Degradation of lignocellulose by fungi based on differential equation

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Abstract. In this paper, we studied the decomposition of plant materials and lignocellulosic fibers by fungi, and the relationship between some characteristics of fungi. And established several mathematical models to solve the problems raised in the problem. Firstly, in the case of the coexistence of different fungi, ignoring the interaction between them, we use differential equation and Monod equation to establish the model. In the model, the decomposition rate of each fungus to plant materials and lignocellulosic fibers at a different time and the mycelial elongation rate of the fungus itself can be obtained. In the model solving, the ode instruction is used to solve the differential equations, and the variable step Runge Kutta Feldberg method is used to improve the accuracy. Then we considered the interactions between different fungi: mutual promotion and mutual inhibition. Different fungi have different moisture tolerance, so they have different decomposition rates of plant materials and wood fibers. By using logistic law, our model can describe the decomposition ability of different interactions between fungi to decompose lignocellulosic fibers. After solving the model, it is found that the fungi that promote each other are the fastest, the fungi that do not affect each other are the second, and the fungi that inhibit each other are the slowest, that is, the decomposition rate is the smallest.

Keywords: Chemostat model, differential equation, Monod model, ode45, logistic model.

1. Introduction

The carbon cycle describes the exchange of carbon throughout the Earth's geochemical cycle and is an important component of life on Earth. Part of the carbon cycle involves the decomposition of compounds, allowing the carbon to be renewed and used in other forms. A key component of this process is the fungal breakdown of plant material and wood fibers. The essence of wood degradation is the degradation of cellulose and lignin in wood fibers. So some of the connections between the shapes of fungi and the processes by which fungi decompose plant materials and wood fibers have been studied.

First, we established a model of fungi decomposing wood fiber, and used differential equation and morrow equation to describe the concentration change of nutrients and the elongation change of mycelium during the process of fungi decomposing wood fiber. Through the calculation, in the form of the image, the nutrient reduction and fungal growth in the process of fungal decomposition of wood fiber were analyzed.

Then we consider the interaction between fungi, which is divided into two parts: mutual promotion and mutual inhibition. When each fungus propagates alone, the number of each fungus follows the logistic law. When they promote each other, the growth rate model of each fungus adds the promoting effect of other fungi to it; on the contrary, when they inhibit each other, the growth of each fungus is blocked, and the growth rate model decreases correspondingly; when they do not affect each other, the growth of each fungus does not increase or decrease.

2. Model Establishment and Solutions

2.1. Model 1

A model can be established to describe the decomposition of lignocellulosic fiber by fungi and the growth of mycelium during the process of lignocellulosic fiber. Describe the change of microbial concentration (i.e. the growth of fungi) and the change of nutrient concentration (i.e. the decomposition rate of lignocellulose by fungi).

2.1.1 Model Establishment

Firstly, fungi decompose lignocellulosic fibers through their activities and obtain energy for their own growth. Fungi grow through the extension of hyphae. The faster the hyphae extend, the higher the growth rate, the stronger the decomposition ability and the higher the decomposition rate. Based on the chemostat, we studied the decomposition of lignocellulosic fiber by fungi and established the model by using the differential equation[1~4] and Monod[1~4] equation.

In the case of the coexistence of different fungi, the relationship of mutual promotion and mutual inhibition between them is not considered. At this time, considering that different fungi do not influence each other, the decomposition rate of wood fiber by fungi is as follows:

$$x'(t)=D\left(x^0-x\right)-\frac{m_1x}{k_1y_1+x}\times\frac{y_1}{\delta_1}-\frac{m_2x}{k_2y_2+x}\times\frac{y_2}{\delta_2}-\cdots\frac{m_nx}{k_ny_n+x}\times\frac{y_n}{\delta_n} \tag{1}$$

D is the outflow rate of the chemostat. The decomposition rate of lignocellulosic fiber is obtained by subtracting the initial concentration of nutrients and nutrients consumed by fungi. The decomposition rate model is obtained by the differential equation.

The growth rate of fungus I was as follows：

$$y_1'(t)=r_1y_1\left[\frac{m_1x}{k_1y_1+x}-D\right] \tag{2}$$

According to the Monod equation, the growth model of fungi was obtained by synthesizing the influence of temperature, humidity and other factors.

The growth rate of fungus 2 was as follows：

$$y_2'(t)=r_2y_2\left[\frac{m_2x}{k_2y_2+x}-D\right] \tag{3}$$

The growth model of fungi was obtained by the same principle.

