Mathematical Model for Deformation Analysis of Guta

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Abstract. In this paper, the data in the appendix were removed from the missing data and the valid data were retained. The following problems were solved by the statistical processing of the valid data. While examining on foot the measured data of tower layers' positions in different years and different classifications, the measured coordinates of each layer of the ancient tower in 1986 will be taken as the basis, and the scatter distribution of each layer of the tower will be made by MATLAB, and the coordinate point positions are found to be on the same basic circular plane. Based on the minimum sum of distances from any point on the same circular plane to the fixed points (a, b, c), a nonlinear programming model is established, and by substituting the data to calculate the The coordinates of the center of each level of the ancient pagoda four years ago were calculated by substituting the data. When analyzing the settlement of the gutta, the change of the horizontal plane is not considered, but the change of the vertical ground coordinates of the gutta, i.e. the change of the Z-axis causes the overall rise and fall of the gutta. Based on the center coordinates of each level of the guttae obtained from problem 1, the three settlement differences, average settlement and annual average settlement from 1986 to 1996,1996 to 2009, and 2009 to 2011 are calculated. It can be seen that the ancient tower is sinking year by year, and the sinking rate increases year by year.

Keywords: Least squares, nonlinear programming, LINGO, fitting, MATLAB.

1. Introduction

Cultural relic is the nation's non-renewable cultural resource, especially the ancient pagoda highly regarded by the world, which not only has extremely high artistic value [1-2], but also provides precious material materials for the study of ancient Chinese architectural history, religious history and local history [3].

Ancient tower due to a long time under the weight, temperature, wind and other effects [4-6], but also occasionally affected by earthquakes, hurricanes, will produce a variety of deformation, such as tilt, bending, twisting, settling and so on. The nonlinear programming mathematical model is established to study the leaning, bending [7], twisting and deformation trend of the ancient pagoda, which can intuitively understand the damage situation of the ancient pagoda and has certain significance for the protection of the ancient pagoda. It is of guiding significance for the department of cultural relics to have a comprehensive understanding of ancient pagodas and formulate necessary protection measures [8-10].

This paper intends to solve the following problems with the given data:

Problem 1: According to the data given in the attachment, analyze the space of ancient pagodas, determine the general method of the center position of each layer of ancient pagodas, and list the center coordinates of each layer of ancient pagodas measured.

Problem 2: Since the ancient pagoda is old, the settlement phenomenon is analyzed according to the data, only the vertical direction is considered, and the horizontal position is not considered.

Problem 3: Earthquakes, hurricanes and other natural disasters have a strong relationship with the damage of ancient pagodas. The data obtained from the above two problems are used to analyze the tilt degree of ancient pagodas.

Problem 4: Analyze the bending degree of the ancient pagoda with the data obtained from the above problems.

2. Analysis of the problem

This is a comprehensive and practical mathematical modeling problem, first of all, Excel is used to screen the data processing and operation, eliminate invalid data, and through the use of LINGO nonlinear programming analysis, and then MATLAB image fitting, the establishment of multivariate linear equation.

2.1. Analysis of Question 1

This question requires a general method to determine the center position of each layer of the ancient pagoda according to the measured data, and the center coordinates of each layer of the ancient pagoda. Taking the measurement data in 1986 as the benchmark, MATLAB software was used to make the scatter diagram of the measurement positions of each layer of the ancient pagoda. According to the diagram, the measurement points were basically located in the same circular plane. According to the minimum sum of the distances from any coordinate (x_i, y_i, z_i) to the fixed point O(a,b,c) on the same circular plane, a nonlinear programming model is established. The original measurement data were substituted into the calculation. For the measurement data of the spire in 2009 and 2011, the coordinates were taken as the standard, and the center coordinates of each layer were obtained by LINGO optimization calculation.

2.2. Analysis of Problem 2

When settling the ancient pagoda, it does not consider the change of the horizontal position, only consider the vertical rise and fall. According to the coordinates of the center of the ancient pagoda calculated in problem 1, the three settlement differences (Δ_i), the average settlement amount and the average annual settlement amount from 1986 to 1996,1996 to 2009, and 2009 to 2011 were calculated using EXCEL tables.

