A Simulation Based Modeling of Multiple Plants Ecosystems

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Abstract. In order to solve the problem of multi-species community growth, a multi-species competitive development model based on Lotka-Volterra equation was established. Second, consider the growth rate of plants under weather conditions. Thirdly, interspecific competition, drought adaptation, drought frequency and pollutants were introduced to consider plant growth. Finally, the optimal population size was simulated by calculating community abundance and stability. The results showed that when the community stability was 0.928, the corresponding species number was 146, and the species abundance was inextricably related to biodiversity and interspecies interaction.

Keywords: Lotka-Volterra Equation, Community abundance, Multi-species development.

1. Introduction

Biodiversity is the diversity of life of different species in a community, and the process of interacting and cycling it together in nature. There is often a general positive relationship between biodiversity and community stability [1]. Biodiversity and community abundance are of great significance to community stability and development.

By simulating the growth and development of different species types in plant communities, considering interspecific relationships, and simulating different drought cycles, long-term interactions between plant communities and the larger environment can be explored.

2. Materials and methods

This paper uses the A question of the 2023 Mathematical Contest In Modeling Competition (https://www.comap.com/) for analysis and research.

3. Multi-Species Competitive Development Model

3.1. Plant Growth Rate Model

Different weather conditions and drought conditions have significant effects on plant growth rate. Under the consideration of interspecific relationship, the transformation of environmental disturbance factors θ can be expressed by the following relation:

$$\theta = \frac{p}{\lambda} - \lambda_a \times \frac{N_a}{N_{\max}} \tag{1}$$

Where N_i（t） represents the number of the ith plant at time t, where i=1,2... n; r_i represents the inherent growth rate of the i-th plant. The competition coefficient of plants is λ_a, N_a is the number of plant population in the current environment; $N_{\max}$ represents the largest population of plants in the current environment; The environmental disturbance factor θ is between（-1，1）, in which the environmental disturbance factor is determined by the competition relationship between precipitation and plants. This model describes the basic law of plant population growth, that is, population size changes over time depending on the inherent growth rate of the species and the current level of competition in the plant community.

While maintaining the basic growth rate of plant population, the growth rate of plant population

fluctuates around the basic growth rate under the influence of external disturbance factors. In order to study the interactions between different plant populations, the following mathematical model is used to describe the model of plant intrinsic growth rate:

$$r = r_{basic} + \theta \tag{2}$$

The intrinsic growth rate of plants is closely related to the metabolic rate of plants[2]. Moreover, the average metabolic rate of boreal forest in China is 0.625[3], so it can be used as a substitute index of growth rate based on it. θ is an environmental disturbance factor affecting growth rate.

3.2. Multi-Species Growth Model

Interspecies interactions are first divided into direct and indirect effects. Species indirectness refers to the influence of one species on a third species through another species (mediator species), existing between species at different trophic levels, producing cascading effects. However, indirect effects are in most cases much smaller than direct effects[4], so they can be ignored in specific simulation situations.

In order to study the common growth of multiple species in limited space under a certain ecosystem, the following multi-species growth model is constructed by referring to the Lotka-Volterra interspecific competition model:

$$\frac{dN_i}{dt} = r_i N_i(t)\left(1 - \sum_{i=1}^{t} \lambda_i N_i(t)\right) \tag{3}$$

Where, λ_i denotes the competition coefficient of the ith plant. If the number of organisms is large, the total competition will increase, thus inhibiting its growth. Therefore, the competition relationship between organisms affects the change of the number of biological populations. When the biological population reaches a certain level, its growth rate will slow down until it stops growing.

3.3. Population Drought Adaptation Model

Drought indices can visualize a drought incident's occurrence, magnitude, lasting time, and intensity. Therefore, the transformation of drought indicators into drought indices is indispensable[5]. There are four main types of droughts: meteorological, agricultural, hydrological, and socio-economic Forces behind the occurrence of agricultural drought.