The growth rate of fungi n was as follows：

$$y_n'(t)=r_ny_n\left[\frac{m_nx}{k_ny_n+x}-D\right] \tag{4}$$

The growth model of fungi n was also obtained.

Among them, r1, r2 and r3 are the growth coefficients of fungi I, II and III respectively, and are the growth coefficients of the comprehensive effects of temperature, temperature width, humidity and humidity width on the growth of fungi. The elongation of mycelium, can be described by using the morrow equation, but the growth of fungi is affected by the change of environment. Therefore, the growth of fungi can be more accurately described by adding a growth factor reflecting the change of environment based on the morrow equation.

The model for the first problem is as follows:

$$\begin{cases} x'(t) = D\left(x^0 - x\right) - \frac{m_1 x}{k_1 y_1 + x} \times \frac{y_1}{\delta_1} - \frac{m_2 x}{k_2 y_2 + x} \times \frac{y_2}{\delta_2} - \cdots \frac{m_n x}{k_n y_n + x} \times \frac{y_n}{\delta_n} \\ y_1'(t) = r_1 y_1 \left[\frac{m_1 x}{k_1 y_1 + x} - D \right] \\ y_2'(t) = r_2 y_2 \left[\frac{m_2 x}{k_2 y_2 + x} - D \right] \\ \vdots \\ y_n'(t) = r_n y_n \left[\frac{m_n x}{k_n y_n + x} - D \right] \end{cases} \tag{5}$$

$$\begin{cases} r_1 = h_1 T_1 + h_2 TD_1 + h_3 M_1 + h_4 MD_1 + rank_1 \\ r_2 = c_1 T_2 + c_2 TD_2 + c_3 M_2 + c_4 MD_2 + rank_2 \\ \vdots \\ r_n = i_1 T_n + i_2 TD_n + i_3 M_n + i_4 MD_n + rank_n \end{cases} \tag{6}$$

2.1.2 Model Solving

In model solving, we solve the model according to the collected data.

We calculated the coexistence of two kinds of fungi and three kinds of fungi

Step 1:

r1 is solved: h_1=0.1,h_2=0.2,h_3=0.1,h_4=0.2,$rank_1$=0.8,T_1=22℃,TD_1=5,M_1=0.8,MD_1=6

$$r_1 = \left(0.1 \times 22 + 0.2 \times 5 + 0.1 \times 0.8 + 0.2 \times 6 + 0.8\right) \times 0.04 \tag{7}$$

r2 is solved: c_1=0.1,c_2=0.2,c_3=0.1,c_4=0.2,$rank_2$=0.4,T_2=16℃,TD_2=2.5,M_2=0.4,MD_2=3

$$r_2 = \left(0.1 \times 16 + 0.2 \times 2.5 + 0.1 \times 0.4 + 0.2 \times 3 + 0.4\right) \times 0.04 \tag{8}$$

In the calculation, R1 and R2 are adjusted and multiplied by a coefficient of 0.04

Step 2:

x^0=1, m_1=0.8, m_2=0.6, k_1=k_2=0.03, δ_1=200

$$\begin{cases} x'(t) = 0.2(1 - x) - \frac{0.8x}{0.03y_1 + x} \times \frac{y_1}{200} - \frac{0.6x}{0.03y_2 + x} \times \frac{y_2}{200} \\ y_1'(t) = r_1 y_1 \left[\frac{0.8x}{0.03y_1 + x} - 0.2 \right] \\ y_2'(t) = r_2 y_2 \left[\frac{0.6x}{0.03y_2 + x} - 0.2 \right] \end{cases} \tag{9}$$

Algorithm: ode45 method is used to solve the differential equation with MATLAB. Here we take 360 days and 10 days as a step. The vector represents the initial value vector, that is, the initial value of nutrients and fungi. It uses variable step fourth and fifth-order Runge Kutta Feldberg method.

$$fun = @\left(t, x\right) \begin{bmatrix} D\left(x^0 - x_1\right) - \frac{m_1 x}{k_1 y_1 + x} \times \frac{y_1}{\delta_1} - \frac{m_2 x}{k_2 y_2 + x} \times \frac{y_2}{\delta_2}; \\ r_1 y_1 \left[\frac{m_1 x}{k_1 y_1 + x} - D \right]; \\ r_2 y_2 \left[\frac{m_2 x}{k_2 y_2 + x} - D \right]; \end{bmatrix} \tag{10}$$

$$[tout, xout] = ode45(fun, [0:10:360], [1; 0.1; 0.1]) \quad (11)$$

The initial values of nutrients, fungi I and fungi II were 1, 0.1 and 0.1 respectively.

