information can be realized to solve the location problem of single target in multi-base passive radar [7]. In this paper, a geometric model is established to solve the above problems in polar coordinates.

2. Model building and solving

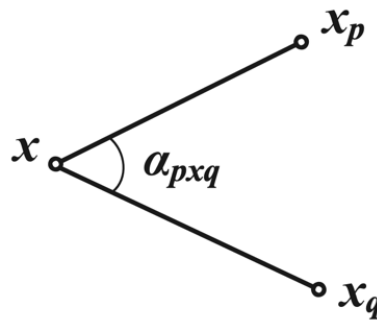


Fig 2. Geometric relationship between known points and unknown points

Based on the principle of trigonometric function, the passive positioning problem is derived, and the geometric model of UAV formation positioning is established [8]. Given any two points $x_p: (x_p, y_p)$, $x_q: (x_q, y_q)$ on the plane, there is a moving point $x: (x, y)$ that does not coincide with them, the position relation of the above three points is shown in Figure 2. In this case, the Angle α_{pxq} used to determine the azimuth information of the unknown moving point satisfies:

$$\cos(\alpha_{pxq}) = \frac{\langle x_p - x, x_q - x \rangle}{|x_p - x| |x_q - x|} \quad (1)$$

Converting the rectangular coordinate form of the above three points into polar coordinate form, the following relation is obtained:

$$\begin{cases} (x_p, y_p) = (r_p \cos \theta_p, r_p \sin \theta_p) \\ (x_q, y_q) = (r_q \cos \theta_q, r_q \sin \theta_q) \\ (x, y) = (r \cos \theta, r \sin \theta) \end{cases} \quad (2)$$

At this time will be (2) into (1) type, you can get information to determine the azimuth Angle α_{pxq} about r, θ function is as follows:

$$\begin{aligned} \alpha_{pxq} &= A_{pq}(r, \theta; r_p, r_q, \theta_p, \theta_q) \\ &= \arccos \left[\frac{-r_p r \cos(\theta_p - \theta) - r_q r \cos(\theta_q - \theta) + r_p r_q \cos(\theta_p - \theta_q) + r^2}{\sqrt{r_p^2 + r^2 - 2r_p r \cos(\theta_p - \theta)} \sqrt{r_q^2 + r^2 - 2r_q r \cos(\theta_q - \theta)}} \right] \end{aligned} \quad (3)$$

According to (3) type and the practical significance of the various physical quantities, easy to prove that: in $\mathbb{R}^2 \setminus \{(r_p, \theta_p), (r_q, \theta_q), (r_0, \theta_0)\}$, is as follows:

$$\sqrt{r_p^2 + r^2 - 2r_p r \cos(\theta_p - \theta)} > 0 \quad (4)$$

And

$$\sqrt{r_q^2 + r^2 - 2r_q r \cos(\theta_q - \theta)} > 0 \quad (5)$$

This is always true. The proof is as follows:

Since point $x_p: (r_p, \theta_p)$ and point $x: (r, \theta)$ are two points that do not coincide, there exists:

$$\cos(\theta_p - \theta) < 1 \quad (6)$$

$$r_q \neq r \quad (7)$$

It's always true. By combining Formula (6) with Formula (7), we can obtain:

$$\sqrt{r_q^2 + r^2 - 2r_q r \cos(\theta_q - \theta)} > \sqrt{(r_q - r)^2} > 0 \quad (8)$$

Similarly, we can prove that equation (5) is constant under this condition.

And from Cauchy's inequality we get

$$|\langle x_p - x, x_q - x \rangle| \leq |x_p - x| |x_q - x| \quad (9)$$

So the function $A_{pq}(r, \theta; r_p, r_q, \theta_p, \theta_q)$ is defined in the R^2 plane except for the point $x_p: (r_p, \theta_p), x_q: (r_q, \theta_q), x_0: (r_0, \theta_0)$.

Aiming at the three-point positioning problem of circular formation, the positioning model of UAV receiving signal is established based on the rigorous mathematical geometry solution triangle calculation, and the results have high accuracy [8].

