

# Exploration of the composition of ancient glass artifacts based on optimization algorithms

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**Abstract.** The Silk Road facilitated exchanges between East and West, where glass played a significant role. However, ancient glass artifacts are susceptible to weathering, leading to changes in their chemical composition. This article explores the relationship between weathering and explanatory variables of glass type, color, and decoration. It presents a method for predicting the weathering degree of glass artifacts and their chemical composition content before weathering. Descriptive statistics are used to calculate the weathering degree and variation of each chemical component content, allowing for quantification of the impact of weathering on the proportion of chemical components. Logistic regression is used to obtain regression coefficients of the weathering degree and other attributes. Various measures, including correlation coefficients and significance tests, are calculated to assess the predictive power of the explanatory variables. The proposed method is valid for predicting the chemical composition proportion data before weathering, which aids further research in the archaeological field. Overall, this article provides a comprehensive approach to understanding the effect of weathering on ancient glass artifacts and predicting their chemical composition content before weathering.

**Keywords:** logistic regression, ancient glass artifacts, variability and weathering degree.

## 1. Introduction

This article examines the effect of weathering on ancient glass artifacts, which played a significant role in the economic and cultural exchanges along the Silk Road<sup>[1]</sup>. Due to storage limitations and technological means, ancient glass artifacts are susceptible to weathering, which can change their chemical composition<sup>[2]</sup>. The aim of this study is to explore the relationship between weathering and the explanatory variables of glass type, color, and decoration. Additionally, the study investigates the use of chemical composition analysis to predict the approximate composition of glass before weathering, aiding further research in the field of archaeology. The article presents a method for predicting the weathering degree of glass artifacts and their chemical composition content before weathering<sup>[2]</sup>. Firstly, a line chart is used to illustrate the relationship between the weathering degree of glass artifacts and their glass type, decoration, and color. Logistic regression is then performed using Stata software to quantitatively obtain the regression coefficients of the weathering degree and other attributes. To assess the predictive power of the explanatory variables, various measures such as correlation coefficient, phi and Cramers V, Lambda coefficient, and the uncertainty coefficient are calculated<sup>[3]</sup>. Difference significance tests are also conducted to test the significance of the predictive power and verify the validity of the proposed method<sup>[4]</sup>. To predict the chemical composition content before weathering, the data is first preprocessed to obtain a more detailed classification. Descriptive statistics are then used to calculate the weathering degree and variation of each chemical component content, allowing for the quantification of the impact of weathering on the proportion of chemical components. Finally, the chemical composition proportion data at each severely weathered point is restored. This method provides a valuable tool for archaeologists in predicting the chemical composition content of glass artifacts before weathering and understanding the impact of weathering on their composition.

## 2. Establishment of the model for the impact of weathering on glass properties

### 2.1. Data profiling

Based on the big dataset provided by archaeological work, this paper conducts data analysis and prediction, assuming that these data are sufficiently reliable and accurate. Due to the large amount of data used in this paper and the strong correlation between the data, it is beneficial to conduct data profiling in advance for data analysis<sup>[5]</sup>.

#### 2.1.1 Data preprocessing

Previous studies have shown that missing value processing is an important step in analyzing color data from archaeological artifacts. In addition, it has been recognized that the sum of component proportions in some data may not be 100%, and data falling within a range of 85% to 105% have been considered valid. Therefore, data preprocessing is crucial for subsequent modeling and analysis.

#### 2.1.2 Inferring the color of missing color glass based on the probability

The absence of color in glass relics can result from inadequate measuring tools or techniques, as well as poor preservation of the relics. These factors can cause rapid oxidation, discoloration, deliquescence, and detachment when exposed to air. It is important to conduct reasonable analysis rather than simply assuming that the missing color is transparent or colorless, taking into account the actual situation. In one study, a heat map filling method was used to fill in the missing data and reasonably infer the missing relic colors based on the characteristics of similar relics. Another study developed a non-destructive method for identifying and restoring missing colors on ancient Chinese glass artifacts<sup>[6]</sup>. This approach involved using a combination of Raman spectroscopy and digital image processing techniques to identify the chemical composition and spectral features of the glass. Then, the missing colors were filled in based on the known coloration mechanisms of ancient glass.

#### 2.1.3 Remove invalid data.

If the proportion of the  $i$ th component in the  $j$ th sampling point of the  $n$ th cultural relic is represented by  $\alpha_{ij}$ , then it should satisfy:

$$85\% \leq \sum_{i=1}^{i=n} \alpha_{ij} \leq 105\% \text{ (where } i = 1, 2 \dots 70, j = 1, 2 \dots 15 \text{)} \quad (1)$$

## 2.2. Correlation analysis

### 2.2.1 Analyze the relationship between surface weathering of cultural relics and their glass type, decoration, and color based on the logistic model

(1) Analyze and explain the explanatory variables and the response variable.

