Experimental Verification of Quantum Entanglement: Insights from EPR Paradox and Bell Tests

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Abstract. Quantum entanglement is an essential principle in quantum mechanics, illustrating the relationship between two or more spatially separated particles.This correlation has been experimentally proven through various techniques, including the entangled photon experiment. Furthermore, the breach of Bell's inequality has confirmed that entanglement represents a genuine physical occurrence, which cannot be accounted for by local hidden variables, thus highlighting the peculiar characteristics of quantum mechanics. This article aims to introduce quantum entanglement and Bell's inequality, and to discuss the experiments conducted by Alain Aspect, John Clauser, and Anton Zeilinger, which paved the way for the development of quantum information science. Additionally, this paper will explore the potential applications of quantum technology in communication, cryptography, and computing, which rely on the exceptional properties of entanglement.

Keywords: Quantum Entanglement; Bell inequality; Hidden variable theory; Quantum information.

1. Introduction

Within the domain of quantum physics, entanglement among two or more particles leads to an intriguing phenomenon. The state of an individual particle is inextricably linked to the states of the other entangled particles. This phenomenon is known as quantum entanglement. Measuring one particle instantaneously influences the state of another particle, irrespective of the separation between them. This seemingly violates Einstein's theory, as it suggests that the state of particles on one side of the universe could be changed by modifying the state of particles on the opposite side. This phenomenon remains inexplicable in classical physics and the macroscopic world, leading scientists to believe that it is an inherent characteristic of quantum mechanics [1]. This has sparked inquiries into the existence of hidden variables that have yet to be discovered, underlying quantum mechanics. In 1964, John Bell formulated the famous Bell inequality [2], but subsequent experiments, such as the one conducted by John Clauser [4], have violated the inequality. Although these experiments have some limitations, such as potential causal relationships between detectors, which could affect the results due to the small spatiotemporal intervals between detectors. To address these limitations, Alain Aspect proposed an experimental plan in 1976[5], which resolved the issues in the Clauser's experiment [4]. Anton Zeilinger further refined the details of quantum entanglement experiments. The results of these experiments have been in opposition to the predictions made by the Bell inequality, confirming the existence of nonlocality in quantum entanglement.

2. EPR Paradox and Bell Inequality

2.1. EPR Paradox

Quantum entanglement is a unique quantum phenomenon that arises when the quantum states of two or more particles interact for a certain duration of time. In this state, even if the particles are far apart, changes in the measurement results of one particle will immediately affect the other particle. This phenomenon fundamentally challenges the intuition and principles of classical physics, as it appears to exceed the information transmission speed limit of light [6]. The wave function between particles through interaction (ψ Function), which means they are no longer separate particles. For example, two particles moving in a straight line can be separated, so here is the wave function:

$$\psi(a,b) = \psi(a) \otimes \phi(b), \tag{1}$$

a and b are the designations of the two particles, while $\psi(a)$ and $\phi(b)$ denote their corresponding wave functions.

The general form of the pure state of two entangled systems is

$$\psi(a,b) = \sum_{i=1}^{n} c_i \psi_i(a) \otimes \phi_i(b), \tag{2}$$

Where c_i is a complex number, and n represents the dimension of a Hilbert space. If it cannot be represented in the form of (1), then it is considered an entangled state. In 1935, Einstein with his teamate introduced the EPR paradox thought experiment to highlight the counterintuitive and irrational nature of this phenomenon [1].

The EPR experiment involves creating an entangled state between two particles and then measuring the particular properties of one particle, such as spin. However, when measuring the spin of one particle, the corresponding property of the other particle will also change, and this change is instantaneous, illustrating the non-local nature of quantum entanglement. This phenomenon seems to have a faster information transmission speed than light, because this effect is instantaneous, just as the connection between two particles seems to be established instantaneously.