In order to study the effects of drought on plant population, based on the growth model, a population drought adaptation model was introduced:

$$\delta = \left(\frac{\sum_{i=1}^{n} D_i N_i(t)}{E}\right)^p \tag{4}$$

D_i represents the drought adaptation coefficient of the i-th plant; E stands for environmental capacity; p represents the saturation parameter of population growth under drought conditions. The model takes into account drought adaptation, which means that plants with higher drought adaptation coefficients are more likely to survive and reproduce under drought conditions.

3.4. Multi-Species Competitive Development Model Under Droughts

A lot of research expenditure, plant hardiness adaptability is present in plant genes. Even using genetic engineering methods, through the study and editing of plant drought tolerance genes, the original various plants can be transformed into drought-tolerant plants.[6]Therefore, the plant drought adaptation coefficient is relatively independent at this time.

By synthesizing the above model and introducing the plant death term, we can get the following

model:

$$\frac{dN_i}{dt} = r_i N_i(t)\left(1 - \sum_{i=1}^{n} \lambda_i N_i(t)\right) \times \left(\frac{\sum_{i=1}^{n} D_i N_i(t)}{E}\right)^p - m_i N_i(t) \quad (5)$$

Where, m_i represents the mortality rate of the i-th plant.When environmental capacity reaches saturation or mortality increases, population growth will slow down or stop. Therefore, plant death needs to be considered after comprehensive model.

3.5. Simulation and Result Analysis

(1) Simulated Forecasts Under Irregular Weather Cycles

First, we assume that there are three different species under the model, and let their intrinsic growth rate, competition coefficient and drought adaptation coefficient be constants.

The benchmark of the intrinsic growth rate is 0.65 based on the average metabolic rate of northern forests in China, assuming that the growth rate of the other two species is greater or less than this benchmark, and after the growth rate is introduced into the periodic function and calculated, the difference is about 0.2 (reserved to one decimal place). Therefore, the competition coefficients of different plants were set to 0.45, 0.65 and 0.85, which together with the intrinsic growth rate affected the environmental interference factors. The drought adaptation coefficients are assumed to be 0.3, 0.6 and 0.8, which represent the saturation parameters of population growth under different drought conditions.

Assuming an environmental capacity of 1000 and the same species mortality rate, it is 0.05.The results are shown in Figure 1.

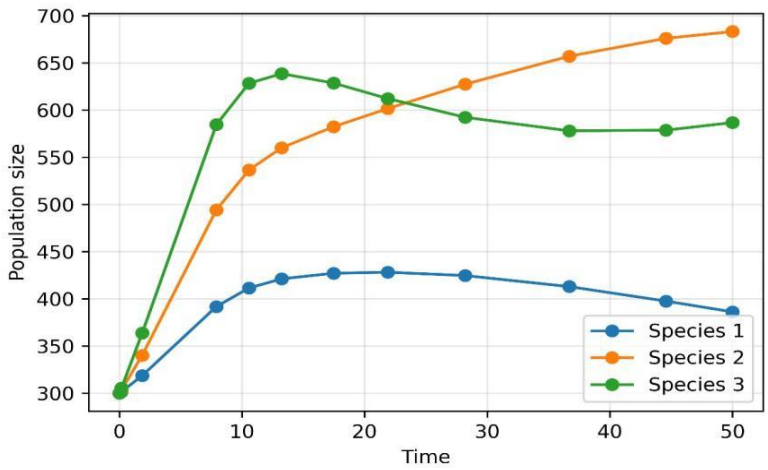


Figure 1. Simulated forecasts under irregular weather cycles

The competition coefficient of species 1 against the other two species was 0.45, the competition coefficient of species 2 against the other two species was 0.65, and the competition coefficient of species 3 against the other two species was 0.85.

The image shows that the species is the smallest: due to its low growth rate and low drought adaptability, this means that it is relatively sensitive to changes in the environment, the living conditions are more demanding, and the number is naturally smaller.