For the issue of whether glass relics are weathered or not, it can be divided into two situations: weathered and non-weathered. We define a random variable  $W$ , which belongs to a two-point distribution (Bernoulli distribution), where 1 represents weathered glass relics, and 0 represents non-weathered glass relics. The specific results are shown in Table 1 below.

**Table 1.** Explanatory and explanatory variables

<b>Events</b>	<b>1</b>	<b>0</b>
<b>Probability</b>	$P$	$(1 - P)$

So in the case of a given explanatory variable  $\bar{X}$ , consider the two-point distribution probability of the explanatory variable  $W$ .

$$\begin{cases} P(W=1|\bar{X}) = F(\bar{X}, \bar{\beta}) \\ P(W=0|\bar{X}) = 1 - F(\bar{X}, \bar{\beta}) \end{cases} \text{ Normally } F(\bar{X}, \bar{\beta}) = F(\bar{X}', \bar{\beta}) \quad (2)$$

$F(\bar{X}, \bar{\beta})$  is called the connecting function, where all three explanatory variables for the nominal variables of the tattoo, type, color (in order  $x_{i1}, x_{i2}, x_{i3}$ ) specific corresponding to take the value of the relationship with the dummy variable please refer to the appendix, with and the explained variable  $W$  connected, for this reason, we want to only need to ensure that  $F(\bar{X}, \bar{\beta})$  is a function defined in  $[0, 1]$ , it can ensure that  $0 \leq \widehat{W} \leq 1$  Therefore, the connecting function is taken to have the following laws [6].

$$\begin{cases} P(W=1|\bar{X}) = F(\bar{X}, \bar{\beta}) \\ P(W=0|\bar{X}) = 1 - F(\bar{X}, \bar{\beta}) \end{cases} \quad (3)$$

When we take  $F(\bar{X}, \bar{\beta})$  as a sigmoid function, then we find the *logistic* function.

$$F(\bar{X}, \bar{\beta}) = S(\bar{X}' \vec{\beta}) = \frac{\exp(\bar{X}' \vec{\beta})}{1 + \exp(\bar{X}' \vec{\beta})} \quad (4)$$

### (2) Solving logistic functions

Since it is a nonlinear model, it is estimated using the maximum likelihood estimation method (*MLE*).[5]

$$\begin{cases} P(W_i=1|\bar{X}_i) = S(\bar{X}_i, \bar{\beta}_i) \\ P(W_i=0|\bar{X}_i) = 1 - S(\bar{X}_i, \bar{\beta}_i) \end{cases} \Rightarrow f(W_i | \bar{X}_i, \bar{\beta}_i) = \begin{cases} S(\bar{X}_i, \bar{\beta}_i), W_i=1 \\ 1 - S(\bar{X}_i, \bar{\beta}_i), W_i=0 \end{cases} \quad (5)$$

The logarithm of this sample is then taken to find the log-likelihood function:

$$\ln L(\beta_i | W_i, \bar{X}_i) = \sum_{i=1}^n W_i \ln [S(\bar{X}_i, \bar{\beta}_i)] + \sum_{i=1}^n (1 - W_i) \ln [1 - S(\bar{X}_i, \bar{\beta}_i)] \quad (6)$$

The gradient descent method is then used to solve for:

$$F(\bar{X}, \bar{\beta}) = S(\bar{X}' \vec{\beta}) = \frac{\exp(\bar{X}' \vec{\beta})}{1 + \exp(\bar{X}' \vec{\beta})} \quad (7)$$

(3) Validation of the rationality of using logistic methods[11] to analyze relationships. Based on the following equation:

$$E(W_i | \bar{X}_i) = 1 \times P(W_i = 1 | \bar{X}_i) + 0 \times P(W_i = 0 | \bar{X}_i) = P(W_i = 1 | \bar{X}_i) \quad (8)$$

So in the case of a given explanatory variable  $\bar{X}$ , considering the probability of the two-point distribution of the explanatory variable  $W$ , this  $\widehat{W}_i$  can be interpreted as the probability that  $W_i = 1$  occurs, i.e., the probability that the first group of weathered artifacts is weathered or not. Therefore, it can be listed as.