The function can be represented as:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle) \tag{3}$$

Where $|\uparrow\rangle$ represent the particle's spin up and $|\downarrow\rangle$ represent the particle's spin down. When measuring along the z-axis in an entangled system of particles A and B, the function collapses into a particular form:

$$|\psi\rangle_A = |\uparrow\rangle_A \otimes |\downarrow\rangle_B, \tag{4}$$

If Alice measures the particle and obtains the result that the particle's spin direction is up, then Bob's (B) particle will definitely spin down along the z-axis when measured. The direction of B's spin appears to be created instantaneously and is related to A, and the correlation between their measurement results can be calculated according to quantum mechanics. In experiments, verification of Bell's inequalities demonstrates the fundamental distinction between quantum entanglement and classical physics [2]. These experiments show that the connection between entangled particles cannot be explained by the concept of local realism, which is what Einstein believed [6]. Local realism is the idea that physical phenomena can be explained by local, pre-determined causal relationships without involving faster-than-light information transmission.

Bell's inequalities were proposed by John Bell in 1964 [2] to test whether quantum entanglement phenomena conform to local realism. Bell's inequalities describe the upper limit of the correlation between the measurement results of two entangled particles. If the measurement results violate this upper limit, local realism does not hold. The CHSH inequality, expressed as $-2 \leq S \leq 2$, The parameter S is related to the correlation between the measurement results of the particles. If S falls within a specific range, then local realism is upheld. Conversely, if S is outside this range, local realism is not upheld.

The results of many experiments do not conform to Bell's inequality description, such as the 1982 experiment by Aspect *et al*. [3]. These experiments confirm the non-locality of quantum entanglement phenomena, i.e., the correlation between entangled particles does not depend on the distance between them, and the measurement results are not pre-determined. This means that Einstein's "hidden variables" theory cannot explain quantum entanglement phenomena.

2.2. The inequality proposed by Bell.

Regarding the fundamental aspects of quantum mechanics, a significant number of physicists lean towards Bohr's perspective. However, there are still researchers exploring the underpinnings and interpretations of quantum mechanics. In 1964, Bell introduced Bell's inequality [2], a principle relevant to systems adhering to local realism. Bell discovered that quantum mechanics' forecasts conflict with this inequality under specific experimental circumstances. However, there were some unreasonable assumptions about the detectors in Bell's derivation of the inequality, making it difficult to verify experimentally [3]. In 1969, Clauser with his teamate solved the difficulty of experimentally verifying the inequality [4]. They proposed the CHSH inequality [8]. The advantage of this inequality is that it can be verified by measuring entangled photons using existing technology [4].

Freedman and Clauser initially confirmed this experimentally [8]. In their experiment, each photon interacts with a dual-channel polarizer, with the direction determined by Alice and Bob. Single-photon detectors D+ and D- detect the signals generated by each channel. The coincident count rate is calculated to obtain the experimental data:

$$E(a,b) = \frac{N_{++} - N_{\pm} - N_{\mp} + N_{--}}{N_{++} + N_{\pm} + N_{\mp} + N_{--}}, \tag{5}$$

Where $N_{i,j}(i,j = +,-)$ correspond to the synchronous event counts recorded by Alice and Bob's detectors. The purpose of the experiment is to test the CHSH inequality, which is related to the EPR thought experiment as follows:

Alice can perform two distinct experiments, A1 and A2 (such as measuring the spin in different directions, a1 and a2), while Bob can conduct either experiment B1 or B2. According to local realism, the outcomes for each quantum system should be predetermined, even without performing any measurements. In contrast to local realism, quantum mechanics cannot predict measurement results. Based on the possible measurement results represented by L(A1), L(B2), etc., the CHSH inequality [8] can be derived:

$$|E(A1,B1) + E(A1,B2) + E(A2,B1) - E(A2,B2)| < 2. \tag{6}$$

To illustrate more specifically the contradiction between Bell's inequality and quantum mechanical predictions, let's consider a pair of entangled photons in the Bell state $|\psi-\rangle$. Calculate the correlation $E(a,b)$ for a given measurement direction, where a and b are the measurement direction vectors for Alice and Bob, respectively. For $|\psi-\rangle$, that $E(a,b) = -a \cdot b$.

Now, let's consider the following four sets of measurement directions: $A1 = (1,0)$, $A2 = (0,1)$, $B1 = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$, $B2 = (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$. Substituting the expression for E(a,b) and plugging the results into the CHSH inequality: $|E(A1,B1) + E(A1,B2) + E(A2,B1) - E(A2,B2)| \leq 2$ and $|S| = \left|-1/\sqrt{2} + 1/\sqrt{2} + 1/\sqrt{2} - \left(-1/\sqrt{2}\right)\right| = 2\sqrt{2}$. In this case, the result predicted by quantum mechanics contradicts the CHSH inequality.

