# Evaluation and Expression of Measurement Uncertainty of Axis Positioning Accuracy of Machine Tools

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**Abstract.** Measure the axis bidirectional positioning accuracy of machine tools with guide rails of 2m, 4m and 10m in length, establish an evaluation model of Measurement uncertainty, and analyze the sources of each uncertainty. Combined with the measurement example, the evaluation process of Measurement uncertainty of axis positioning accuracy of machine tools is expounded, and the linear expression formula of Measurement uncertainty is given.

**Key words:** metrology; machine tool; axis positioning accuracy; uncertainty in measurement.

## 1. Introduction

The coordinate positioning and measurement system is an important component of machine tools, which is used to accurately provide the coordinate positions of moving components and achieve positioning units [1]. The bidirectional positioning accuracy of the axis of a machine tool is one of its important metrological characteristics, which determines the type of positioning performance of the machine tool. The measurement method and Measurement uncertainty evaluation of axis bidirectional positioning precision of machine tools have been studied in relevant standards [2-3], specifications [1] and literature [4], but the evaluation example of Measurement uncertainty only analyzes the machine tool guideway with a measurement length of less than 2m, lacking Measurement uncertainty evaluation of axis bidirectional positioning precision with a larger measurement range. Therefore, on the basis of existing research, this paper measures the axis bidirectional positioning accuracy of machine tools with guide rails of 2m, 4m and 10m respectively, and evaluates their Measurement uncertainty.

## 2. Measuring method

Machine tool positioning measurement system linear axis. Evenly select several measurement points within the working stroke of each axis, and cycle 5 times in both positive and negative directions in a standard cycle or step cycle. Record the difference between the actual position and the target position. The measured lengths L are 2m, 4m, and 10m, respectively. The measuring process is shown in Figure 1.



**Figure 1.** Measurement process diagram of machine tool axis positioning accuracy

The uncertainty evaluation examples provided in this section are examples for improving the evaluation under industrial conditions. The improvement of industrial conditions includes: (1) precise adjustment of the optical path. (2) the working environment temperature is 20℃±1℃. (3) temperature measurement deviation band $R$ for machine tools $R(\theta)$ is 0.2℃. (4) expansion coefficient deviation $R(\alpha)$ $2.0\times10^{-6}$℃$^{-1}$. (5) the environmental change deviation $E_{VE}$ is 1.7μm ($L$=2m), 2.0μm ($L$=4m), and 4.0μm ($L$=10m).

## 3. Evaluation and expression of Measurement uncertainty

### 3.1 Measurement model

According to GB/T17421.2, the measurement model for the bidirectional positioning accuracy $A$ of machine tool axes is

$$A = \max[\bar{X}_i \uparrow +2S_i \uparrow; \bar{X}_i \downarrow +2S_i \downarrow] - \min[\bar{X}_i \uparrow -2S_i \uparrow; \bar{X}_i \downarrow +2S_i \downarrow] \tag{1}$$

The deviation $E$ of the bidirectional positioning system for the axis of machine tools is

$$E = \max[\bar{X}_i \uparrow; \bar{X}_i \downarrow] - \min[\bar{X}_i \uparrow; \bar{X}_i \downarrow] \tag{2}$$

The formula for calculating the bidirectional repeated positioning accuracy $R$ of machine tool axes is shown in equation (3), which can be derived from equations (4) to (8)

$$R = \max[R_i] \tag{3}$$

$$R_i = \max[2S_i \uparrow +2S_i \downarrow + |B_i|; R_i \uparrow; R_i \downarrow] \tag{4}$$

$$S_i \uparrow = \sqrt{\frac{1}{n-1}\sum_{j-1}^{n}(X_{xi} \uparrow -\bar{x}_i \uparrow)^2} \tag{5}$$

$$S_i \downarrow = \sqrt{\frac{1}{n-1}\sum_{j-1}^{n}(X_{xi} \downarrow -\bar{x}_i \downarrow)^2} \tag{6}$$

$$B_i = \bar{X}_i \uparrow -\bar{X}_i \downarrow \tag{7}$$

$$\bar{X}_i \uparrow = \frac{1}{n}\sum_{j=1}^{n} X_{ij} \uparrow; \bar{X}_i \downarrow = \frac{1}{n}\sum_{j=1}^{n} X_{ij} \downarrow \tag{8}$$