$$\widehat{W}_i = P(W_i = 1 | \bar{X}_i) = S(\bar{X}'_i \vec{\widehat{\beta}}) = \frac{\exp(\bar{X}'_i \vec{\widehat{\beta}})}{1 + \exp(\bar{X}'_i \vec{\widehat{\beta}})} = \frac{e^{\widehat{\beta}_0 + \widehat{\beta}_1 x_{i1} + \widehat{\beta}_2 x_{i2} + \dots + \widehat{\beta}_k x_{ik}}}{1 + e^{\widehat{\beta}_0 + \widehat{\beta}_1 x_{i1} + \widehat{\beta}_2 x_{i2} + \dots + \widehat{\beta}_k x_{ik}}} \quad (9)$$

Which when we predict out  $\widehat{W}_i \geq 0.5$ , it is considered that its prediction for  $\widehat{W}_i = 1$ , that is, in the actual archaeology, we through the naked eye to judge the place of the decoration, color and type that can be initially judged that the glass artifacts appear weathering; otherwise when  $\widehat{W}_i < 0.5$  is considered that its prediction of  $\widehat{W}_i = 0$ .

### 2.2.2 The relationship between surface weathering of glass artifacts and their intrinsic properties is explored based on a control variable chi-square method

Glass type, decoration, color, etc. are the own properties of the artifacts, and each property has an impact on the weathering of the artifacts [7], so we use the idea of controlling variables, and then use the method of column coefficients to select the impact of these three inherent properties on the weathering of the artifacts one by one.

(1) Introduction to the chi-square test:

The logic diagram of the chi-square test is shown in Figure 1 below:

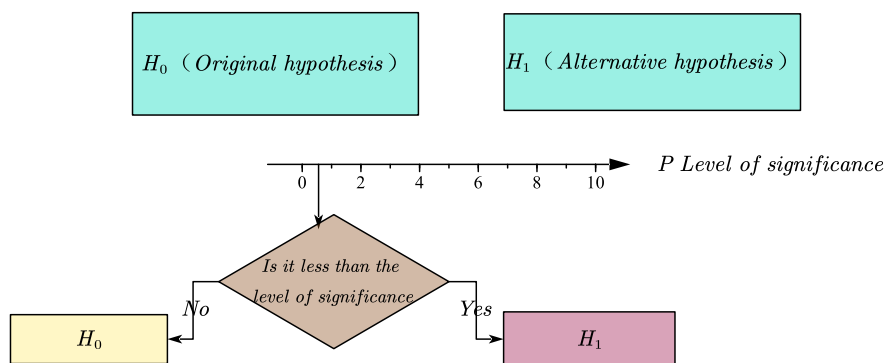


Figure 1. chi-square test logic diagram

(2) A study of glass artifact types:

$H_{01}$  : When controlling for the same two other variables, the effect of a change in the color of the glass artifact on the weathering of the glass artifact is assumed.

(3) Studies on the decoration of glass artifacts:[8]

$H_{02}$  : The change in color of the glass artifact has a significant effect on the weathering of the glass artifact when controlling for two other variables that are the same.

(4) A study of the color of glass artifacts:

$H_{03}$  : The change in color of the glass artifact has a significant effect on the weathering of the glass artifact when controlling for two other variables that are the same.

### 2.2.3 Prediction of pre-weathering chemical composition content based on variability and weathering degree

According to the category, we can record high potassium weathering and high potassium unweathered as group  $P_{decency}$ , where  $P_{ij}$  vector represents the record vector of 14 chemical components in the case of weathering state j at sample point i of high potassium group, j is taken as 1, 2. 1 means unweathered, 2 means weathered. The lead-barium weathering and lead-barium unweathered are recorded as group  $Lb_{decency}$ . where  $LP_{ij}$  vector represents the recorded vector of 14 chemical components in the weathering condition j at sampling point i of the lead-barium group. From the above matrix scatter plot and box plot of the proportion of each chemical composition of the two glasses, it can be seen that the distribution of each chemical composition at different i

sampling points shows a more concentrated distribution, so we choose the mean value of the proportion of chemical composition as the study object, and predict the content of composition  $k$  at the  $i$ -th sampling point before weathering ( $j = 2 \rightarrow j = 1$ ) by defining the weathering degree.

### 3. Results and analysis

#### 3.1. Inferring the color of the missing color glass based on the probability size

Through the observation data and calculation, it can be seen that the maximum total number of species should be 84, and the actual number of classification in the text is 21, among which the groups with missing color are the weathering pattern A lead-barium group, which is recorded as type  $B_1$ , and the weathering pattern C lead-barium group is recorded as type  $B_2$ . The specific results are shown in Table 2 below.

