Study on the Particle Precipitation Phenomenon Based on the Boycott Effect

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Abstract. There is a well-known Boycott effect in particle precipitation. In order to further study the Boycott effect, this passage first analyzed the force on the particle, drew a diagram of the liquid flow consistent with the actual situation, and analyzed its settlement process from the perspective of dynamics. Based on the PNK theoretical model, this passage derived the proportional relation function between the average settling velocity v and the vertical settling velocity v_0, and obtain the relationship between the settling velocity and several variables. These variables include particle concentration range, particle radius, solvent viscosity coefficient, initial solvent volume concentration, container tilt angle, etc. According to this formula, passage designed related experiments to analyze the conditions of settlement strengthening effect, so as to further clarify the applicable conditions of settlement strengthening effect. It can be seen from the passage's work that the experimental data is in good agreement with the theoretical value, which proves that this passage's theory is very valuable, and this work also provides a certain reference value for industrial sewage settlement and river sand treatment.

Keywords: PNK, Boycott, Particle Precipitation, Reinforcing Effect.

1. Introduction

Nature never ceases to amaze us with its countless phenomena, and one that has recently captivated people's attention is the peculiar behavior of particles in liquid when the container they are in is tilted. Upon observing this, one would be surprised to find that the settling rate of particles in the container is significantly faster. This extraordinary phenomenon is attributed to the differing convection conditions between particles and liquids, and it has sparked the interest of numerous scholars who have undertaken extensive research on the matter.

The historical origins of this phenomenon can be traced back to 1920 when Boycott first observed cell sedimentation in test tubes filled with blood. He noticed that the percentage of cell-free liquid present in tilted tubes was greater than in vertical ones[1]. Eight years later, in 1928, Lundgren pointed out that the liquid displaced by settling particles could flow downward through the inclined plane and bypass the falling particles to form seepage flow, thus accelerating the particle settling rate[2]. On the other hand, in 1949, Kinosita observed convection during sedimentation under inclined tubes and inclined boundaries, and he observed that some particles moved at 100 times their settling velocity[3].

Further research on this effect was conducted by W.D. ill et al in 1976. They studied particle settlement in a container with a specific shape, a conical container, and found that the particle settlement rate may be several times faster than that in a vertical test tube with the same height. This enhancement is caused by natural convection[4]. With the development of computer technology, Z. J. Xu and E. E. Michaelides et al. conducted lattice Boltzmann simulations on the process of particle deposition in 2005[5].

In 2018, S. Palma et al. began using this effect to predict under what conditions a pipeline would be blocked. They found that the initial particle volume concentration was a crucial factor[6]. The following year, V. Braranets and N. Kizilova proposed that there is an optimal angle when the tube is tilted at a small angle, which makes the particle settlement the fastest. If the tilted angle exceeds this optimal angle, the particle settlement rate will be weakened[7]. However, it is worth noting that the optimal angle of settlement is still a problem to be solved.

Although the Boycott effect has been studied extensively from many aspects, there are still many shortcomings, mainly reflected in the fact that most of the models are macroscopic and cannot accurately describe the motion state and concentration distribution of particles in the suspension. In light of this, our study seeks to delve further into this issue based on previous research.

Firstly, we analyzed the force of the particles, drew a diagram of the liquid flow consistent with the actual situation, and analyzed its settlement process from the perspective of dynamics. Based on the Ponder, Nakamura and Kuroda (PNK) Theoretical Model, we derived the proportional relation function between the average settling velocity v and the vertical settling velocity v_0, and obtain the relationship between the settling velocity and several variables. These variables include particle concentration range, particle radius, solvent viscosity coefficient, initial solvent volume concentration, container tilt angle, etc. According to the theory we derived, we designed related experiments to analyze the conditions of settlement strengthening effect, so as to further clarify the applicable conditions of settlement strengthening effect.

Through the above work, we can also analyze the application prospect of particle accelerated settlement effect in industrial sewage settlement and river sand treatment, which is helpful to design a practical sewage purification model.

2. Theoretical Analysis

2.1. Model Construction

We use the method of constructing a fluid mechanics model system to describe the settling state of particles in a container, and use Stokes law to calculate the settling rate of particles in a homogeneous liquid at a certain angle. Table 1 shows the notations we used in our analysis. Based on the sedimentation process photos, we simplified it to the physical model shown in the Fig. 1 below, where the symbol meanings are described in the variable description below. The test tube is divided into three parts. The part with the green arrow is the low-density clarified liquid formed after particle settlement, the middle is the suspension mixed with particles and liquid, and the bottom is the precipitation area. A-B-C is the area of settlement enhancement, where the effect of settlement enhancement occurs.