2.1.3 Analysis and Evaluation of results

According to the calculation results, as shown in the figure

Case: two kinds of fungi

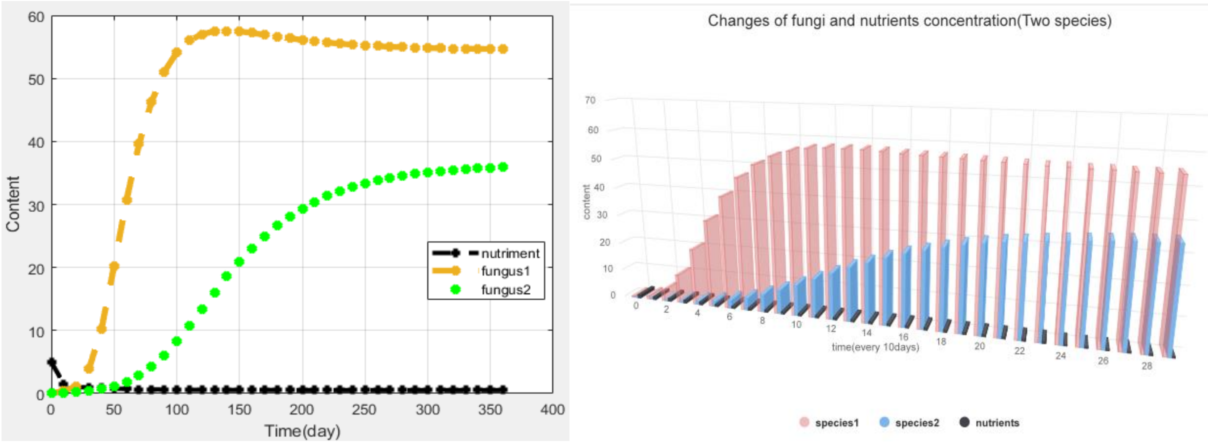

Fig. 1 Changes of nutrients and fungi concentration and quantity in the presence of two kinds of fungi

Some calculation results are listed:

Table 1. Decomposition of two fungi

day	Nutrient decomposition rate	The growth rate of fungi	The growth rate of fungi
10	0.994	0.319	0.164
20	0.984	1.003	0.271
30	0.958	2.978	0.444
…	…	…	…
360	0.545	54.645	35.931

According to the figures and tables, the growth rate of fungus I increased first and then gradually stabilized, the decomposition rate of wood fiber also increased first and then gradually stabilized, the growth rate of fungus II increased first and then gradually stabilized, the decomposition rate of wood fiber also increased first and then gradually stabilized. But the growth rate of fungus II was less than that of fungus I, and the water tolerance of fungus I was higher than that of fungus I. Therefore, the decomposition rate of the lignocellulosic fiber of fungus II was higher than that of fungus II.

2.2. Model 2

In the model, different fungi have different growth rates and water tolerance, so the interaction between them will affect the decomposition of wood fiber.

2.2.1 Model Establishment

Based on model one, we add the influence of different fungal interactions. Fungal interactions can be divided into two situations: first, they promote each other's growth; second, they inhibit each other's growth

Case 1: mutual promotion:

Fungi decompose wood fibers together, and different fungi belong to different populations. Taking two kinds of fungi as an example, the change of the number of fungi I and II followed the logistic [5] law. Fungus one promotes the growth of fungus two, and fungus two promotes the growth of fungus one.

The model is as follows:

$$\begin{cases} x'(t) = D\left(x^0 - x\right) - \frac{m_1 x}{k_1 y_1 + x} \times \frac{y_1}{\delta_1} - \frac{m_2 x}{k_2 y_2 + x} \times \frac{y_2}{\delta_2} - \cdots \frac{m_n x}{k_n y_n + x} \times \frac{y_n}{\delta_n} \\ y_1'(t) = r_1 y_1 \left[\frac{m_1 x}{k_1 y_1 + x} - D \right] + \frac{m_2 x}{k_2 y_2 + x} \times \frac{y_2}{\delta_2} \\ y_2'(t) = r_2 y_2 \left[\frac{m_2 x}{k_2 y_2 + x} - D \right] + \frac{m_1 x}{k_1 y_1 + x} \times \frac{y_1}{\delta_1} \\ \vdots \\ y_n'(t) = r_n y_n \left[\frac{m_n x}{k_n y_n + x} - D \right] + \frac{m_{n-1} x}{k_{n-1} y_{n-1} + x} \times \frac{y_{n-1}}{\delta_{n-1}} + \cdots \frac{m_1 x}{k_1 y_1 + x} \times \frac{y_1}{\delta_1} \end{cases} \tag{12}$$

$$\begin{cases} r_1 = h_1 T_1 + h_2 TD_1 + h_3 M_1 + h_4 MD_1 + rank_1 \\ r_2 = c_1 T_2 + c_2 TD_2 + c_3 M_2 + c_4 MD_2 + rank_2 \\ \vdots \\ r_n = i_1 T_n + i_2 TD_n + i_3 M_n + i_4 MD_n + rank_n \end{cases} \tag{13}$$