At the same time, species II is the most abundant: due to its moderate growth rate and moderate drought adaptation, this means that it is relatively adapted to changes in the environment, and its living conditions are relatively relaxed and therefore more abundant.

Third, species three are moderately numerous: because of their largest growth rate and greatest drought adaptability, this means that it has the strongest ability to change the environment and has relatively relaxed living conditions, but it is not the largest in number.

In summary, ecological characteristics such as growth rate and drought adaptation have important influences on the number of different species. The higher the degree of drought adaptation, the larger the species population, and too high drought adaptation degree may have a suppressive effect on the species population [7].

(2) Impacts of Higher Frequency And Wider Variation of Droughts

When droughts occur more frequently and more widely, they can have a significant impact on plant growth rates. Research now shows that increasing drought should reduce population growth rates[8]. And, if wet years are more likely to be followed by dry years, it has larger variation in vital rates. Therefore, according to the negative impact of drought on growth rate and its positive effect on plant mortality, adjusting the growth rate and mortality rate in the model can simulate the impact of species number on the population as a whole when drought is more frequent.

Here, considering the impact of drought events, the corrected growth rate is obtained after calculation according to the growth rate model. We simulated the three scenarios: in which the number of species is present, and the growth rate was calculated to be between 0.2 and 0.3, so we set it to be a random number selected in this interval. The increased mortality rate is taken in between, and a value is randomly selected.

Due to the different species and large number, the adjusted environment capacity is 150 for each species. The results are shown in Figure 2.

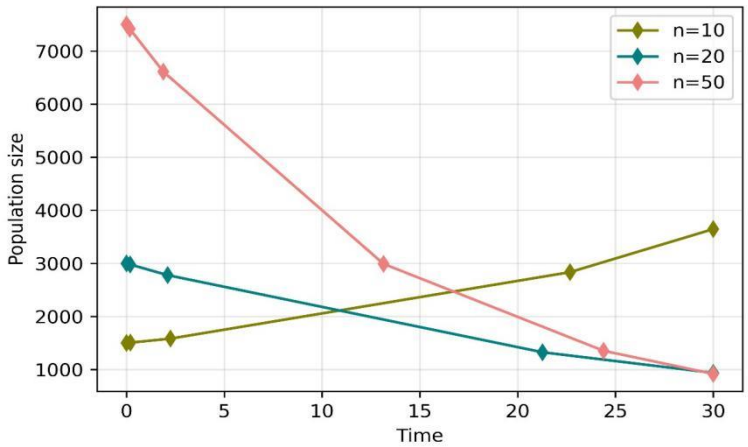


Figure 2. Impacts of higher frequency and wider variation of droughts

As shown in the figure, when the frequency of drought is higher and the range of change is larger, the number of species has a significant impact on the population size of species in the population, and the maximum value is significantly smaller than the environmental capacity under different circumstances. In addition, population size fluctuates more dramatically over time.

When the number of species$n=50$, there are too many of them, resulting in particularly fierce competition, so the population size declines faster. When it declines to a certain extent, competition within the community decreases, and the decline slows down thereafter. When the number of species $n=20$, population size still slowly declines because the growth rate is less than the competition rate under drier conditions, and the mortality rate is high. When the number of species$n=10$, whether viewed from the perspective of competition or environmental capacity, the community has sufficient resources to provide conditions for growth, so it shows a growth trend. Plant species with similar ecological habits in communities will unite to form ecological species groups, and several species in a heterogeneous environment will have similar adaptations to environmental conditions, which will have a positive association effect[9]. Therefore, after growing to a certain extent, with more resources, some species may develop interspecific cooperation to cope with extreme weather conditions, so the growth trend accelerates.

(3) Simulation Forecasts When Droughts are Less Frequent

If droughts are less frequent, it means that growth r and mortality rates m will return to normal values. In this case, r is the value randomly selected in the interval $[0.6,0.7]$, and the value

randomly selected in the interval $[0.05,\ 0.1]$.