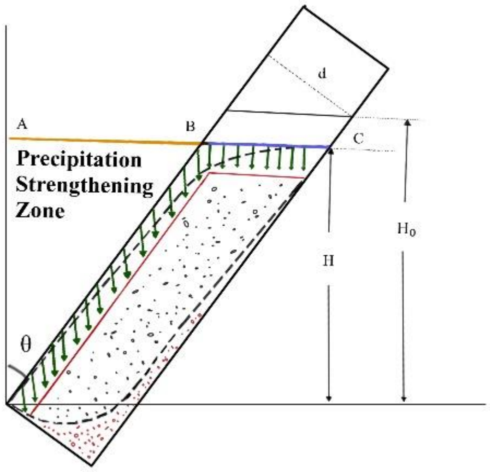

Figure 1. Settlement System Model [2]

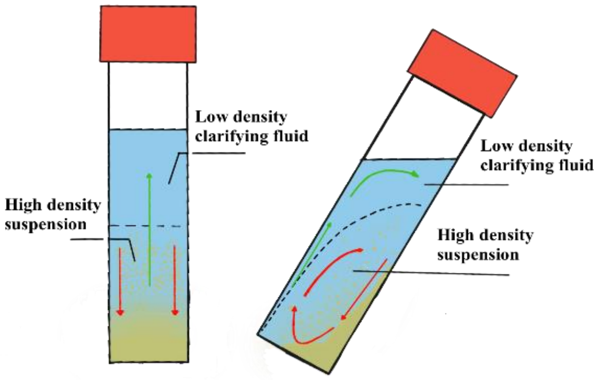

Figure 2. Comparison Diagram of Vertical and Inclined Settling Fluid Flow

2.2. Hydromechanical Analysis

For vertical fluid, as shown in the flow state in the Fig. 1 above, the green arrow in the figure is the low-density clarifying liquid without obvious particles formed after particle settlement; on the contrary, the part with more particles is the high-density suspension. Due to the relative motion of the particles and liquid, the clarified liquid with low density separates from the turbidity and moves upward, thus forming the upwelling. At the same time, the dense suspension moves downward under the force of gravity(see Fig. 2). However, the upwelling and the settling of particles interfere with each other, thus slowing down the downward movement of particles.

Table 1. Notation

Symbol	Physical Meaning
v	Inclined settlement rate of particle population
v_0	Vertical settlement rate of particle population
u_0	Vertical sedimentation rate of a single particle
H	Liquid level
H_0	Initial liquid level
d	Container diameter
θ	Container tilt angle (vertical angle)
μ	Liquid viscosity coefficient
r	Particle radius
C_0	Initial volume concentration of particles
ρ_1	Liquid density
ρ_2	Particle density
β	Settlement strengthening coefficient
k	Container height to diameter ratio
γ	Particle shape correction coefficient (approximately 0.7)
n	Constant related to Reynolds number (approximately 4.65) [8]
Re	Reynolds number

For inclined fluid, particles will be settled on the tilted side of the container due to vertical gravity, and a low-density clarified liquid region will be generated on the other side. The low-density clarified liquid will gather upward due to pressure difference, and the shunt of high and low density flow slows down the resistance of particle settlement.

According to literature[9], the potential energy lost by particles is transformed into convective kinetic energy, which is transformed into internal energy through the viscous dissipation of the flow around the convection and settling particles. Therefore, the influence of particle rotation and expansion tension is ignored in the equilibrium.

2.3. Particle Stress Analysis

We analyze the force of particles in liquid and establish a simplified fluid mechanics system model.

The small particles are idealized into a spherical shape with regular shape. In this settling process, the force analysis of the particle group is carried out. Due to the large number of particle groups, the collision between particles and the friction between particles are reduced due to the fact that the particles are filled with liquid, and this movement phenomenon is random. Therefore, for particles with low concentration, the interaction between particles has little influence, so the collision and friction between particles are temporarily excluded.