3. Experimental Falsification of Bell's Inequality

Since Bell's inequality was proposed, many experiments have successfully verified the contradiction between quantum mechanical predictions and Bell's inequality [2], like Freedman and Clauser's experience [7]. These experimental results support the non-locality of quantum mechanics and conflict with the realist viewpoint. However, these experiments cannot completely rule out all potential influences. For example, there may exist "hidden variable" theories that still adhere to realism but can explain these experimental results.

To address this issue, physicists continue to conduct more precise and rigorous experiments to test Bell's inequality. One noteworthy experiment is the Aspect experiment, which was conducted in 1982 [9]. This experiment used a more efficient entangled photon source and more reliable detector technology, and the results still support the non-locality of quantum mechanics and contradict Bell's

inequality. Finally, Anton Zeilinger and his collaborators conducted the first quantum interference experiments with neutrons, atoms, and large molecules, achieving a test of quantum nonlocality without the locality loophole and the detection efficiency loophole[10].

3.1. Freedman and Clauser's Experiment

In Freedman and Clauser's experiment [7]. The goal of this experiment was to test whether the correlation between entangled particles described by quantum mechanics was consistent with local hidden variable theories. The experiment used calcium atoms in their ground state. The calcium atoms were excited to the 6^1P_1state by a hydrogen arc lamp with a wavelength of 227.5 nm, and then underwent a transition from 6^1P_1 to 6^1S_0. During this process, the atoms emitted two photons, which were entangled. The energy level diagram of calcium is shown in Fig. 1.

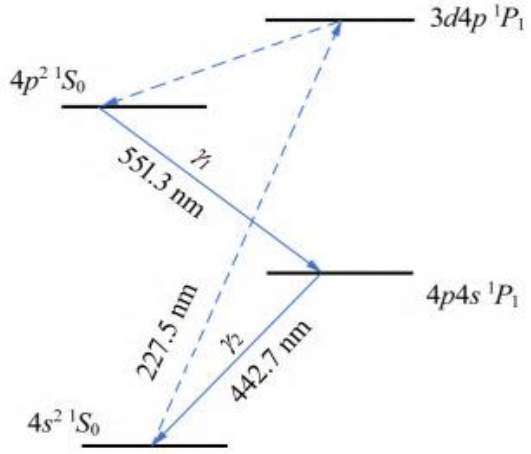


Fig. 1 Energy level diagram of calcium, with Reference [7]

The experiment used two antiparallel photon detectors, Alice and Bob, to detect the photons emitted by the atoms. The distance between the two detectors was 5 meters. They measured the linear polarization correlation of the two photons (γ1 and γ2). Alice and Bob's coincident counting rate for detecting anti-parallel photons is given by R = (1/2)(cos(φ))^2.

In the initial experiment, the polarizers were positioned at angles of 0° and 90° relative to one another. Assuming that all incident photons have an equal probability of being detected, irrespective of whether they pass through a polarizer or not, the above assumption can be constrained using an inequality on the coincident counting rate $-1 \leq \Delta(\varphi) \leq 0$, where

$$\Delta(\varphi) = 3R(\varphi)/R_0 - R(3\varphi)/R_0 - (R_1 + R_2)/R_0. \tag{7}$$

Based on the experimentally measurable quantities, rewrote the inequality as:

$$\left|\frac{R(22.5°)}{R_0} - \frac{R(67.5°)}{R_0}\right| - \frac{1}{4} \leq 0 \tag{8}$$

In this case, R(22.5°) and R(67.5°) correspond to the coincident count rates when the polarizer angles are set at 22.5° and 67.5°, respectively [7].

The experimental results showed that the measured values violated the CHSH inequality, implying that local hidden variable theories could not explain the phenomenon of quantum entanglement. Freedman and Clauser's experiment laid the foundation for subsequent experiments that further confirmed the quantum entanglement phenomenon, supporting the non-locality of quantum mechanics.

3.2. The Aspect Experiment

In 1969, John Clauser proposed an experimental scheme for testing local hidden variable theories in quantum mechanics. It was also a test of Bell's inequalities. Although Clauser's experiment was an important study at the time, subsequent research revealed some potential loopholes in the experiment.