2.3. Analysis of Problem 3

According to the coordinates of the center of the ancient tower calculated in problem 1, MATLAB software was used to make the x-y-z scatter images of the center lines in 1986, 1996, 2009 and 2011. It can be seen from the images that the coordinates of the center lines of the ancient tower are almost in the same line, so we established the least square fitting model of the spatial line. By substituting the data of the center of the ancient pagoda into the model, MATLAB was used to calculate the equation of the center line of the ancient pagoda for four years, and the change Angle of the center line of the ancient pagoda for three times from 1986 to 1996, from 1996 to 2009, and from 2009 to 2011 was calculated by using the vector product, so as to judge the tilt degree of the ancient pagoda.

2.4. Analysis of Problem 4

The bending analysis is based on the coordinate data of the ancient pagoda center calculated in the first question. The direction vectors of the coordinates of the center points of the two adjacent layers of the ancient pagoda in 1986, 1996, 2009 and 20114 are calculated respectively. Then, the offset angles of the two adjacent layers are calculated according to the cross-product calculation, and the bending degree of the ancient pagoda is judged from the change of the offset angles.

3. Model building and solving

3.1. Data processing and analysis of Question 1

3.1.1. Model building

Ignoring the missing data in the measurement data and taking the measurement data in 1986 as the benchmark, MATLAB software was used to make the scatter diagram of the measurement positions of each layer of the ancient pagoda. According to the diagram, the measurement points were basically located in the same circular plane. The nonlinear programming model is established according to the

minimum sum of the distances between any coordinate (x_i, y_i, z_i) and the fixed point O(a,b,c) on the same circular plane.

$$\boldsymbol{min} = \sum_{i=1}^{n} \sqrt{(x_i - a)^2 + (y_i - b)^2 + (z_i - c)^2} \; ; (\boldsymbol{i} = \mathbf{1}, \mathbf{2}, \mathbf{3} \ldots\ldots \mathbf{13}; \boldsymbol{n} = \mathbf{1}, \mathbf{2}, \mathbf{3} \ldots\ldots \mathbf{8}) \quad (1)$$

3.1.2. Using LINGO software to solve

The original measurement data are substituted into the calculation. For the measurement data of the spire in 2009 and 2011, their coordinates shall prevail. For the specific LINGO optimization calculation source program and operation results, see Annex 2 (source program) and Annex 3 (operation result).

For example, to calculate the coordinates of the center point of the first layer in 1986, the calculation process is as follows:

min=((565.454-a) ^2+(528.012-b)^2+(1.792-c)^2)^1/2
+((562.058-a) ^2+(525.544-b)^2+(1.818-c)^2)^1/2
+((561.39-a) ^2+(521.447-b)^2+(1.783-c)^2)^1/2
+((563.782-a)^2+(518.108-b)^2+(1.769-c)^2)^1/2
+((567.941-a)^2+(517.407-b)^2+(1.772-c)^2)^1/2
+((571.255-a)^2+(519.857-b)^2+(1.77-c)^2)^1/2
+((571.938-a)^2+(523.953-b)^2+(1.794-c)^2)^1/2
+((569.5-a)^2+(527.356-b)^2+(1.801-c)^2)^1/2;
a<572.
b<516;
c<1.82.

$$a=566.6648,\ b=516.000,\ c=1.787375 \quad (2)$$

By analogy, the center coordinates of other layers can be calculated.

The center coordinates of each layer of the ancient pagoda are shown in Table 1 below.