As the suspended particles in the container sink, they create small areas of clarified liquid on the upper sloping wall. The upward flow in this area will flow to the liquid area of the opening due to the restriction of the inclined wall. Meanwhile, the particles will fill the clear liquid area of the inclined wall in time, and thus the particles at the opening will be sunk. In the Fig. 3 below, the area with dense lattice is the suspension mixed with liquid and particles, while the area with sparse lattice is the low-density clarifying liquid. The green head in the test tube represents the downward movement of the particles in the first part to form clarifying liquid.

Because the liquid flow is continuous, it is not easy to analyze. Therefore, we divide particle descent into two processes. The first process is vertical deposition of particles, as shown in process a in Fig. 3. The second process is for particles to fill the void left by vertical settlement, as shown in process b in Fig. 3.

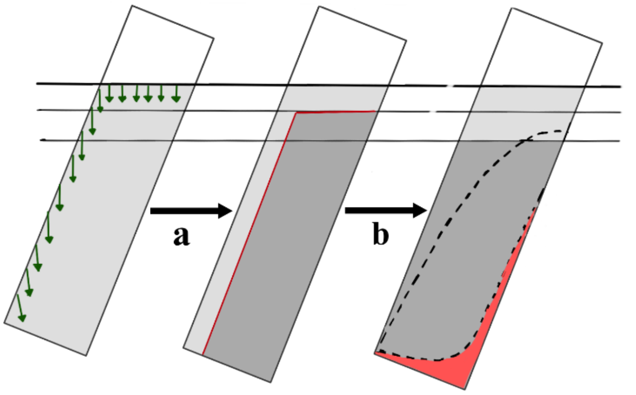

Figure 3. Two Sub-Processes of Settlement
Process a: Particles Settle Freely
Process b: Particles Fill the Clear Liquid Area

2.3.1 Vertical Settling Process of Particles

We first discuss the vertical deposition process of particles, as shown in process a. The settling particles are subject to gravity $G = \rho_2 \frac{4}{3}\pi r^3 g$, buoyancy $F = -\rho_1 \frac{4}{3}\pi r^3 g$, and viscous resistance $f = -6\,\pi\mu r u$, therefore we can get its dynamic equation as follows

$$\rho_2 \frac{4}{3}\pi r^3 g - \rho_1 \frac{4}{3}\pi r^3 g - 6\pi\mu r u = \rho_2 \frac{4}{3}\pi r^3 a \tag{1}$$

$$a = \frac{\rho_2 - \rho_1}{\rho_2} g - \frac{9}{2}\frac{\mu}{\rho_2 r^2} u \tag{2}$$

When $a = 0$, the speed of the particles reached the maximum, we can get a single particle vertical subsidence rate u_0

$$\frac{4}{3}\pi r^3(\rho_2 - \rho_1)g - 6\pi\mu r u_0 = 0 \tag{3}$$

$$u_0 = \frac{2(\rho_2 - \rho_1)gr^2}{9\mu} \tag{4}$$

During the movement before reaching the maximum velocity u_0, we know that

$$\frac{du}{dt} = \frac{\rho_2 - \rho_1}{\rho_2} g - \frac{9}{2}\frac{\mu}{\rho_2 r^2} u \tag{5}$$

The solution is

$$\ln\left(\frac{\rho_2 - \rho_1}{\rho_2} g - \frac{9}{2}\frac{\mu}{\rho_2 r^2} u\right) = -\frac{9\mu}{2\rho_2 r^2} t + c \tag{6}$$

In the initial conditions: $t = 0,\ u = 0$, assuming that starting from the static settling

$$c = \ln \frac{\rho_2 - \rho_1}{\rho_2} g \tag{7}$$

$$u_t = \frac{2(\rho_2 - \rho_1)gr^2}{9\mu}\left(1 - e^{-\frac{9\mu}{2\rho_2 r^2}t}\right) = u_0\left(1 - e^{-\frac{9\mu}{2\rho_2 r^2}t}\right) \tag{8}$$

Then, we discuss the Eq. 4 and Eq. 8, calculate the time t takes when u_t is equal to u_0

$$e^{-\frac{9\mu}{2\rho_2 r^2}t} = 0 \tag{9}$$

When $t = 1.01 * 10^{-3} s$, $e^{-\frac{9\mu}{2\rho_2 r^2}t} = 0.01$, u_t tend to be u_0. That is, the acceleration time is very short and negligible.

