

Plant Community Prediction Based on Lotka-Volterra Model and Statistical Analysis

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Abstract. The change process of plant communities is an important issue involving botany, environmental science and system science. This paper mainly studies the state changes of plant communities under irregular weather cycles, and establishes a dynamic model of species interaction based on Lotka-Volterra equations. The Euler method, statistical analysis are used to solve the problem. we extend the basic Lotka-Volterra model to the generalized Lotka-Volterra model, which can describe the change of a community with multiple species. Based on this, by introducing parameters such as drought intensity, species sensitivity to drought, pollution level and habitat reduction into the equations, a model for predicting the change of plant community status under the influence of irregular weather cycles is obtained. The statistical method is creatively used to do Sensitivity analysis for the model. The results show that: (1) pollution and habitat reduction do not affect the advantages of highly drought-resistant plants, but extend the gain range of species number on plant community stability. (2) the influence of some negative factors on the gain interval is the comprehensive result of the influence of two opposite factors, competition intensity and species diversity.

Keywords: Phytocoenosis, Irregular Weather, Lotka-Volterra Model, Statistical Analysis.

1. Introduction

The Earth is rich in natural resources and has a high degree of ecosystem diversity that provide a suitable environment for the survival of different organisms. As a result, not only humans, but also other beings thrive on this planet and make up our colorful world. However, in recent years, the speed of global warming has accelerated obviously^[1], and extreme weather such as drought has become more frequent than in the past^[2]. Worldwide, in 2019, there were 396 disasters that killed 11,755 people, affected 95 million others, and cost nearly US\$130 billion. In the northern areas of United States, the warm extremes are increasing on all of the timescales in the past few years^[3]; Groisman et al.^[4] show that the frequency of freezing rain in northern North America has increased by approximately one day per year during the 2005-2014 decade compared to the three preceding decades; the United Kingdom was hit by storms which caused rare winter floods; the New South Wales region, where the Australian city of Sydney is located, was hit by the most severe flooding in 60 years. Not only are the production of these phenomena influenced by human factors^[5], but their impacts are closely related to the living environment of all organisms, including humans, which makes it a meaningful question how to more effectively help plant populations cope with extreme weather. To address this issue, this paper explores the potential factors that enable plant populations to thrive better in extreme weather.

To date, mathematicians and biologists have conducted several studies on the quantitative development of populations and have continuously updated and refined the results of previous efforts. In 1798, Thomas Robert Malthus pointed out that the population grows geometrically under the condition of no limitation of subsistence resources in his famous works *An Essay on the Principle of Population*^[6], and correspondingly developed a simple exponential model,^[7] which is widely regarded as the first principle of population dynamics in the field of population ecology. To reflect the limitation of a certain amount of natural resources on the infinite exponential growth of the

population, Pierre proposed the Logistic Equation^[8] by introducing a parameter to represent environmental resources. Lotka and Volterra^[9] further proposed a new model by introducing the competition coefficients to portray the competitive effects and superiority-disadvantage relationships among different species, which is named after them together. Song^[10] improved the original Lotka-Volterra model by studying predation systems with continuous time lag or disturbance based on Krasnoselskii's immobility point theorem and overlap degree theory. However, we point out that these proposed models are no longer a good fit for today's natural environment or fail to provide help for the environmental problems we face. On the one hand, their models are static and do not take into account environmental dynamic changes over time; on the other hand, their portrayal of the complex and variable natural environment is too coarse, for example, they simply use a constant K to describe natural resources, ignoring refined but important factors such as pollution intensity, habitat reduction, and interspecies competition, which could be dynamically changing over time; finally, they do not provide reasonable and feasible countermeasures for realistic problems based on the predictions of the models. In view of this, in this paper, a more refined and rational dynamic modeling of the environment based on the Lotka-Volterra model is conducted, so as to explore the factors affecting the growth of plant populations under extreme weather. The results are further analyzed using statistical methods to provide feasible suggestions for how to more effectively help plant populations to improve their resistance under extreme weather.

2. Establishment of the Model

In the following process, X_i represent the population size of species i , K_i represent the environmental capacity of species i , r_i represent the population growth rate of species i , α_{ij} represent the influence of the species on the species, α_{ii} represent the interspecific competition of the species, θ_i represent the average influence of irregular weather on an individual in population i , N represent species richness, t_d represent the beginning time of drought, k represent the change rate of drought environment.