In equations (3) to (8), $X_{ij}$ is the actual measured position deviation when the moving component approaches the i-th target position for the j-th time, $\bar{X}_i \uparrow$, $\bar{X}_i \downarrow$ are the mean unidirectional position deviation at a position, $B_i$ is the reverse value at a position, $S_i \uparrow$, $S_i \downarrow$ are the estimator for the unidirectional axis repeatability of positioning at a position, $R_i$ are the bidirectional repeatability of positioning at a position, and $R_i \uparrow$, $R_i \downarrow$ are the unidirectional repeatability of positioning at a position.

From formula (1)~(3), it can be seen that the bidirectional positioning accuracy of machine tool axis A is mainly calculated by the bidirectional positioning system deviation E and the bidirectional repeated positioning accuracy R. If the correlation between the two is ignored, the measurement uncertainty of the bidirectional positioning accuracy of machine tool axis $u(A)$ is [5]

$$u^2(A) = u^2(E) + u^2(R) \tag{9}$$

### 3.2 Factor analysis of Measurement uncertainty

The factors affecting the measurement uncertainty are measuring device, cosine error, temperature compensation and environmental change deviation.

#### 3.2.1. Standard uncertainty introduced by measuring devices $u_D$

If the MPE of the linear measurement system of the laser interferometer is known to be $\pm\frac{R_{\mathrm{Drel}}}{2}$, assuming a rectangular distribution, the standard uncertainty uD introduced by the measurement device is shown in equation (10)

$$u_D = \frac{R_{\mathrm{Drel}}}{2\sqrt{3}} \cdot L \tag{10}$$

where $R_{\mathrm{Drel}}$ is the relative MPE of the linear measurement system of the laser interferometer, $L$ is measure the length.

#### 3.2.2. Standard uncertainty introduced by misalignment of measurement device to machine axis under test $u_{MS}$

With a laser interferometer, the laser beam should be parallel to machine axis under test. A misalignment is generally observed only in the change of the intensity of the return beam. Let the misalignment be $\delta$, The cosine error introduced by it $\Delta L_{\mathrm{MS}}$ is

$$\Delta L_{\mathrm{MS}} = L \cdot [1 - \cos(\frac{\delta}{L})] \tag{11}$$

Assuming that the cosine error follows a rectangular distribution, the standard uncertainty $u_{\mathrm{MS}}$ can be obtained as

$$u_{\mathrm{MS}} = \frac{\delta^2}{4\sqrt{3} \cdot L} \tag{12}$$

#### 3.2.3. Standard uncertainty introduced by temperature measurement $u_{MM}$

The impact on temperature measurement is the selection of temperature measurement points, which should represent the temperature of the machine tool. Another impact comes from the uncertainty of temperature sensors measuring temperature.

In practical applications, these effects can be seen as included in a temperature measurement error band $R(\theta)$ medium. This includes the temperature difference between the actual temperature of the machine tool and the temperature measurement point, as well as the uncertainty of the sensor measurement temperature. The uncertainty introduced by temperature measurement is divided into two items. One is the standard uncertainty $u_{\mathrm{MM}}$ introduced by measuring the temperature of the machine tool (the temperature difference between the actual temperature of the machine tool and the temperature measurement point). One is the standard uncertainty $u_{\mathrm{MD}}$ introduced by the temperature measuring device. Because the Measurement uncertainty $u_{\mathrm{D}}$ of the laser interferometer already includes this uncertainty, $u_{\mathrm{MD}}$=0. The temperature measurement error band can be regarded as a rectangular distribution, so the standard uncertainty $u_{\mathrm{MM}}$ introduced by measuring the temperature of the machine tool is

$$u_{\mathrm{MM}} = \frac{R(\theta)}{2\sqrt{3}} \cdot \alpha \cdot L \tag{13}$$

where $\alpha$ is expansion coefficient of axis system.