Table. 1 The central coordinates of the layers of the Ghouta

i	1986	1996	2009	2011
1	(566.6648,516.000,1.787375)	(566.6649,522.7102,1.783000)	(566.7268,522.7015,1.7645)	(566.7269,522.7014,1.763250)
2	(566.7196,522.6684,7.320256)	(566.7205,522.6674,7.314625)	(566.7640,522.6693,7.30900)	(566.7642,522.6690,7.2905)
3	(566.7735,522.6272,12.75525)	(566.7751,522.6256,12.7507)	(566.8001,522.6384,12.73225)	(566.8004,522.6388,12.72687)
4	(566.8161,522.5944,17.07825)	(566.8183,522.5922,17.07512)	(566.8293,522.6132,17.06975)	(566.8297,522.6127,17.052)
5	(566.8621,522.5591,21.72050)	(566.8649,522.5563,21.71600)	(566.8603,522.5866,21.70937)	(566.861,522.586,21.70387)
6	(566.9084,522.5244,26.23513)	(566.9118,522.5210,26.22950)	(566.9471,522.5342,26.211)	(566.9478,522.5335,26.2045)
7	(566.9467,522.5081,29.83912)	(566.9505,522.5042,29.83225)	(566.9792,522.5123,29.82463)	(566.9809,522.5115,29.817)
8	(566.9842,522.4924,33.35088)	(566.9884,522.4881,33.34538)	(567.0305,522.4797,33.33987)	(567.0312,522.4788,33.33662)
9	(567.0217,522.4764,36.80000)	(567.0265,522.4714,36.80000)	(567.0816,522.4466,36.84375)	(567.0825,522.4457,36.82225)
10	(567.0569,522.4624,40.17212)	(567.0620,522.4572,40.16763)	(567.1370,522.3937,40.16113)	(567.1381,522.3936,40.23)
11	(567.1045,522.4230,44.43113)	(567.1102,522.4173,44.43537)	(567.1804,522.3549,44.43175)	(567.181,522.3535,44.42614)
12	(567.1517,522.3836,48.71187)	(567.1578,522.3775,48.70737)	(567.2224,522.3160,48.691)	(567.2237,522.3246,48.69308)
13	(567.0850,522.7403,52.83429)	(567.0912,522.7340,52.83000)	(567.2716,522.2715,52.81875)	(567.2725,522.2701,52.81312)
spire	(567.2472,522.2437,55.12325)	(567.2544,522.2367,55.11975)	(567.336,522.2148,55.0910)	(567.3375,522.2135,55.0870)

3.2. Data processing and analysis of Question 2

When analyzing the settlement of ancient pagodas, the horizontal position change is not considered, only the vertical rise and fall is considered. According to the coordinates of the center of the ancient pagoda calculated in Problem 1, EXCEL tables are used to calculate the settlement difference of three times ($\Delta_v = z_{(n+1)i} - z_{ni}$) from 1986 to 1996,1996 to 2009 and 2009 to 2011,(n represents the measurement times), the average settlement amount $\overline{\Delta_v} = \frac{\sum_{i=1}^{14} \Delta_i}{14}$, and the annual average settlement amount $\overline{\Delta_s} = \frac{\sum_{i=1}^{14} \Delta_i}{14t}$, $t = 10,13,2$.

The settlement difference results are shown in Table 2 below.

Table. 2 The settlement difference results.

Year/ Differential settlement Δ_v	1986-1996	1996-2009	2009-2011
Δ_1	-0.00438	-0.0185	-0.00125
Δ_2	-0.00563	-0.00563	-0.0185
Δ_3	-0.00455	-0.01845	-0.00538
Δ_4	-0.00313	-0.00537	-0.01775
Δ_5	-0.0045	-0.00663	-0.0055
Δ_6	-0.00563	-0.0185	-0.0065
Δ_7	-0.00687	-0.00762	-0.00763
Δ_8	-0.0055	-0.00551	-0.00325
Δ_9	0	0.04375	-0.0215
Δ_{10}	-0.00449	-0.0065	0.06887
Δ_{11}	0.00424	-0.00362	-0.00561
Δ_{12}	-0.0045	-0.01637	0.00208
Δ_{13}	-0.00429	-0.01125	-0.00563
Δ_{spire}	-0.0035	-0.02875	-0.004
$\overline{\Delta_v}$	-0.00377	-0.00778	-0.00225
$\overline{\Delta s}$	-0.000377	-0.0006	-0.00113

As can be seen from the above table:

(1) During 1986-1996, the 10-year average subsidence was -0.00377, that is, the ancient pagoda sank 3.77mm, and the 10-year average subsidence was 0.377mm.

(2) From 1996 to 2009, the average subsidence in 13 years was -0.00778, that is, the ancient pagoda sank 7.78mm, and the average subsidence in 13 years was 0.6mm.