In 1976, the Aspect experiment implemented an acousto-optic device that switched photons between two different branches of the experimental setup in 20 ns [3], which for the first time avoided the "loophole" of locality. Then in 1981-1982, Aspect, Philippe Grangier, Gerard Roger, and Jean Dalibard used more advanced instruments and techniques to conduct this experiment [3,8,9]. The experiment [3], depicted in Figure 2, utilized a calcium cascade light source to generate entangled photon pairs, which initially arrived at optical switches C1 and C2. The photons could be either directed towards polarizers and detectors PM1 and PM2 or reflected towards a separate set of polarizers and detectors, PM1' and PM2'. The channel switching occurred approximately every 10 nanoseconds, and the polarizers were 12 meters apart. The optical switch resulted from the interference of ultrasonic standing waves created by two electro-acoustic transducers moving in opposing directions. This allowed for rapid changes in polarizer orientation. With a 6-meter distance between the calcium photon source and each polarizer, the time needed to alter the polarizer's orientation had to be less than 20 ns.

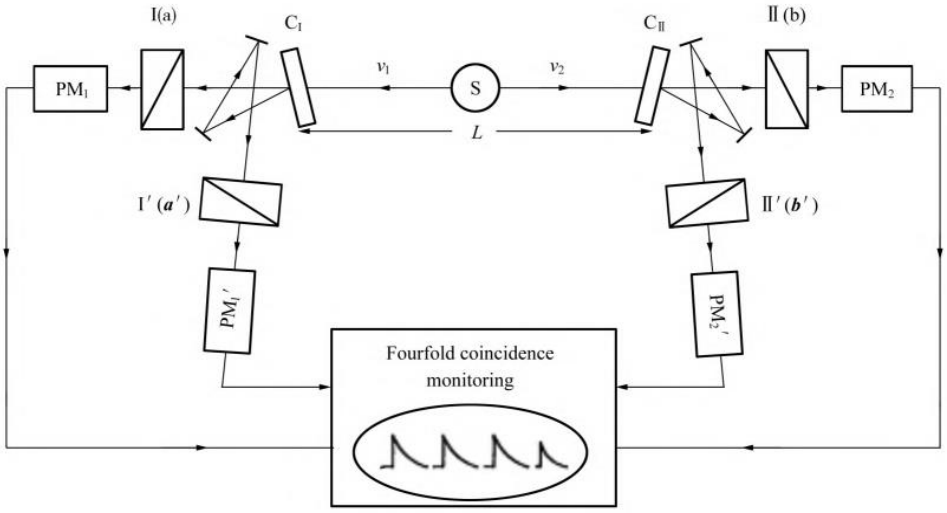


Fig. 2 Aspect experimental design diagram, with Reference [3]

The Shimony-Holt inequalities $-1 \leq S \leq 0$ with

$$S = \frac{N(\vec{a},\vec{b})}{N(\infty,\infty)} - \frac{N(\vec{a},\overrightarrow{b'})}{N(\infty,\infty')} + \frac{N(\overrightarrow{a'},\vec{b})}{N(\infty',\infty)} + \frac{N(\overrightarrow{a'},\overrightarrow{b'})}{N(\infty',\infty')} - \frac{N(\overrightarrow{a'},\infty)}{N(\infty',\infty)} - \frac{N(\infty,\vec{b})}{N(\infty,\infty)}. \quad (9)$$

Two experiments were conducted to test Bell's inequalities. In each experiment, directions were chosen that would produce the maximum conflict between the predictions of quantum mechanics and Bell's inequalities $[(a,b) = (b,a'),(a',b') = 22.5°;\ (a,b') = 67.5°]$. The average value of the two experiments was $S = 0.101 \pm 0.020$, which clearly violated Bell's inequality $S \leq 0$ (five standard deviations) and was almost consistent with the value predicted by quantum mechanics, 0.112.

Nonetheless, the experiment had its constraints, as the limited distance between the polarizers did not provide a genuinely random setting. In 1998, Zeilinger's group carried out an experiment with observers 400 meters apart, testing the inequality under stringent locality conditions [10]. Despite these adjustments, the outcome still demonstrated a violation.