### 3.2.4. Standard uncertainty introduced by expansion coefficient $u_{EM}$

During the measurement process, the expansion coefficient of the measured axis and measuring device generally adopts the values in the manual, but their actual values may deviate from this value, which is called the expansion coefficient deviation. The interval is $(-\frac{R(\alpha)}{2},+\frac{R(\alpha)}{2})$. Assuming a rectangular distribution, the standard uncertainty $u_{\mathrm{EM}}$ caused by the expansion coefficient deviation of the tested machine tool is

$$u_{\mathrm{EM}}=\frac{R(\alpha)}{2\sqrt{3}}\cdot\Delta T\cdot L \tag{14}$$

where $\Delta L$ is temperature deviation relative to 20℃.

### 3.2.5. Standard uncertainty introduced by environmental variation error $u_{EVE}$

During the time of measurement, the environment, the instrument and/or the machine tool might drift or change, influencing the readout of the measuring system. This environmental variation error can be checked by a drift test. By setting up the measurement equipment on the machine tool under test and looking at the change of readout in the extreme position during the time necessary to do the positioning test, the magnitude of this influence, $E_{\mathrm{VE}}$, can be obtained. Assuming that the range of $E_{\mathrm{VE}}$ variation is $(-\frac{E_{\mathrm{VE}}}{2},+\frac{E_{\mathrm{VE}}}{2})$,which follows a rectangular distribution, the standard uncertainty $u_{\mathrm{EVE}}$ introduced is

$$u_{\mathrm{EVE}}=\frac{E_{\mathrm{VE}}}{2\sqrt{3}} \tag{15}$$

## 3.3 Calculation of standard uncertainty

### 3.3.1. Calculation of u*(E)*

The system deviation depends on the average value of each point in the five measurement cycles, so the accuracy of the measurement device, cosine error, temperature compensation, and environmental change deviation are the main sources of uncertainty. The first three remain unchanged in the five test cycles, so they are correlated. Only the standard uncertainty $u_{\mathrm{EVE}}$ introduced by the environmental change deviation $E_{\mathrm{VE}}$ is not correlated. The uncertainty $u$(E) of the system deviation can be obtained from the relevant calculation formula of the composite uncertainty as follows

$$u(E)=\sqrt{u_{\mathrm{D}}^2+u_{\mathrm{MS}}^2+u_{\mathrm{MM}}^2+u_{\mathrm{MD}}^2+u_{\mathrm{EM}}^2+u_{\mathrm{ED}}^2+\frac{1}{5}u_{\mathrm{EVE}}^2} \tag{16}$$

### 3.3.2. Calculation of u*(R)*

From the definition of bidirectional repeated positioning accuracy, it can be seen that its uncertainty comes from the standard uncertainty of the unidirectional repeated positioning accuracy and the standard uncertainty of the reverse difference in five measurement cycles, which are not related. According to the formula for calculating the accuracy of bidirectional repeated positioning, the standard uncertainty $u$(R) is

$$u(R)=2\cdot\sqrt{\frac{6}{5}}\cdot u_{\mathrm{EVE}} \tag{17}$$

## 3.4 Combined standard uncertainty and expanded uncertainty

The evaluation results of standard uncertainty, composite standard uncertainty, and expanded uncertainty for axis positioning accuracy measurement are shown in Table 1.

**Table 1.** Source of standard uncertainty components for CMM's fourth axis error calibration results

| *Uncertainty Name* | *L*=2m | *L*=4m | *L*=10m |
|---|---|---|---|
| *u*(E) | 0.9μm | 1.1μm | 2.8μm |
| *u*(R) | 1.8μm | 3.4μm | 8.6μm |
| *u*(A) | 2.0μm | 3.6μm | 9.1μm |
| *U*(A)(*k*=2) | 4.0μm | 7.2μm | 18.2μm |

By linearizing the above results, the expanded uncertainty in measuring the positioning accuracy of machine tool axes can be expressed as

$$U(A) = 0.4\mu m + 1.8 \times 10^{-6} L \ (k=2) \tag{18}$$

## 4. Conclusion

The bidirectional positioning accuracy of machine tool axis with guide rails of 2m, 4m and 10m in length is measured. The evaluation process of measurement uncertainty of machine tool axis positioning accuracy is described, and the linear expression result of measurement uncertainty is given. The research contents can provide reference for the evaluation of measurement uncertainty of the axis bidirectional positioning accuracy of machine tools with larger measurement length

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