3. Research on location of unknown UAV

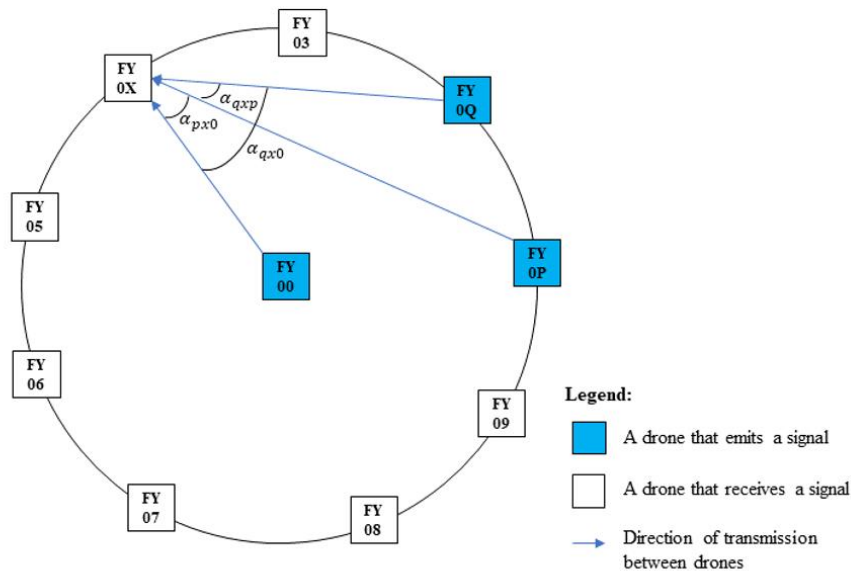


Fig 3. Schematic diagram of position relationship between transmitting signal UAV and receiving signal UAV

As shown in Figure 3, the coordinates of the location of the UAV numbered $FY00$ are selected as the origin coordinates, and then there is a formula $r_0 = 0$. According to the conditions that the position of the UAV used for transmitting signals is unbiased and the number is known, it is assumed that the UAV $FY0P$ and $FY0Q$ numbered p, q are accurately used for transmitting signals. Thus obtain:

$$\theta_p = \frac{(p-1)2\pi}{9} \quad (10)$$

$$\theta_q = \frac{(q-1)2\pi}{9} \quad (11)$$

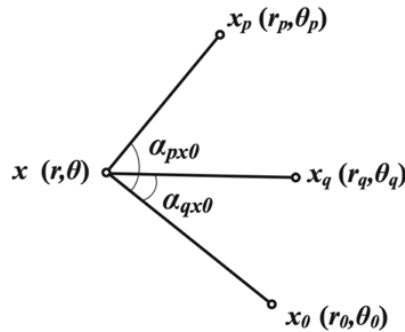


Fig 4. Simplified position diagram of transmitting signal UAV and receiving signal UAV

Since there is no deviation in the position of the UAV used for transmitting signals, its radius is also determined, which can be obtained as follows:

$$r_p = r_q = R \quad (12)$$

According to the polar coordinate points in Figure 4, equations (10), (11) and (12) are substituted into equations (3) to obtain the two Angle values:

$$\begin{cases} \alpha_{px0} = A_{p0}(r, \theta; r_p, \theta_p) = \arccos \left[\frac{-r_p r \cos(\theta_p - \theta) + r}{\sqrt{r_p^2 + r^2 - 2r_p r \cos(\theta_p - \theta)}} \right] \\ \alpha_{qx0} = A_{q0}(r, \theta; r_q, \theta_q) = \arccos \left[\frac{-r_q r \cos(\theta_q - \theta) + r}{\sqrt{r_q^2 + r^2 - 2r_q r \cos(\theta_q - \theta)}} \right] \end{cases} \quad (13)$$

The azimuth information $\alpha_{px0}, \alpha_{qx0}$ is known as the signal received by the UAV, so equation (13) above can be denoted as $(\alpha_{px0}, \alpha_{qx0}) = A_1(r, \theta)$.

At this point, it is easy to verify that the following conditions are satisfied in local region $G \subset \mathbb{R}^2 \setminus \{(r_p, \theta_p), (r_q, \theta_q), (r_0, \theta_0)\}$:

$$\begin{cases} f \in C^1(G; \mathbb{R}^2) \\ \text{When } (r', \theta') \in G, (\alpha'_{px0}, \alpha'_{qx0}) = A_1(r', \theta') \\ \bigcup_{A_1}(r, \theta) \text{ is reversible} \end{cases} \quad (14)$$

By formula (14), and according to the inverse function theorem is function can be determined $B_1: (\alpha_{px0}, \alpha_{qx0}) \mapsto (r, \theta)$. Thus, the polar coordinates of unknown numbered UAV $FY0P$ and $FY0Q$ can be determined, namely, the passive receiving signal UAV positioning model. The numerical calculation and solution of this model will be discussed later.