2.1. Introduction of Lotka-Volterra Basic Model

Assuming that there are two populations of species 1 and species 2, x_1 and x_2 represent the population size of species 1 and species 2, K_1 and K_2 represent the environmental capacity of species 1 and species 2, r_1 and r_2 represent the population growth rate of species 1 and species 2. According to the Logistic law, there are the following relationships:

$$\frac{dx_1}{dt} = r_1 x_1 \left(1 - \frac{x_1}{K_1}\right) \quad (1)$$

$$\frac{dx_2}{dt} = r_2 x_2 \left(1 - \frac{x_2}{K_2}\right) \quad (2)$$

Among them: $\frac{x_1}{K_1}$ is understood as the space that has been utilized, and $1 - \frac{x_1}{K_1}$ is understood as the unused space. When two species compete or use the same space, the 'used space item' should also add the space occupation of the K_2 population. Then:

$$\frac{dx_1}{dt} = r_1 x_1 \left(1 - \frac{x_1}{K_1} - \alpha_{21} \frac{x_2}{K_2}\right) \quad (3)$$

$$\frac{dx_2}{dt} = r_2 x_2 \left(1 - \frac{x_2}{K_2} - \alpha_{12} \frac{x_1}{K_1}\right) \quad (4)$$

Among them, α_{21} : the competition coefficient between species 2 and species 1, that is, the space occupied by each K_2 individual is equivalent to the space occupied by $\alpha_{21}K_1$ individuals. α_{12} : the competition coefficient between species 1 and species 2, that is, the space occupied by each K_1 individual is equivalent to the space occupied by $\alpha_{12}K_2$ individuals.

Species 1 species 2 cannot inhibit each other: eventually tend to species 1 species 2 The number of both populations does not change, that is:

$$\frac{dx_1}{dt} = r_1 x_1 \left(1 - \frac{x_1}{K_1} - \alpha_{21} \frac{x_2}{K_2} \right) = 0 \quad (5)$$

$$\frac{dx_2}{dt} = r_2 x_2 \left(1 - \frac{x_2}{K_2} - \alpha_{12} \frac{x_1}{K_1} \right) = 0 \quad (6)$$

Satisfying (5) and (6) is the population equilibrium point.

We can extend the Lotka-Volterra model to more complex population dynamics. When there are n populations, considering the interaction of different populations, in this paper we use the parameter α_{ij} to represent the influence of the species on the species, and α_{ii} to represent the intraspecific competition of the species, so the Lotka-Volterra equation can be expressed as follows:

$$\frac{dx_i}{dt} = r_i x_i \left(1 - \frac{\sum_{j=1}^S \alpha_{ij} x_j}{K_i} \right) \quad (7)$$

To predict the succession of plant communities under the influence of irregular weather cycles in the presence of drought, this paper aims to develop a model to predict the evolution of a plant community over time in the face of various irregular weather cycles, including drought periods that should have sufficient precipitation. At the same time, the interaction between different species is a factor that should be considered in the model.

Considering the influence of irregular weather on population i , the weather state parameter d is introduced, with $d = 0$ indicating normal state and $d = 1$ indicating irregular state. Correspondingly, the weather influence parameter θ_i ($\theta_i \in (0, 1)$) of population i is introduced, which represents the average influence of irregular weather on an individual in population i , and reflects the adaptability of species i to irregular weather from the negative side. Finally, considering that the extent to which the population is affected by irregular weather is related to its size, $d\theta_i x_i$ can be used to characterize this overall impact. Thus, the final representation of the model is obtained:

$$\frac{dx_i}{dt} = r_i x_i \left(1 - \frac{x_i + \sum_{j \neq i} \alpha_{ij} x_j}{K_i} \right) - d\theta_i x_i \quad (i = 1, 2, 3, 4, 5) \quad (8)$$

Although the previous model based on the Lotka-Volterra equation has considered the interaction of different plant populations and the influence of irregular weather, there are still some defects. In terms of the combination of the two, Model 1 uses a simple addition, which cannot achieve a good combination of the two. The new model assumes that the interaction between species will be affected by irregular weather. For example, arid environment will reduce the natural growth rate of plant populations, thus affecting the interaction between species. By multiplying the two, we achieve this. Furthermore, from the description of irregular weather, model 1 introduces a simple weather state parameter d and weather influence parameter δ_i , which depicts a static irregular weather and community system. The new model will more realistically assume that the irregular weather cycle is a dynamic process. Specifically, in the model, the intensity D of the drought environment is a function of time, and its value decreases during the drought period, thus affecting the natural growth rate of the plant population; t_d and k respectively represent the beginning time of drought and the change rate of drought environment, which can be adjusted for sensitivity analysis.