In the case of less drought, the impact of the number of species on community population size will first be reflected in the following aspects:

Population size will increase significantly. This is because plants have a suitable growing environment. **2.** When the population of species grows, its growth rate is faster, and when the population declines, its downward trend is flatter. **3.** When the species is moderate, the population size changes completely opposite to the conditions with high drought frequency. The results are shown in Figure 3.

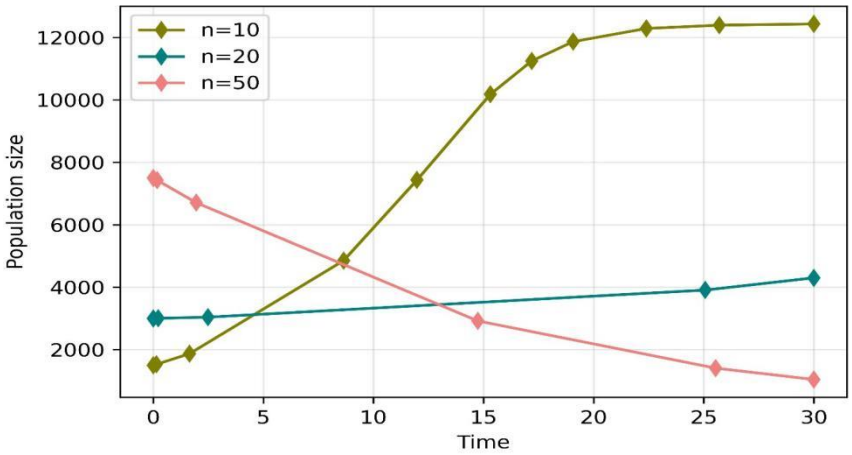


Figure 3. Simulation forecasts with weakening drought frequency

The details are as follows:

When the number of species is$n=50$,the number of communities is close to saturation, and in a particular community, resources are limited, although the growth rate is high, the impact of competition between species has exceeded its growth, so it shows a downward trend. When the number of species is $n=20$, due to the small impact of drought, high growth rate and low mortality rate, there is still room for development and cooperation of different species of plants in the community, so the total number of communities shows an increase state. When species is$n=10$, their number is much lower than the maximum environmental capacity of the community, and there is a high growth rate at this time, so the species will grow rapidly and tend to flatten when the maximum environmental capacity is reached.

(4) Pollution and Introduction of Habitat Variables

Based on the drought adaptive quantitative growth model of multi-species plant communities, we introduce pollution and habitat variables as follows:

$$\frac{dN_i}{dt}=r_iN_i(t)\left(1-\sum_{i=1}^{n}\lambda_i\,N_j(t)\right)\cdot\left(\frac{K-\sum_{i=1}^{n}D_i\,N_i(t)-\gamma(t)}{E}\right)^{p}-m_iN_i(t)-\epsilon_i(t)N_i(t) \quad (6)$$

$\epsilon_i(t)N_i(t)$ Indicates an external threat to the species. $N_i(t)$ Represents the number of i-th species at time t. It is usually considered as an additional mortality rate.

After we introduced the polluted and habitat reduction variables, we left other parameters unchanged and adjusted for pollutant and habitat impact factors. It is evident that plant responses to individual pollutants in isolation largely differ from responses to the complex multiple pollutant scenario[10]. Therefore, the pollutant impact factor should be a combination of multiple complex pollutants. Habitat is a space where a plant community thrives, so it has a greater and more direct impact on the community. Here, we will set the ϵ with the value between $[0.2,0.4]$,γ is a random number between $[0.3,0.5]$ values .This value can take into account the impact on plant communities

under different degrees of pollutants and habitat loss.

The results are shown in Figure 4.