2.3.2 Particles Fill the Void of Vertical Settlement

Next, we will discuss the second process of settlement, where particles fill the void left by vertical settlement, as shown in process b in Fig. 3.

Due to liquid flow and other reasons, the whole particle system will always ensure that there is no vacancy in the particle system at the inclined wall surface, so the particle system will fill the vacancy on the inclined wall surface, and the final settlement is only reflected on the surface of the horizontal plane. From this we will be able to calculate the true rate of decline.

Let the velocity of vertical settlement be v_0; $S(t)$ is the area rate of decline of the clarifying liquid (sedimentation effect). According to PNK theory, it can be obtained:

$$v = \frac{dH}{dt} = -v_0\left(1 + \frac{H}{d}\sin\theta\right) \tag{10}$$

By deforming and integrating the Eq. 10, we can get

$$\int \frac{1}{v_0\left(1 + \frac{H}{d}\sin\theta\right)} dH = \int -d\,t \tag{11}$$

When θ is not equal to zero

$$\frac{d}{v_0 \sin\theta} \ln v_0 \left(1 + \frac{H}{d}\sin\theta\right) = -\mathrm{t} + \mathrm{C} \tag{12}$$

When $t = 0$, set the initial height of H_0, constant C can be calculated as follows

$$C = \frac{d}{v_0 \sin\theta} \ln\left[v_0\left(1 + \frac{H_0}{d}\sin\theta\right)\right] \tag{13}$$

Therefore, the Eq. 13 can be rewritten as

$$\ln\left[v_0\left(1+\frac{H}{d}\sin\theta\right)\right]=-\frac{v_0\sin\theta}{d}t+\ln\left[v_0\left(1+\frac{H_0}{d}\sin\theta\right)\right] \tag{14}$$

When $H=0$, the settlement is completed, and the time taken is as follows

$$t=\frac{d}{v_0\sin\theta}\ln\left(\frac{H_0\sin\theta}{d}+1\right) \tag{15}$$

The average velocity of settlement is

$$v=\frac{\frac{H_0}{d}\sin\theta}{\ln\left(\frac{H_0\sin\theta}{d}+1\right)}v_0 \tag{16}$$

2.3.3 Settlement Time of Particle Group in Inclined Vessel

According to literature [10], we can know that the free sedimentation rate v_0 of particle group is

$$v_0=u_0(1-C_0)^n \tag{17}$$

Substitute Eq. 4 into Eq. 17

$$v_0=\frac{2(\rho_2-\rho_1)gr^2}{9\mu}(1-C_0)^n \tag{18}$$

The time for the vertical container to settle under the same conditions is

$$H=v_0t' \tag{19}$$

And $H\cos\theta=H_0$, $\frac{t}{t'}=\beta$ is defined as settlement strengthening coefficient, then

$$\beta=\frac{\ln(k\sin\theta+1)}{k\sin\theta\cos\theta} \tag{20}$$

In the Eq. 20, $k=\frac{H_0}{d}$.

The smaller the settlement strengthening coefficient β is, the shorter the clarification time of the mixture is, the better the effect of accelerated settlement is, and the faster the particles sink under the same conditions. Meanwhile, in the previous analysis, the particle is idealized into a sphere, but the actual calcium carbonate particle is not spherical and has a certain sharp angle, so the particle shape correction coefficient should be considered. Here, the correction coefficient is determined to be 0.7[8], then we have $\bar{v}=\gamma v$.

In summary, the true average rate of inclined sedimentation of particle groups can be written as

$$\bar{v}=\gamma\cdot\frac{H_0\sin\theta}{dln\left(\frac{H_0\sin\theta}{d}+1\right)}\cdot\frac{2(\rho_2-\rho_1)}{9\mu}\cdot gr^2\cdot(1-C_0)^n \tag{21}$$

The settling time of particle groups in a tilted container with high H_0 satisfies the following equation:

$$t=\frac{d\cdot\ln\left(\frac{H_0\sin\theta}{d}+1\right)\cdot 9\mu}{\gamma\cdot\sin\theta\cdot 2(\rho_2-\rho_1)\cdot gr^2\cdot(1-C_0)^n} \tag{22}$$

3. Experiment

3.1. Preparation and Procedure

In order to explore the conditions affecting the settlement enhancement effect and seek the optimal conditions for precipitation, experiments were designed to verify the correctness of the above theoretical analysis, and the influences of liquid viscosity coefficient, initial solvent volume concentration, particle radius and container tilt angle on the settlement enhancement effect were analyzed.