$\frac{1}{1+e^{-k(t-t_d)}}$ is a sigmoid function that can transform time t into a value between 0 and 1. With the help of it, the change rate k of arid environment can be combined with time t , so as to realize the dynamic change rate of arid environment and make it very practical. Finally, the improved Lotka-Volterra equation is obtained:

$$\frac{dx_i}{dt} = r_i x_i \left(1 - \frac{x_i + \sum_{j=1, j \neq i}^N \alpha_{ij} x_j}{K_i} \right) \left(1 - \frac{D}{1+e^{-k(t-t_d)}} \right) \quad (9)$$

Where N stands for species richness.

Considering the important dynamic factor of time t , the dynamic new model can predict more accurate and reasonable results. However, with the increase of community size, the calculation amount of the new model will increase rapidly. In contrast, the former model can achieve relatively

accurate results with less computation. This shows that the new model is not superior to the previous model in any case.

2.2. Extended Model

Pollution and habitat reduction will affect the natural conditions (such as climate) of the environment in which the plant community is located, the amount of available resources, and even directly endanger the life of the plant, thus having a profound impact on the evolution and development of the community.

In order to explore the influence of these factors on community development, they are included in the Lotka-Volterra model. Considering that more common pollution does not kill plants immediately, but subtly restricts their growth and reproduction; at the same time, different plants have different adaptability to pollution. Therefore, this paper introduces the pollution parameter q_i , which exerts its influence by directly acting on the natural growth rate r_i of species i . In order to be more realistic, q_i is actually a dynamic parameter, which will gradually decrease with the dilution parameter p . This is due to the gradual improvement of human environmental awareness and control methods and the fact that plant populations will bear and absorb part of the pollution. Therefore, for a long time, we believe that the impact of environmental pollution will be gradually diluted at a certain ratio. The dilution parameter depicts the rate of contamination dilution. Finally, considering that the existing environmental capacity K_i in the existing model can better describe the habitat, and in order to maintain the good computational performance of the existing model, this paper no longer introduces new variables for this purpose. Thus, a new model is obtained:

$$\frac{dx_i}{dt} = r_i x_i \left(1 - \frac{x_i + \sum_{j \neq i} \alpha_{ij} x_j}{K_i} \right) (r_i - q_i) \quad (10)$$

3. Result Analysis

3.1. The Impact of Pollution and Habitat Reduction

The initial species richness was set to 1000, and sensitivity analysis was performed under different pollution intensities. The other conditions were kept the same, which were set as 0.3, 0.6 and 0.9 times of the initial value of pollution intensity. From the histogram Figure 1, it can be seen that when the pollution increases, the range of species richness increases to enhance the adaptability of the community, that is, the rising range moves to the right to a certain extent. This is because as the species richness increases, its positive effect on the adaptability of the community is higher than the negative effect of more complex communities on the environment and resource demand. In a more polluted environment, the effect of this positive effect is more obvious.

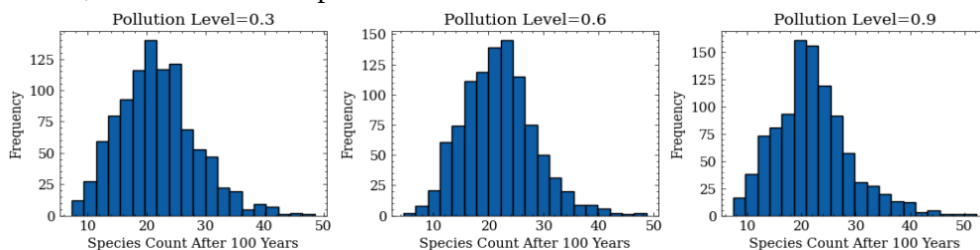


Figure 1. Sensitivity Analysis of Pollution Level

Then, the plant habitat was reduced to 0.9, 0.6 and 0.3 of the initial value respectively, and the histogram was used to explore the effect of habitat reduction on the role of species richness improvement in promoting community development. In Figure 2, It can be seen that as the habitat decreases, the rising range shifts to the left. This is because the habitat is closely related to the total amount of resources in the environment. Under the condition of less habitat, the dominant role of increasing species richness in promoting community development is quickly replaced by limited resources. Under the condition of limited resources, this substitution is more likely to occur.

