Plant Community Changes Based on The Lotka-Volterra Model

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Abstract. Globally, ecosystem functions are threatened by repeated droughts and declining plant diversity. Due to frequent droughts, the species in the plant community show different adaptions towards the extreme weather conditions. The aim of this report is to explore how the plant community changes over time. Considering the growth law of population, establish the model of precipitation and plant growth rate based on the Logistic function. Moreover, considering the competition and interaction between populations, this paper chooses the Lotka-Volterra model and adapt it, then use the improved Euler method to obtain numerical solutions of these nonlinear ordinary differential equations of plant population over time under the interaction of multiple populations. The observations are obtained: For the population of a plant community, when the rainy season intersects with droughts, the decline rate of the population at the inflection point slows down, the continuity of population change is interrupted. And the plant population shows an increasing trend in the rainy season, and a decreasing trend in droughts.

Keywords: Improved Eulerian method, Lotka -Volterra model, simulation.

1. Introduction

Different species of plants respond to stress in different ways. Grassland is an important ecosystem type, an important national ecological barrier and a green animal husbandry production base. The grasslands play an important role in safeguarding national ecological security, food security and even global ecological balance [1]. Yet grasslands are quite sensitive to drought. The current intensification of human activities affects the global atmospheric cycle pattern, and the frequency of drought events increases significantly. The frequency and severity of droughts vary. Numerous observations have shown that the number of different species plays a role in how plant communities adapt to successive generations of drought cycles. In some communities with only one plant species, the offspring were less resilient to drought than individual plants in communities with four or more species [2]. It is difficult for plants with weak adaptive ability to survive in extreme weather conditions, which will lead to large- scale degradation of temperate grassland and damage of ecosystem function. In other communities with multiple plant species, the number of plants increases interspecific competition, leading to a trend of population increase and then decrease. Therefore, it is necessary to study and research the ecological adaptation mechanism of grassland plants to drought events.

2. Establishment and solution of Lotka-Volterra model

2.1. The structure of the model

In a community, competition is always present. For populations within a community, both intraspecific and interspecific competition exist. Therefore, to predict the behavior of a plant community over time during various irregular weather cycles, we need to model the mechanisms of competition among different populations and the relationship between climate change and the growth rates of each plant population in a grassland ecosystem [3]. For the interspecific competition, this paper first applied the Lotka-Volterra competition model to construct the following set of nonlinear ordinary differential equations for the competition relationship between two different populations.

$$\begin{cases} \frac{dN_1}{dt} = r_1 N_1 \left(1 - \frac{N_1}{K_1} - \frac{a_{12} N_2}{K_2}\right) \\ \frac{dN_2}{dt} = r_2 N_2 \left(1 - \frac{N_2}{K_2} - \frac{a_{21} N_1}{K_1}\right) \end{cases} \tag{1}$$

And it is unrealistic that only two populations exist in a community, so for interspecific competition among n populations in a community, we expand the set of the above two equations to n equations and construct a multiple intergroup competition-symbiosis model.

$$\begin{cases} \frac{dN_1}{dt} = r_1 N_1 \left(1 - \frac{N_1}{K_1} - \sum_{k=1,k\neq i}^{n} \frac{a_{1k} N_k}{K_1}\right) \\ \frac{dN_2}{dt} = r_2 N_2 \left(1 - \frac{N_2}{K_2} - \sum_{k=1,k\neq i}^{n} \frac{a_{2k} N_k}{K_2}\right) \\ \vdots \\ \frac{dN_n}{dt} = r_n N_n \left(1 - \frac{N_n}{K_n} - \sum_{k=1,k\neq i}^{n} \frac{a_{nk} N_k}{K_n}\right) \end{cases} \tag{2}$$

In addition, due to limited resources, the formation of grassland ecosystems is closely related to the pattern of climate change over time, and the growth of plants is often influenced by two major climatic factors, temperature, pollution and precipitation[4]. Plants growing in grasslands have various adaptation strategies to the grassland climate and have certain adaptability to the fluctuation of precipitation. With the increase of precipitation, the natural growth rate of plants gradually increases and the growth rate gradually slows down. In order to investigate the change in the community over time, this paper establish the relationship between precipitation and the natural growth rate of plant populations within the community. The growth rate of plants will not increase indefinitely with the increase of precipitation. When the precipitation reaches a certain value, the natural growth rate will reach a maximum and then will not increase anymore[5]. Even, in extreme cases, when there is too much precipitation, the plant growth rate may tend to decrease. Based on this characteristic, model the relationship between the two using the logistic equation.

$$r(h) = \frac{R}{1+e^{-k(h-h_0)}} \tag{3}$$

When the precipitation is 0, according to this equation, the corresponding natural growth rate is R/2. This shows a high growth rate of plants even under extreme drought conditions. This obviously does not match the actual situation. In fact, when there is no precipitation at all, plants lose the opportunity to take water from the soil and can only survive by consuming their own stored nutrients, and their growth rate is infinitely close to zero at that time[6]. Therefore, this paper improves this model as follows.