According to the requirements of the experiment, we used the following experimental equipment:

(1) Round bottom test tubes of different diameters;
(2) Calcium carbonate particles of different diameters;
(3) 8 mesh, 16 mesh, 26 mesh, 40 mesh, 70 mesh screen;
(4) Syringe, Timer, Runner, Copper Wire, Rubber Sleeve, Measuring Cylinder.

Calcium carbonate particles were used to simulate insoluble impurities such as sediment in sewage, and a mixture of water and glycerol was used as a liquid to carry the particles. When we need to detect the container tilt angle, we use the built-in protractor in Tracker software to measure.

Our specific experimental steps are as follows:

The limestone particles were smashed with a hammer, and then screened with a certain number of mesh samples to control the radius of the particles within a relatively small range. Particles in different radius ranges were grouped, as shown in the Table 2 below.

Table 2. Relationship between mesh and diameter

Group	**Mesh**	**Diameter (mm)**
1	8-16	3.000-1.250
2	16-26	1.250-0.710
3	26-40	0.710-0.450
4	40-70	0.450-0.224

We used a measuring cylinder to prepare a mixture of glycerin and water, and thoroughly stirred it, and the viscosity coefficient of the solvent was measured by falling ball method and recorded.

A mixture of glycerin and water is mixed with particles to form a suspension.

In order to make the particles and liquid fully and evenly mixed, and to ensure the same mixing degree of particles and liquid in each experiment to the maximum extent, we bound the test tube on the runner for 10 turns.

According to the four conditions affecting the particle settlement time, divided into 4 groups of experimental groups.

Record the video in slow motion, adjust the video picture into color negative with Adobe Photoshop, and record the particle settling time with a timer.

Record the data, do five experiments for each group, and take the average value of the experimental data for analysis.

3.2. Description of experimental conditions

Unless otherwise specified, all the following experiments were performed under the following experimental conditions in Table 3.

Table 3. Experimental conditions

Temperature (℃)	20
Tilt angle (°)	40
Limestone granules (mm)	1.000
Total solvent volume (mL)	50
Initial particle volume concentration (%)	3.6
Solvent viscosity coefficient (Pas)	137.15

The initial particle volume concentration is 5g of limestone particle concentration, which is about 3.6%. In a mixture of glycerol and water, the viscosity of 85% glycerol by volume is 137.15 Pas (20℃).

In order to study the influence of solvent viscosity coefficient, tilt angle, particle radius and initial particle volume concentration on sedimentation effect, four groups were set according to the above variables. At the same time, five experiments were conducted in each group to record the corresponding sedimentation time, and the theoretical value obtained by using the Eq. 21 was compared.

4. Result

For different solution viscosity coefficients, the experiment was divided into three groups (except for different liquid viscosity coefficients, other irrelevant variables were consistent with the Table 3). The viscosity coefficients of the three groups were 14.79Pas, 66.81Pas and 137.15Pas, respectively, and the corresponding glycerol-water mixed liquid volume ratios were 3:2, 4:1 and 17:3. The experimental results are shown in Fig. 4. For different tilt angles, we divided the experiment into five groups. The tilt angles of the five groups are 30°, 40°, 50°, 60° and 70° respectively, and the experimental results are shown in Fig. 5. According to different particle radius, we divided the experiment into five groups. The particle diameter of these five groups is 0.32mm, 0.6mm, 1mm and 4mm respectively. The experimental results are shown in Fig. 6. For different initial particle volume concentrations, we divided the experiment into four groups. We use the weight of the particle to calculate the volume of the particle, and therefore the volume concentration. The initial particle weights of the four groups are respectively 2.5g, 5.0g, 7.5g and 10.0g. The experimental results are shown in Fig. 7. Where t_s is the theoretical settlement time, $\bar{t}$ is the average settling time of the five experiments, δ is the relative error between $\bar{t}$ and t_s, U represents the uncertainty.