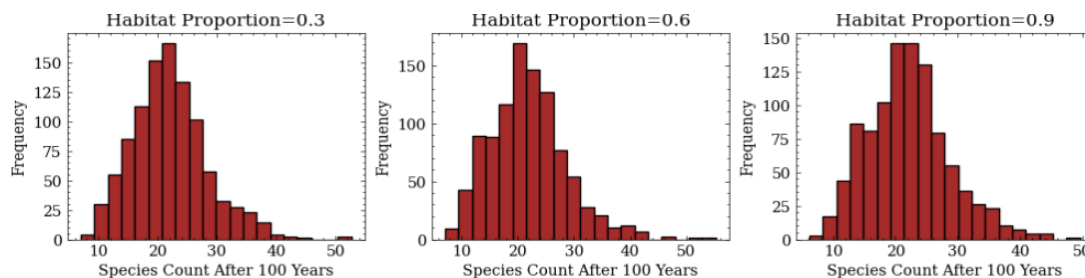


Figure 2. Sensitivity Analysis of Habitat Proportion

3.2. Different Reactions to Drought

This paper use the total number of individuals in the population as a measure to explore the performance of three different species in the same polluted environment. Their ability to resist pollution is high, medium and low.

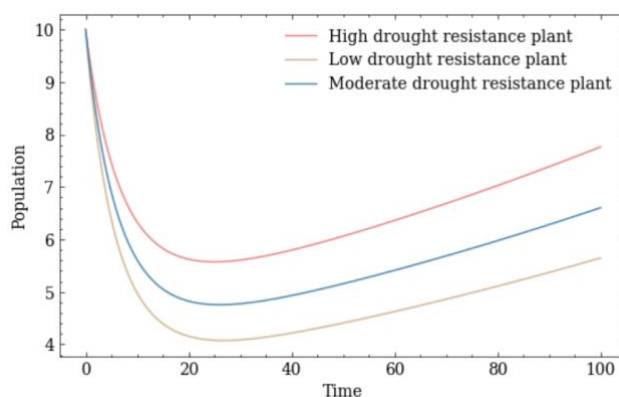


Figure 3. Pollution of Plants over Time

From Figure 3, It can be seen that in the same environment, species with stronger resistance can obviously survive and develop better. The results of the model reflect the significant impact of species type on the development of plant populations, which is also consistent with our previous conclusions.

3.3. Cause Analysis

Statistical histogram is an important method for us to study the effect of increasing species richness on the whole community under certain factors. Under the factors of pollution and habitat change, the rising range of species richness shifted to the right and left respectively with the aggravation of adverse factors. It was noted that in the previous study on the impact of species richness on community development under low drought frequency, we concluded that higher drought frequency is not conducive to species richness to play a positive role. Therefore, it can be concluded that under the influence of pollution factors, the effect of species richness on the development of plant communities is opposite to the previous conclusion; under the influence of habitat reduction, the effect of species richness on plant community development is similar.

4. Conclusion

4.1. Suggestions

Based on the results of the model, we can take the following measures to improve the long-term viability of plant communities:

(1) Reducing pollution: Pollution is one of the main factors leading to the decrease of plant community adaptability. Therefore, we should take measures to reduce pollution to improve the adaptability and viability of plant communities.

(2) Protecting habitat: habitat destruction will lead to a decrease in the number and adaptability of plant communities. Therefore, we should take measures to protect habitats to improve the adaptability and viability of plant communities.

(3) Our model results show that increasing species diversity and regulating the interspecific relationship of plant communities are feasible approaches as well.

4.2. Future Work

1. The numerical solution of differential equations can be calculated by using the approximation method of higher order derivatives to improve the calculation accuracy.

2. More factor parameters can be introduced, and multivariate analysis such as principal component analysis or factor analysis can be used to clarify the relationship between the various factors and their impact on plant communities.

3. The Lotka-Volterra equation that describes the relationship between competition, predation, symbiosis and mutual benefit can be integrated to establish a more complete community system dynamics model.

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