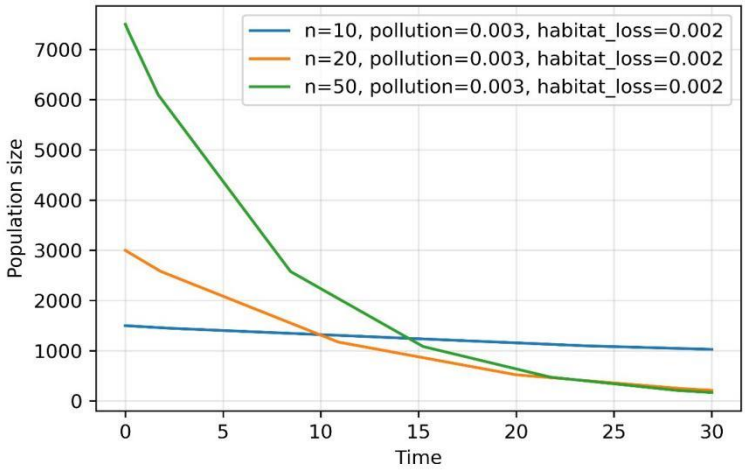


Figure 4. The number of species affected by habitat and pollution

Pollution and habitat impacts can directly alter species trends. At this point, the population size no longer increases. In the three cases of different species, population size showed a downward trend. It can be seen that pollutants and habitat loss are fatal to plant survival.

3.6. Sensitivity Analysis

Through the above research, we found that under different drought conditions, different species numbers in the community will affect the community population.

In this section, we change the pollutant and habitat impact parameters several times and combine them to obtain the final population size of the community, and compare the results with the original results with the final change of population size under different parameters to test the sensitivity of the model after introducing the new variables.

The figure above shows in detail the change of the community reaching the end point over time when n=10, 20, 50.The results are shown in Figures 5, 6, and 7