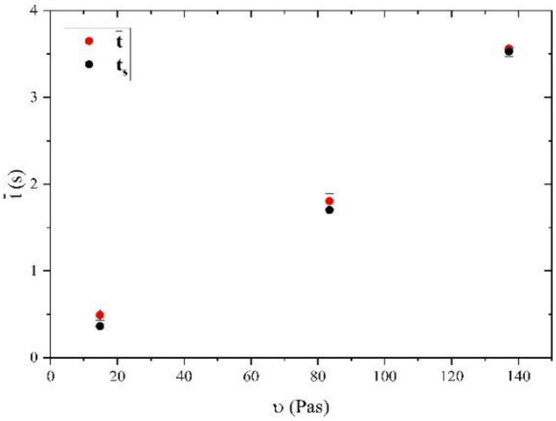

Figure 4. Influence of Different Viscosity Coefficients on Settling Time

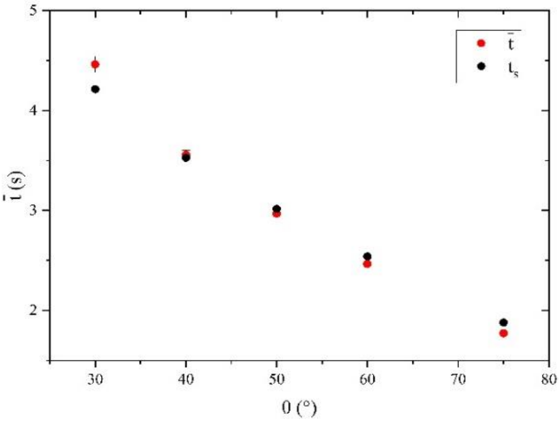

Figure 5. Influence of Different Tilt Angles on Settling Time

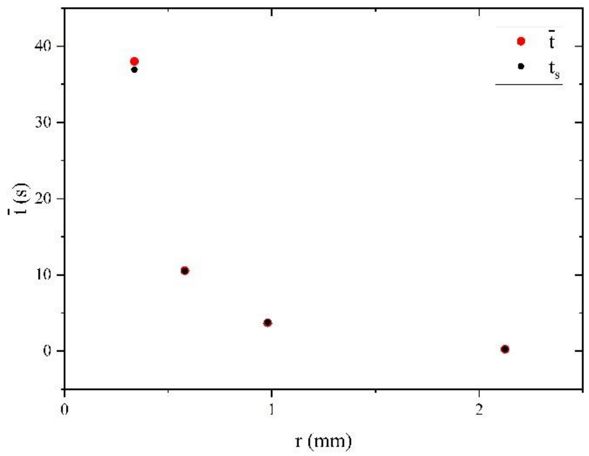

Figure 6. Influence of Particle Diameter on Settling Time

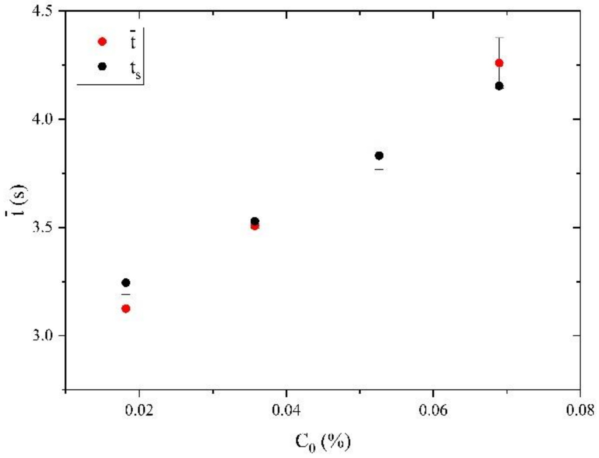

Figure 7. Influence of Initial Particle Volume Concentrations on Settling Time

5. Conclusions

This passage first conducted a force analysis on the particles and analyze their settlement process from the perspective of dynamics. Based on the PNK theory model, the proportional relation function between the average settling velocity v and the vertical settling velocity v_0 is derived. This passage got the relationship between settling velocity and multiple variables. According to Eq. 21, conditions of settlement strengthening effect were analyzed, such as particle concentration range, particle radius, solvent viscosity coefficient, initial solvent volume concentration, container tilt angle, etc., in order to further clarify the applicable conditions of settlement strengthening effect. From the above work, you can see that the experimental data is in good agreement with the theoretical values, which proves that this passage's theory is very valuable. Through the above work on particle settlement, this passage has made certain contributions to industrial sewage settlement and river sand treatment, and further laid a solid foundation for designing a practical sewage purification model.

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