(3) During 2009-2011, the two-year average subsidence was -0.00225, that is, the ancient pagoda sank 2.25mm, and the two-year average subsidence was 1.13mm

To sum up: the ancient pagoda is sinking year by year, and the rate of decline is increasing year by year.

3.3. Data processing and analysis of Question 3

According to the coordinates of the center of the ancient tower calculated in problem 1, MATLAB software was used to make the x-y-z scatter images of the center lines in 1986, 1996, 2009 and 2011, as shown in Fig. 1-4 below.

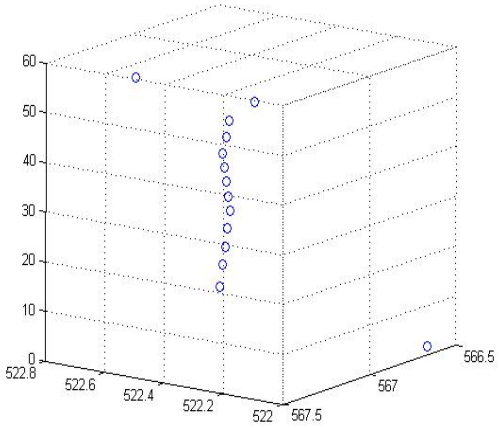


Fig. 1 Ancient tower center coordinates in 1986

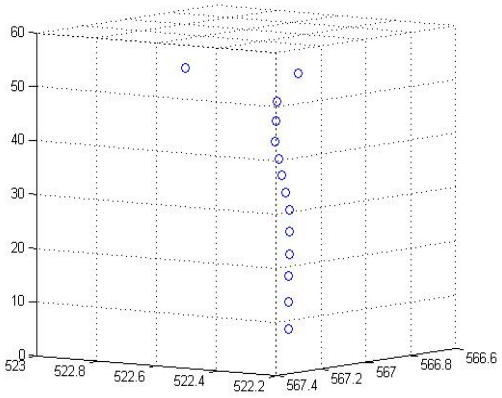


Fig. 2 Ancient tower center coordinates in 1996

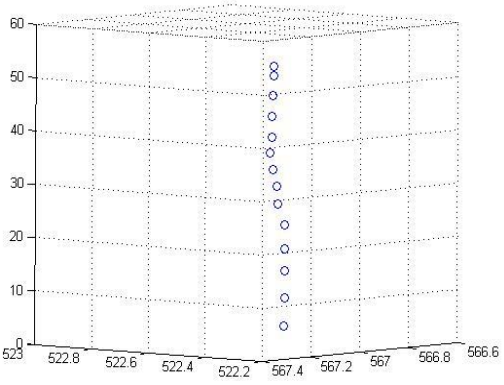


Fig. 3 Ancient tower center coordinates in 2009

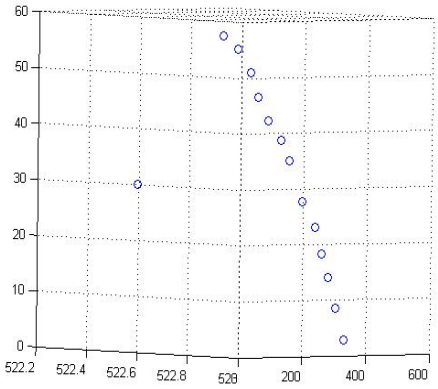


Fig. 4 Ancient tower center coordinates in 2011

It can be seen from the image that the coordinates of the center line of the ancient tower are almost in the same line, so we establish the least square fitting model of the spatial line.

```
>> X= [ …… ];
>> Y= [    ];
>> Z=[    ];
>> F=[Z;1 1 1 1 1 1 1 1 1 1 1 1 1 1];
>> M=F*F'
>> N=F*X'
>> O=F*Y'
>> A=(M\N)'
```

$$>> B=(M\backslash O)' \tag{3}$$

By substituting the data of the center of the Ghouta into the model, MATLAB was used to calculate the equation of the center line of the Ghouta for four years. The specific process is as follows:

Year 1986

```
>> X=[566.6648 566.7196 566.7735 566.8161 566.8621 566.9084 566.9467 566.9842 567.0217
567.0569 567.1045 567.1517 567.0850 567.2472];
>> Y=[516.0000 522.6684 522.6272 522.5944 522.5591 522.5244 522.5081 522.4924 522.4764
522.4624 522.4230 522.3836 522.7403 522.2437];
>> Z=[1.787375 7.320256 12.75525 17.07825 21.72050 26.23513 29.83912 33.35088 36.80000
40.17212 44.43113 48.71187 52.83429 55.12325];
>> F=[Z;1 1 1 1 1 1 1 1 1 1 1 1 1 1];
>> M=F*F'
M =
   1.0e+004 *
      1.6819      0.0428
      0.0428      0.0014
>> N=F*X'
N =
   1.0e+005 *
      2.4278
      0.0794
>> O=F*Y'
O =
   1.0e+005 *
      2.2370
      0.0731
>> A=(M\N)'
```

```
A =
    0.0100 566.6480
>> B=(M\O)'
B =
```

$$0.0468\ 520.6180 \tag{4}$$

The center line equation of the ancient pagoda in 1986.

$$\begin{cases} x = 0.01z + 566.648 \\ y = 0.0468z + 520.618 \end{cases} \tag{5}$$

Symmetric equation: $\frac{x-566.648}{0.01} = \frac{y-520.6180}{0.0468} = \frac{z}{1}$; Direction vector $\overrightarrow{s_1} = (0.01,0.046,1)$

The calculation process of the equation in 1996, 2009 and 2011 is the same as that in 1986 and will not be shown here. Table 3 shows that from 1986 to 1996, the ancient pagoda tilted. From 1996 to 2009, the pagoda was tilted; From 2009 to 2011, there was no tilting. From the annual average Angle difference, it can be seen that the tilting of the ancient pagoda slowed down year by year.

Table. 3 Annual mean Angle difference analysis

Year Differential settlement	1986-1996	1996-2009	2009-2011
$\Delta\theta$	0.018277^o	0.189678^o	0
Annual mean Angle difference	0.0018277^o	0.014591^o	0

3.4. Data processing and analysis of Question 4

Based on the coordinate data of the ancient tower center calculated in the first question, the direction vectors of the center coordinates of each layer of the ancient tower and the center coordinates of the previous layer in 1986, 1996, 2009 and 20114 were calculated respectively. Then, the offset Angle of the center of each layer was calculated according to the cross-product calculation, and the bending degree of the ancient tower was judged from the change of the offset Angle.

$$\vec{S}_{i(i+1)} = (x_{i+1} - x_i, y_{i+1} - y_i, z_{i+1} - z_i),\ \ cos\,\theta_i = < \overrightarrow{s_{i(i+1)}}, \overrightarrow{s_{(i+1)(i+2)}} > = \frac{|x_i x_{i+1} + y_i y_{i+1} + z_i z_{i+1}|}{\sqrt{x_i{}^2 + y_i{}^2 + z_i{}^2}\sqrt{x_{i+1}{}^2 + y_{i+1}{}^2 + z_{i+1}{}^2}}$$

$$\theta_i = arccos < \overrightarrow{s_{i(i+1)}}, \overrightarrow{s_{(i+1)(i+2)}} > \tag{6}$$

Vector coordinates and offset angles are shown in Table 4 below.