4. Applications of Quantum Entanglement

4.1. Quantum Teleportation

Quantum teleportation is a significant application in the realm of quantum information that enables the transmission of a quantum state over large distances without directly transferring the quantum system carrying that state. The mathematical feasibility of quantum teleportation can be demonstrated [11]. This will be explained by briefly reviewing the relevant formulas.

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \tag{10}$$

Here, $|00\rangle$ and $|11\rangle$ represent the ground and excited states of the two quantum bits, respectively. Next, consider the unknown quantum state Alice possesses $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ where α and β are complex numbers, satisfying $|\alpha|^2 + |\beta|^2 = 1$.

Alice performs a Bell state measurement on this unknown quantum state with the EPR entangled particle she maintains. During this process, the quantum states become entangled:

$$|\Psi_{total}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{AB} + (\alpha|0\rangle \otimes \beta|1\rangle)_C. \tag{11}$$

Through the Bell measurement, Alice obtains a classical bit pair (x, y), which needs to be sent to Bob. Then represent the entangled state after the measurement with the following formula:

$$|\Psi_{total}\rangle = U_y V_x(\alpha|0\rangle + \beta|1\rangle)_B \tag{12}$$

Here, U_y and V_x are quantum operations defined according to the classical bits (y, x) sent by Alice. Bob performs the corresponding quantum operations based on the received classical bit information. After these operations, the particle (Bob) will acquire the same state as the original unknown quantum state. The feasibility of this process relies mainly on quantum entanglement (EPR pairs) and the transmission of classical information.

4.2. Quantum Cryptography

Quantum Key Distribution is an essential application of quantum cryptography, aiming to securely transmit keys through a quantum channel, thereby achieving encrypted communication. A famous QKD protocol is the BB84 protocol, proposed by Charles H. Bennett and Gilles Brassard in 1984 [12]. The core idea of the BB84 protocol is to utilize the no-cloning property of quantum bits (such as photons) and the irreversibility of single measurements to achieve unconditionally secure key distribution. In the BB84 protocol, Alice and Bob each choose a set of orthogonal bases. Alice then randomly generates a string of quantum bits and encodes them using randomly chosen bases. Afterward, Alice sends the encoded quantum bits to Bob, who also randomly chooses a set of bases to measure the received quantum bits.

During the transmission process, if there is an eavesdropper (Eve), she cannot copy the quantum bits without causing errors due to the quantum no-cloning theorem. Simultaneously, because of the irreversibility of quantum measurement, Eve cannot obtain information without affecting the quantum bits. Therefore, in QKD, any eavesdropping attempt will lead to a significant increase in the error rate. After Alice and Bob complete the transmission and measurement of quantum bits, they compare a portion of the measurement results through a public channel to detect potential eavesdropping. If they detect a high error rate, there might be an eavesdropper, and they need to restart the key distribution process. Otherwise, they can generate a secure key through subsequent information processing steps (such as information reconciliation and privacy amplification) for encrypted communication. In summary, Quantum Key Distribution takes advantage of quantum entanglement and the properties of quantum bits to achieve unconditionally secure key distribution, offering broad application prospects in quantum communication and quantum cryptography fields.

5. Conclusion

Over the past few decades, quantum entanglement has evolved from a mere thought experiment and theoretical discussion to a foundation for practical applications. From the initial questioning by Einstein and his colleagues to the proposal of Bell's inequality, researchers have been able to overcome traditional physical limitations through quantum entanglement, achieving more powerful computing capabilities, secure information transmission, and unconditional security. However,

despite the significant progress made in both theory and practice of quantum entanglement, many challenges and questions still need to be addressed. Future research will require breakthroughs in the generation, manipulation, and measurement of quantum entanglement, as well as overcoming technical issues related to practical applications, such as noise suppression, efficient use of entanglement resources, and scaling up to macroscopic dimensions.

In conclusion, as one of the core characteristics of quantum mechanics, quantum entanglement has already demonstrated its immense potential in multiple fields. As scientists deepen their understanding of quantum entanglement, more breakthroughs in the development of quantum technologies can be expected in the future. Only by continuing to explore the fundamental principles of quantum entanglement, improving experimental techniques, and discovering new application scenarios, can the true revolution of quantum science be ultimately realized.

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