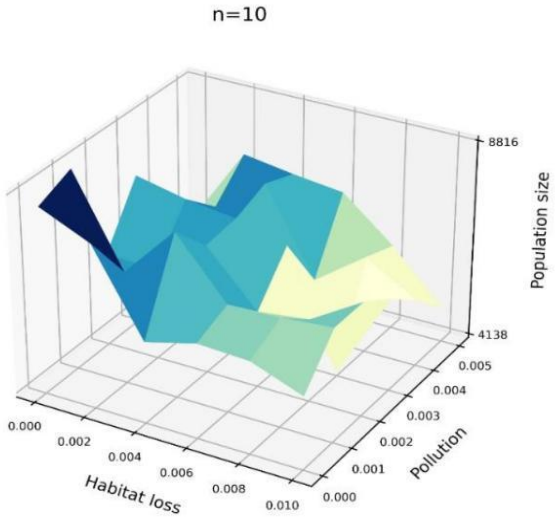


Figure 5. Sensitivity analysis 1

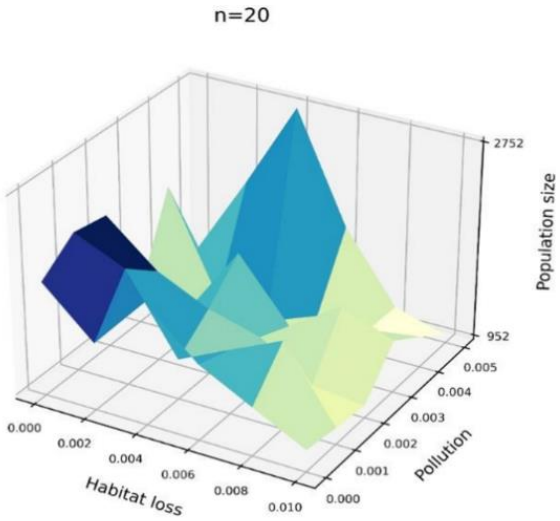


Figure 6. Sensitivity analysis 2

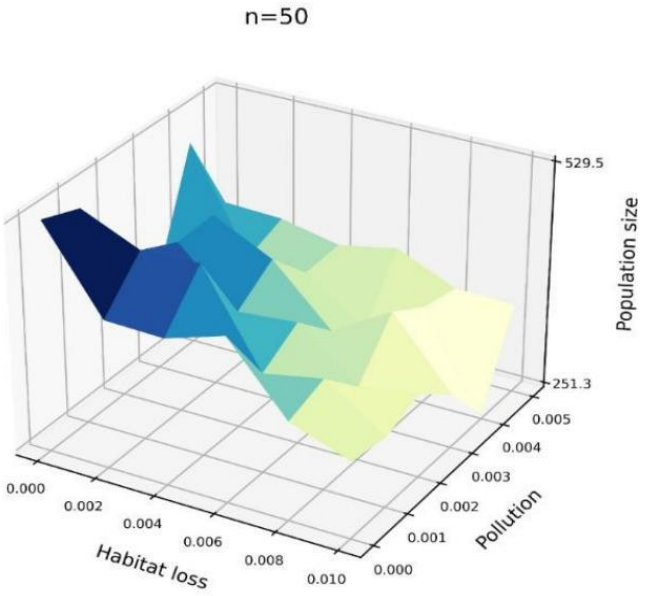


Figure 7. Sensitivity analysis 3

The original results were 0.03 and 0.02 for the pollutants and habitat variables, respectively, and the 3D plots better showed the response of the community under other influence combinations. It can be found that when the values of pollutant impact and habitat impact parameters increase, the final population size shows a significant downward trend with the influence of interspecific relationship, and the pollutant impact and habitat impact significantly exceed other variables.

4. Community Abundance and Stability model.

The model assumes that the interactions between species within the plant community are stable and do not change significantly in arid environments. Since the growth process is divided into different stages, and the continuous process of each stage does not change much, the competitiveness index can remain roughly unchanged in a fixed stage.

4.1. Species Adaptation and Ecological Balance

Species distribution models are vital for predicting species' adaptation to the environment in a specific ecosystem. They help identify factors influencing species distribution and richness, quantify species-environment relationships, predict species responses to land use and climate change, and estimate potential population distributions.

Plant communities' adaptability relies on species number and diversity. Increased species abundance leads to higher competition and greater susceptibility to drought. As species diversity grows, competition intensifies, reducing resource utilization and compromising survival under drought conditions. Thus, maintaining a balance between species diversity and competition is crucial for ensuring community adaptability and ecosystem stability in the long run.

4.2. Models of Species Abundance and Stability

Species diversity and stability are affected by ecosystems, and under current conditions, they can be simplified to species diversity and arid environments, and competition between different species. Assuming that the number of distinct species in the plant community is defined as n, we can build the following simple mathematical model to study the effect of species abundance on community stability.

$$\frac{dx_i}{dt} = x_i(a_i - \lambda_i x) \tag{7}$$

Among them x_i is the relative abundance of the i-th plant, a_i is the adaptability of the plant, λ_i is the

competitiveness of the plant, and $x = \sum_{i=1}^{n} x_i$ is the total relative abundance of all species in the community. The first part $x_i \beta_i$ indicates the growth rate of the plant without competitive pressure, that is, its own growth capacity. The second part $-x_i \lambda_\alpha x$ indicates that the plant is subject to competitive pressure from other species, and its growth rate x slows as the relative abundance of other species in the community increases. In this differential equation, the weights of these two parts are determined by the relative abundance of the plant x_i, i.e. the larger the plant's population, the stronger its own growth capacity, but at the same time the greater the influence of competitive pressures.

The results show that the level of species diversity has two effects: unimodal and U-shaped on the overall stability of ecosystems. That is, when biodiversity is low, it can increase overall ecosystem stability, and when biodiversity is high, it weakens stability; The opposite is a U-shaped relationship. If diversity has a positive effect on some functions and a negative impact on others, it also has unimodal and U-shaped effects on ecosystem versatility [11].

Ecosystem stability refers to the stability of ecosystem functions, which describes the ability of ecosystems to maintain or restore their original state after being disturbed. Theoretically, community stability is affected by factors such as species stability, and the relative abundance of dominant species [12].