Table. 4 Vector coordinates and offset angles

	1986	1996	2009	2011
$\vec{S}_{12}$	(0.0548,6.6684,5.532881)	(0.0556, -0.0428,5.531625)	(0.0372,-0.0322,5.5445)	(0.0373,-0.0324,5.52725)
$\vec{S}_{23}$	(0.0539,-0.0412,5.434994)	(0.0546,-0.0418,5.436075)	(0.0361,-0.0309,5.42325)	(0.0362,-0.0302,5.13637)
$\vec{S}_{34}$	(0.0426,0.5322,4.323)	(0.0432,-0.0334,4.32442)	(0.0292,-0.0252,4.3375)	(0.0293,-0.0261,4.32513)
$\vec{S}_{45}$	(0.0460,-0.0353,4.64225)	(0.0466,-0.0359,4.64088)	(0.0310,-0.0266,4.63962)	(0.0313,-0.0267,4.65187)
$\vec{S}_{56}$	(0.0463,-0.0347,4.51463)	(0.0469,-0.0353,4.5135)	(0.0868,-0.0524,4.50163)	(0.0868,-0.0525,4.50063)
$\vec{S}_{67}$	(0.0383,-0.0163,3.60399)	(0.0387,-0.0168,3.53725)	(0.0321,-0.0219,3.61363)	(0.0331,-0.0220,3.6125)
$\vec{S}_{78}$	(0.0375,-0.0157,3.51176)	(0.0379,-0.0161,3.51313)	(0.0513,-0.0326,3.51524)	(0.0503,-0.0327,3.519620)
$\vec{S}_{89}$	(0.075,-0.016.,3.44912)	(0.0381,-0.0167,3.45462)	(0.0511,-0.0331,3.50388)	(0.0513,-0.0331,3.48563)
$\vec{S}_{910}$	(0.0352,-0.0140,3.37212)	(0.0355,-0.0142,3.36763)	(0.0554,-0.0529,3.31738)	(0.0556,-0.0521,3.40775)
$\vec{S}_{1011}$	(0.0476,-0.0394,4.25901)	(0.0482,-0.0399,4.26774)	(0.0434,-0.0388,4.27062)	(0.0429,-0.0401,4.19614)
$\vec{S}_{1112}$	(0.0472,-0.0394,4.28074)	(0.0476,-0.0398,4.272)	(0.0420,-0.0389,4.25925)	(0.0427,-0.0289,4.26694)
$\vec{S}_{1213}$	(-0.0667,0.3567,4.12242)	(-0.0666,0.3565,4.12263)	(0.0492,-0.0445,4.12775)	(0.0488,-0.0545,4.12004)
$\vec{S}_{1314}$	(0.1622,-0.4966,2.28896)	(0.1632,-0.4973,2.28975)	(0.0644,-0.0567,2.27225)	(0.0653,-0.0566,2.27388)

Based on the above calculation, it can be seen from Table 5 that most of the offset angles are in the upper and lower levels, while the Angle offset of each layer is in the upper and lower levels, so the bending degree of the ancient tower is small and basically in a stable state.

Table. 5 The Ghouta shifts in Angle every year.

	1986	1996	2009	2011
θ_1	50.74987	0.002779	0.006984	0.018304
θ_2	7.452274	0.003673	0.007742	0.028248
θ_3	7.453624	0.003025	0.005252	0.017097
θ_4	0.020402	0.02063	0.797135	0.795374
θ_5	0.182477	0.178766	0.675971	0.661983
θ_6	0.004188	0.012945	0.375364	0.346315
θ_7	0.633943	0.019931	0.009913	0.027101
θ_8	0.648212	0.045066	0.391505	0.344176
θ_9	0.295175	0.297188	0.5428	0.479115
θ_{10}	0.009014	0.008891	0.017491	0.159939
θ_{11}	5.689151	5.482908	0.151035	0.384462
θ_{12}	17.85606	14.84307	1.241939	1.168089

4. Conclusions

4.1. Model to promote

(1) Due to the error in the hypothesis, many factors inconsistent with the reality are simplified in real life. Therefore, in order to obtain accurate data, a better scheme can be established to track and measure the ancient pagoda for a long time, and the measured data can be used to optimize the scheme design again by using the method of this model.

(2) If the orientation of the measurement point is given, then the orientation of bending and torsion can be better analyzed, so as to better analyze the changing trend of the ancient pagoda.

(3) This model can study the tilt and torsion of other buildings, which has certain research significance for the protection of ancient buildings.

(4) In the establishment of the center coordinate point model, there is a certain fitting error, which has a certain impact on the bending and twisting Angle.

4.2. Evaluation of the model

(1) The established model can be closely related to the reality and solve the proposed problem in combination with the actual situation, which has practicality, universality and popularization.

(2) In the process of data processing, due to human factors, there will be certain errors, and the data is limited, cannot meet all requirements.

(3) Due to the fact that no accurate natural conditions are given in the attached data, the estimation can only be made in a specific environment.

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