According to the mechanism analysis of the first question, the numbered information is the target Angle adjusted by UAV. It can also make the UAV get the position of the signal source aircraft against the reference circle so as to locate.

4. Solve the minimum number of transmitting signal UAVs

As can be seen from the model in question 1, three points are needed to realize positioning when the numbering is known. Therefore, question 2 can be transformed as follows: In addition to $FY00$

and FY01, how many signals from unknown numbered UAVs can be received to determine the number of one of them?

The triangulation in passive location is studied, including multi-station azimuth only triangulation and multi-station azimuth only least square method [9]. The UAVs numbered *FY00* and *FY01* used for transmitting signals are known. The coordinate position of *FY00* is selected as the origin, and then $r_0 = 0, r_1 = R$. According to the model of equation (3), the following equation can be obtained.

$$\alpha_{1x0} = A_{10}(r, \theta; R, \theta_1) = \arccos \left[\frac{-R \cos(\theta_1 - \theta) + r}{\sqrt{R^2 + r^2 - 2Rr \cos(\theta_1 - \theta)}} \right] \quad (15)$$

The position of the UAV transmitting signal with known number and unknown number is accurate. Suppose its number is p , then $r_p = R$ and the following equation is also satisfied.

$$\begin{cases} \alpha_{1x0} = A_{10}(r, \theta; R, \theta_1) = \arccos \left[\frac{-R \cos(\theta_1 - \theta) + r}{\sqrt{R^2 + r^2 - 2Rr \cos(\theta_1 - \theta)}} \right] \\ \alpha_{px0} = A_{p0}(r, \theta, \theta_p; R) = \arccos \left[\frac{-R \cos(\theta_p - \theta) + r}{\sqrt{R^2 + r^2 - 2Rr \cos(\theta_p - \theta)}} \right] \\ \alpha_{px1} = A_{p1}(r, \theta, \theta_p; R, \theta_1) = \arccos \left[\frac{-R \cos(\theta_p - \theta) - R \cos(\theta_1 - \theta) + R \cos(\theta_p - \theta_1) + r^2}{\sqrt{R^2 + r^2 - 2Rr \cos(\theta_p - \theta)} \sqrt{R^2 + r^2 - 2Rr \cos(\theta_1 - \theta)}} \right] \end{cases} \quad (16)$$

In order to solve the nonlinear equations in the model of problem 2, Newton iteration method and exhaustion method are used in this paper. This algorithm combines the advantages of simple and easy implementation of least square method and high convergence accuracy of Newton method.[10].

We might as well write the formula (16) as $(\alpha_{1x0}, \alpha_{px0}, \alpha_{px1}) = A_2(r, \theta, \theta_p)$, and as we did in the first question, we can know that there is a function $B_2: (\alpha_{1x0}, \alpha_{px0}, \alpha_{px1}) \mapsto (r, \theta, \theta_p)$, from which we can determine the coordinates of *FY0X*. Therefore, as long as one more UAV transmits signals, the effective positioning of UAV can be realized.

5. Conclusions

This paper mainly studies the problem of using effective signals to reverse coordinate of UAV in plane, and optimizing the transmission times and quantity of signals between UAV.

First of all, in view of the information of UAV at the center of the circle and the two points around the circle in problem 1 of this paper, the coordinate problem is solved according to the Angle signal, from which the general formula of any three points on the plane and the Angle between the lines are abstracted. On this basis, the model for solving the problem is established to further explore the relationship between the coordinates of the digging points and their geometric forms. By using polar coordinate programming, the Angle between different UAV lines is calculated, and using this information as known information, the nonlinear equations are solved by inverse derivation of their own coordinates. Secondly, regarding the adjustment scheme of UAV swarm, we believe that it is essentially a graph theory problem, that is, to transmit the least signals so that each UAV can make effective adjustment through the received signals, which involves the inflow and outflow of signals, whether the signals are repeated and so on. Based on the model we established, we found that on the basis of the background of the second question, adding another UAV can realize the effective positioning of UAV.

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