$$r_i(t) = \frac{R_i}{1+e^{-k_i h(t)}} - \frac{R_i}{2} \tag{4}$$

Substitute the expression for the natural growth rate into equation 1, obtain the equation for the population change over time.

$$\frac{dN_i(t)}{dt} = \left(\frac{R_i}{1+e^{-k_i h(t)}} - \frac{R_i}{2}\right) N_i(t) \left(1 - \frac{N_i(t)}{K_i} - \sum_{k=1,k\neq i}^{n} \frac{a_{1k} N_k}{K_1}\right) \tag{5}$$

To predict how a plant community changes over time as it is exposed to various irregular weather cycles, this paper evaluates the biodiversity of a region. The Biodiversity Index is used to evaluate the biodiversity of a region, which includes Shannon-Wiener Index(H), Simpson Index(D), Pielou's Evenness Index(J). S represents the number of species in the region and Pi represents the relative importance of each species in the environment (in this paper, the ratio of each species' biomass to the total biomass)[7].

$$P_i = \frac{N_i}{\sum_{i=1}^{n} N_i} \tag{6}$$

$$H = -\sum_{i=1}^{S} P_i \ln P_i \tag{7}$$

$$D = 1 - \sum_{i=1}^{S} P_i^2(t) \tag{8}$$

$$J = \frac{H}{\ln(S)} \tag{9}$$

2.2. Numerical solution

Find the numerical solution to solve the system of differential equations, and the most concise way to find the numerical solution is Euler's method.

$$\begin{cases} y'(t) = f(t, y) \\ y(0) = y_0 \end{cases} \tag{10}$$

Divide the time equally, the length of the interval is denoted as h.

$$\begin{cases} y_i = y(t_i) \\ y_{i+1} = y_i + hf(t_i, y_i) \end{cases} \tag{11}$$

The differential equation is:

$$\begin{cases} y_{n+1} = y_n + hf(x_n, y_n) \\ y(0) = a \end{cases} \tag{12}$$

Since the Eulerian method has a large significant error, the improved Eulerian method is used to solve the equation. Use the explicit Euler formular as a prediction.

$$\bar{y}_{n+1} = y_n + hf(x_n, y_n) \tag{13}$$

To facilitate programming calculations, the modified Euler formula is written as:

$$\begin{cases} y_p = y_n + hf(x_n, y_n) \\ y_c = y_n + hf(x_{n+1}, y_n) \\ y_{n+1} = \frac{1}{2}(y_n + y_n) \end{cases} \tag{14}$$

3. Analysis of results

3.1. Establishment and visualization of the simulation

The relation between growth rate and precipitation is shown in the figures as followed. The changes in rate of growth for different species are consistent with logistic regression when the number(n) of species are different.