**Table 2.** Percentage of different types of glass according to color classification

	Number of categories	Ornamentation	Type	Color	Surface weathering
Maximum number of categories	84	3	2	8	2
Actual number of categories	21	3	2	8	2

According to the principle of hot card filling, we find the object that is most similar to the type to which the missing value data belongs among all the data, and then fill it with the known data with the highest probability among them. The types and percentages obtained through Excel pre-processing are shown in Table 3 below.

**Table 3.** Percentage of type  $B_1$  vs.  $B_2$

POC		Color							
Surface weathering	Ornamentation	Type	Black	Blue-Green	Pale Blue	light green	Dark Green	Purple	Total
Weathering									
	A								
		Lead Barium	22.22%	11.11%	66.7%	0.00%	0.00%	0.00%	100.00%
	C								
		Lead Barium	0.00%	13.33%	40.00%	6.67%	26.67%	13.33%	100.00%

By the above proportional relationship, we can deduce that the color of the glass numbered 19, 48 is light green, and the glass numbered 40, 58 should be (light blue and dark green). The results are shown in Table 4 below.

**Table 4.** Inference of glass color based on probability percentages

Counting items: Object number			Color		Inferring colors
Surface weathering	Ornamentation	Type	(blank)	Total	
Weathering					
	A				
		Lead Barium	2	2	2 Light green (19, 48)
	C				
		Lead Barium	2	2	1 light blue + 1 dark green
Total			4	4	

### 3.2. The relationship between the surface weathering of artifacts and their glass type, decoration and color is analyzed based on logistic models

(The explanatory parameters of the logistic model for the weathering relationship on the surface of the artifacts were obtained as  $x_{i1}, x_{i2}, x_{i3}$ , nominal variables denoted in turn as ornament, type, and color take the following values:

$$\widehat{W}_i = \frac{e^{0.2700566 + 0.1418352 \times x_{i1} - 1.314018 \times x_{i2} + 0.0644001 \times x_{i3}}}{1 + e^{0.2700566 + 0.1418352 \times x_{i1} - 1.314018 \times x_{i2} + 0.0644001 \times x_{i3}}} \quad (10)$$

In above equation,  $x_{i1}, x_{i2}, x_{i3}$ , ( $x_{i1} = 1, 2, 3$   $x_{i2} = 1, 2$ ,  $x_{i3} = 1, 2, 3 \dots 8$ ) variables are expressed as the nominal variables of ornament, type and color in order.

In the upper section,  $x_{i1}$  is appointed to numbers 1, 2, or 3, which are used to represent different patterns.  $x_{i2}$  can be appointed to numbers 1 and 2 which can be used to indicate whether the pattern is weathered or not. According to the RGB color model, the  $x_{i3}$  which can be replaced by integers from 1 to 8 can be represented by black, teal, green, light blue, light green, dark blue, dark green, and purple. The accuracy of the test results is  $Pr(+|D) = 82.35\%$ , and the predictive performance of the test is good.

### 3.3. The relationship between surface weathering of glass artifacts and their intrinsic properties is explored based on a control variable chi-square method

The column association coefficients, including Phi, Cramers V, Lambda, and uncertainty coefficient, were calculated using SPSS and are presented in the Table 5 below:

**Table 5.** Various coefficients for different analysis terms

Analysis items	Number of column links	Phi	Uncertainty factor
Ornamentation	0.285	0.297	0.77
Type	0.288	0.301	0.07
Color	0.29	0.303	0.5

From the table, it can be seen that the uncertainty coefficient of patterns and colors is much greater than 0.05, and only the uncertainty coefficient of type is relatively close to 0.05. However, due to the small amount of data, we believe that it is only for reference.

## 4. Conclusion

After incorporating the references, the updated paragraph with numbered citations is as follows: The glass system in China consists mainly of the PbO-SiO<sub>2</sub> and PbO-BaO-SiO<sub>2</sub> systems, with glass products becoming more durable with the introduction of sodium-calcium formulas via the Silk Road<sup>[9]</sup>. However, weathering of glass is affected by factors such as temperature, humidity, and storage methods, making it difficult to study the attributes of ancient Chinese glass products. This paper explores the effects of weathering by establishing correlation and regression models and proposes methods for predicting glass attributes before weathering. The innovative concepts of weathering degree, variation degree, and correlation degree are introduced to improve the model's rationality and objectivity<sup>[10]</sup>, and practical analyses help understand archaeological work's rigor. The methods used have good expansibility and research value and can be applied to situations requiring voting or studying the impact of factors on diseases. The logistic regression model will have broad applications in healthcare, production capacity, and education, helping more people effectively prevent diseases and develop effective drugs.

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