To study the effect of species diversity on community stability, we need to calculate the overall adaptability of the community, and we can calculate the overall adaptability of the community using the ratio of the adaptability of all plants to the relative abundance of all species in the community:

$$S = \frac{\sum_{i=1}^{n} xD_i}{\sum_{i=1}^{n} x_i} \tag{8}$$

We can study how the stability of communities varies with species diversity under different parametric conditions, such as different adaptive and competitive distributions. Typically, community stability reaches its highest level of mid-species diversity and then gradually declines, as too many or too few species increase community instability

4.3. Simulation Prediction of Optimal Species Number, Community Stability and Outcome Analysis

We simulated the initial abundance, species competition coefficient and drought adaptation coefficient as N-dimensional random arrays, and normalized the competition coefficient matrix, and other parameters were consistent with the previous to better simulate the growth environment of multiple types of plants in the community.

The simulation shows that when the number of species is 145, the maximum value S is: 0.9278, at which time the community benefits from the species. The reason for this may be interspecific interactions. When the environmental resources are sufficient, cooperative relations will be generated between some species to promote each other's growth, and when the environmental capacity is saturated, the competition relationship will be highlighted and the stability of the community will decrease. Due to the decline of the number of species, there will be a surplus of environmental resources, so the stability of the community increases, and its overall fluctuation and decline trend. When it reaches its lowest point, it will fluctuate and rebound and gradually moderate.
The results are shown in Figure 8.

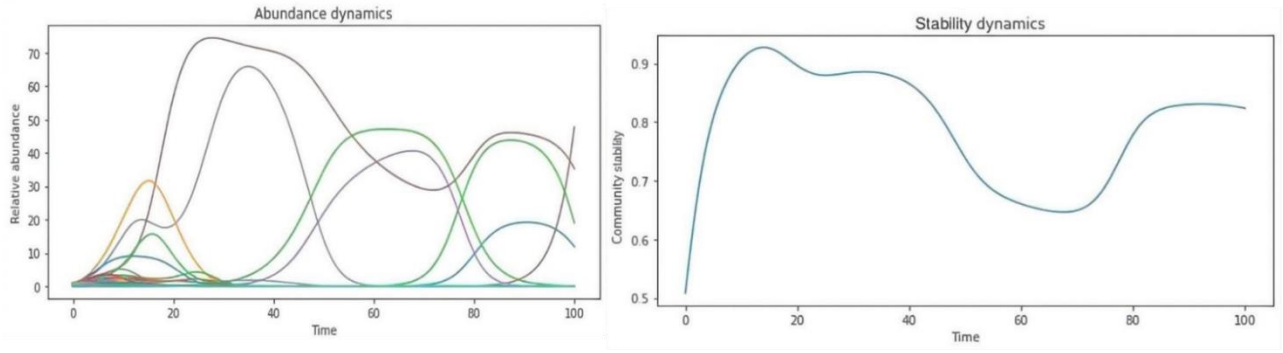


Figure 8. Simulation prediction of optimal species number, community stability and outcome analysis

4.4. Community Abundance and Community Stability

Since the 80s of last century, a large number of field experiments have consistently found a positive correlation between biodiversity and ecosystem (community) stability.

Specifically, if a species is relatively abundant in a community, it will be more adaptable and competitive, and thus have a greater impact on stability. Conversely, if the relative abundance of a species in the community is low, then its adaptability and competitiveness will be relatively weak, and thus have less impact on stability.

5. Conclusions

By establishing two models: Multi-Species Competitive Development Model and Community Abundance and Stability model, we have led to following conclusions.

Excessive species can significantly impact plant growth due to interspecific interactions and other factors. In contrast, a lower number of species allows for abundant environmental resources, promoting favorable growth conditions for plants. Moderate species diversity can moderate the impact of drought on community development. The introduction of new variables had a substantial influence on all quantitative species types, resulting in a downward trend in community abundance, with more severe declines. Multiple experiments revealed that community stability was 0.928, corresponding to a species count of 146. Overall, species abundance is closely tied to interspecific interactions.

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