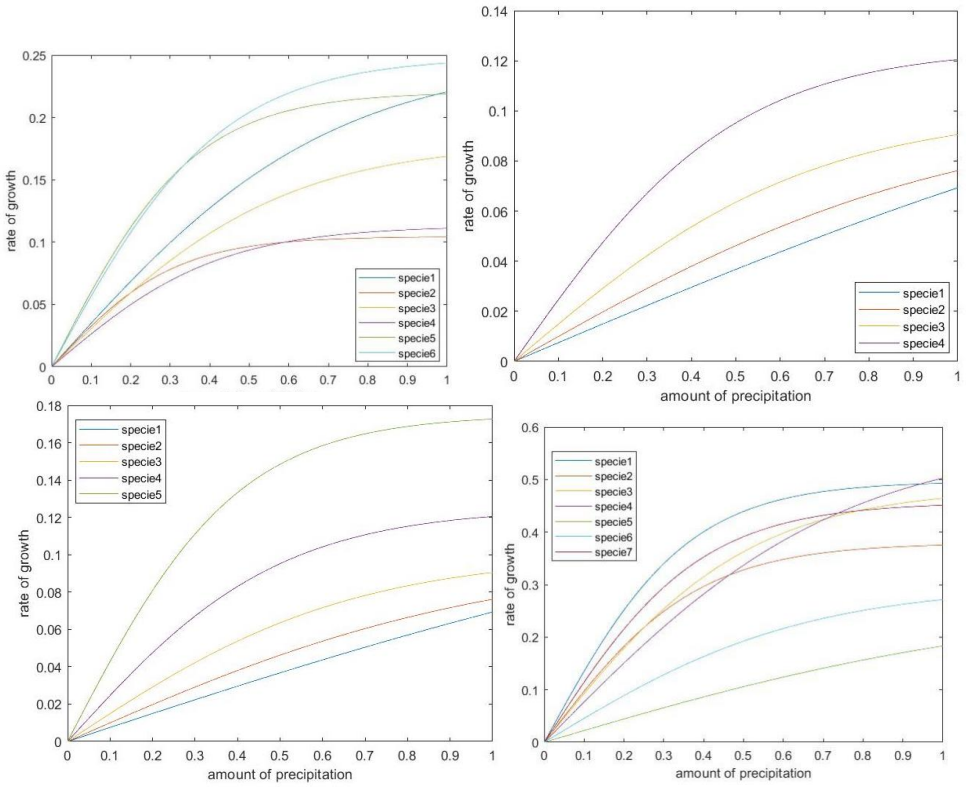


Figure 1: The relationship between rate of growth and amount of precipitation

To plot the figure of the population of each species with respect to time, this paper set the rainfall as a segmented function, dividing the year into a dry season and a rainy season. The starting point of time is January 1 of each year. The following Figure 2 showed the change in precipitation over time. Figure 2 simulates the precipitation of the grassland in a year, taking the zero point of the horizontal axis as the starting point of the year. The figure simulates the precipitation conditions that the grassland precipitation is mainly concentrated in April to August.

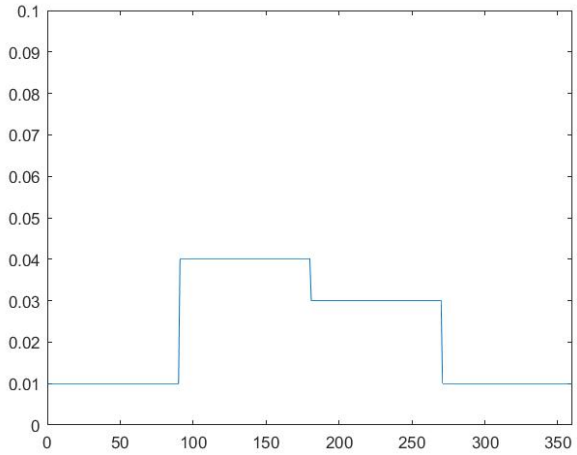


Figure 2: Precipitation versus Time

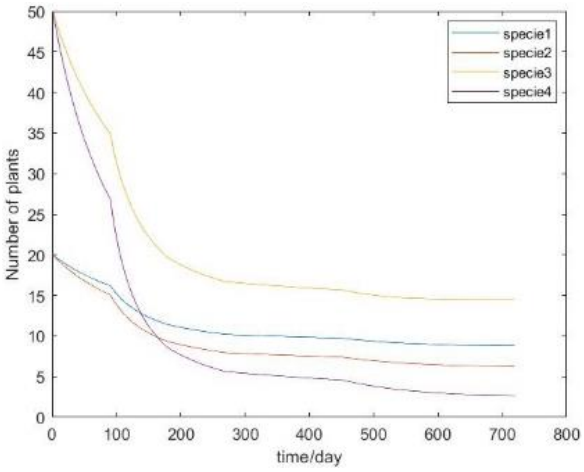


Figure 3: Population growth curves of 4 species

3.2. Analysis of simulation results

Precipitation determines the water condition of vegetation growth and is an effective index to determine vegetation productivity [8]. The growth, development and yield formation of natural herbage are closely related to precipitation factors, and vegetation growth conditions are different under different precipitation conditions.

In Figure 3, the populations of all species are declining. When the dry season ended and entered the first rainy season, the rate of decline in the population decreased significantly. Subsequently, the species populations gradually stabilized as the plants gradually adapted to the environment. During the gradual stabilization of the species populations, there are some organisms whose numbers increased significantly, some animals lost in the competition, and some others whose populations are maintained at an equilibrium level.

Combined with Figure 2 and Figure 3, grassland growth has an obvious lag to precipitation, generally within 50 to 60 days. Different types of grasslands have different lag times, but there is no significant difference [9]. At the same time, the code numerical analysis shows that the lag time of grassland vegetation response to precipitation in different time periods from April to August is 64, 80, 40, 40, 56 and 64 days, respectively. In particular, the response of grassland vegetation growth to precipitation change in June and July is the most sensitive.

4. Conclusions

For the population of a plant community, when the rainy season intersects with droughts, the decline rate of the population at the inflection point slows down, the continuity of population change is interrupted. And the plant population shows an increasing trend in the rainy season, and a decreasing trend in droughts. The growth rate of plant population is affected by precipitation.Both intraspecific and interspecific competition limit the infinite growth of plants in order to achieve a state of equilibrium. Without considering the pollution of grassland vegetation, the plant growth rate is sensitive to the condition of precipitation [10]. According to the Lotka-Volterra model, there are approximately 4 species in the community will lead to the most beneficial community.When the degree of interspecific and intraspecific competition was relatively stable, precipitation became the main factor affecting plant growth rate.

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