Case 2: mutual inhibition

There are two species of fungi, fungi one and fungi two. When fungus one and fungus two coexist, the inhibition of fungus two on the growth of fungus one is directly proportional to the number of fungus two, and fungus one has the same effect on fungus two.

The model is as follows:

$$\begin{cases} x'(t) = D\left(x^0 - x\right) - \frac{m_1 x}{k_1 y_1 + x} \times \frac{y_1}{\delta_1} - \frac{m_2 x}{k_2 y_2 + x} \times \frac{y_2}{\delta_2} - \cdots \frac{m_n x}{k_n y_n + x} \times \frac{y_n}{\delta_n} \\ y_1'(t) = r_1 y_1 \left[\frac{m_1 x}{k_1 y_1 + x} - D \right] - \frac{m_2 x}{k_2 y_2 + x} \times \frac{y_2}{\delta_2} \\ y_2'(t) = r_2 y_2 \left[\frac{m_2 x}{k_2 y_2 + x} - D \right] - \frac{m_1 x}{k_1 y_1 + x} \times \frac{y_1}{\delta_1} \\ \vdots \\ y_n'(t) = r_n y_n \left[\frac{m_n x}{k_n y_n + x} - D \right] + \frac{m_{n-1} x}{k_{n-1} y_{n-1} + x} \times \frac{y_{n-1}}{\delta_{n-1}} + \cdots \frac{m_1 x}{k_1 y_1 + x} \times \frac{y_1}{\delta_1} \end{cases} \tag{14}$$

$$\begin{cases} r_1 = h_1 T_1 + h_2 TD_1 + h_3 M_1 + h_4 MD_1 + rank_1 \\ r_2 = c_1 T_2 + c_2 TD_2 + c_3 M_2 + c_4 MD_2 + rank_2 \\ \vdots \\ r_n = i_1 T_n + i_2 TD_n + i_3 M_n + i_4 MD_n + rank_n \end{cases} \tag{15}$$

2.2.2 Model Solving

Fungus 1 and fungus 2 have different water tolerance and growth rate. Suppose that their m and MD have different values, the solving process and algorithm are the same as the first model.

2.2.3 Analysis and Evaluation of results

Case 1: mutual promotion

Table 2. Growth and decomposition of lignocellulose by two fungi when they promote each other

day	Nutrient decomposition rate	Fungus l hyphal elongation	Fungus 2 hyphal elongation
10	0.994	0.353	0.165
20	0.981	1.225	0.272
30	0.946	4.030	0.453
…	…	…	…
360	0.418	89.532	71.759

Case 2: mutual restraint:

Table 3. Growth and decomposition of lignocellulose by two fungi under mutual inhibition

day	Nutrient decomposition rate	Fungus l hyphal elongation	Fungus 2 hyphal elongation
10	0.994	0.352	0.164
20	0.981	1.220	0.270
30	0.947	3.997	0.439
…	…	…	…
360	0.472	55.523	29.234

It can be seen from the calculation results in the next three tables that the decomposition rates of lignocellulosic fibers caused by different interactions of fungi are as follows:

mutual promotion≥They don't affect each other≥Mutual restraint (competition)

The following images show the growth and decomposition of fungi with different interactions:

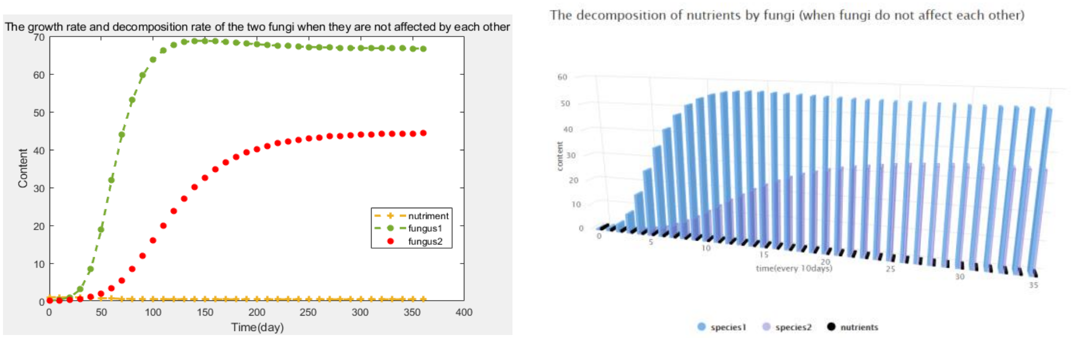

Fig. 2 Mycelial elongation and decomposition rate of two fungi without mutual influence

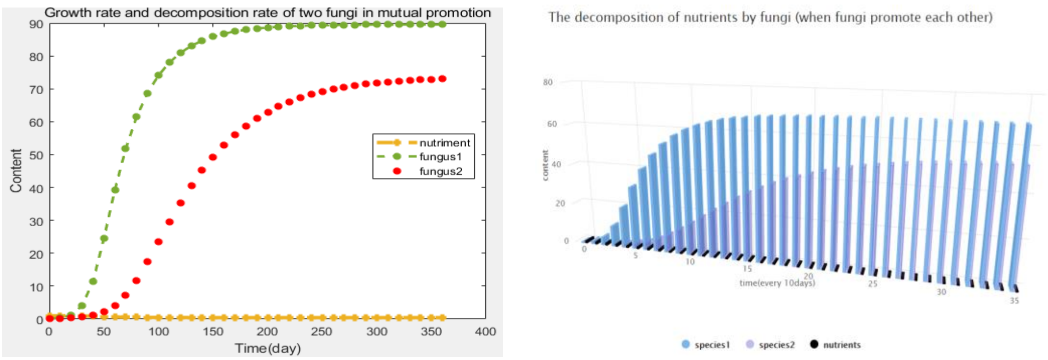

Fig. 3 Mycelial elongation and decomposition rate of two kinds of fungi when they promote each other

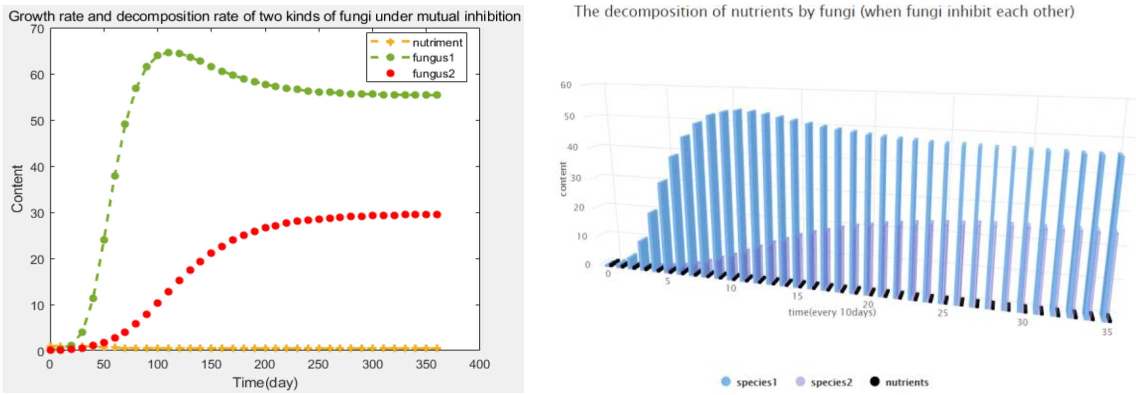

Fig. 4 Mycelial elongation and decomposition rate of two fungi under mutual inhibition

The histogram can help to observe the changes in the number of fungi as a whole, as well as the quality of wood fiber decomposition.

3. Evaluation of the model

3.1. Strengths

Although the two models are slightly different, their common advantage is strong stability, which can accurately depict that in the case of coexistence of various fungi, different fungi have different water tolerance, different mycelial elongation, decomposition process of wood fiber, and high efficiency, which are also the advantages of the model We used the coexistence of two and three kinds of fungi to study. If more bacteria are coexisting, this model can also quickly describe the decomposition of wood fiber by fungi.

3.2. Weaknesses

The common disadvantage of the two models is that the accuracy may not be very high. In the process of research, we set the hyphal elongation rate and decomposition rate of each fungus caused by its environmental changes as constants. In order to further improve the accuracy of the model, we can set different changes.

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