Research On the Distribution of Water and Electricity in Eastern Mother River Based on Multi-Objective Programming

Jiacheng Wu [1, #], Aokun Dong [2, #], Jiaxiang Pan [3, #], Jingjing Wang [1, #], Yuheng Wang [1, *]

[1] School of Ecology and Environment, Northwestern Polytechnical University, Xi'an, China, 710072

[2] The School of Economics and Management, Chang'an University, Xi'an, China, 710064

[3] Shenzhen Campus, Jinan University, Shenzhen, China, 518053

* Corresponding Author Email: yuheng.wang@nwpu.edu.cn

[#]These authors contributed equally

Abstract. This paper collected five states involved in the demand of water resources and hydropower resources, primarily hydrological conditions, lake Powell and lake mead. According to lake location, involves the basin and factors such as evaporation of lake Powell and mead lake water resources supply situation is analyzed, five states can get the final supply of water. Then, the water demand and daily water supply data of five states are used to solve the problem of the time to meet the fixed demand without additional water supply. The extra water supply is solved by considering the precipitation factor. Then, this paper solved the weight of different supply types in each state through Analytic Hierarchy Process to determine the water consumption priorities of five states and get the optimal allocation of different water types in each state. Finally, this paper ranks the water supply priority levels of five states and each industry in a state. The water supply scheme is given according to the priority levels.

Keywords: hydropower; multi-objective programming; Sensitivity analysis; Analytic Hierarchy Process.

1. Introduction

Reservoir plays an important role in flood control, power generation, irrigation, and water supply and guarantees urban development and people's survival.

However, for now, more than 95 percent of the western United States remains in a severe drought crisis. Water levels at the Hoover Dam, the largest in the United States, have dropped to record lows, threatening hydropower supplies for about 1.3 million people in California, Nevada, and Arizona. However, at Glen Canyon, the situation is even direr: The dam's generating capacity has been reduced by 41 percent as Lake Powell has set a record low for the reservoir. According to projections, if lake Powell continues to decline on current trends, the Glen Canyon Dam could stop working by 2023 due to lack of water.

As the climate and population change, the Colorado River Agreement of 1922, signed by the seven states in the Colorado River basin, no longer applies to the current situation. Therefore, it is essential to reformulate the distribution plan according to the current situation [1].

2. Two Dams Supply and Demand Model

2.1. Model Establishment

Water Demand

Suppose that the total water demand of Arizona, California, Wyoming, New Mexico, and Colorado is, residential water of each state is WD_i, agricultural water is RW_i, industrial water is IW, and power production water is PW_i, then the water resource demand model is:

$$WD_i = RW_i + AW_i + IW + PW_i \quad (1)$$

Water Supply

The water supply from Lake Powell and Lake Mead to the five states is represented by WS. The water storage capacity of Lake Powell is W_B, the water storage capacity of Lake Mead is W_M, the evaporation capacity of Lake Powell is E_B, the evaporation capacity of Lake Mead is E_M, and the evaporation capacity of rivers between Lake Powell and Lake Mead is E_C. The following model can represent the water supply relationship of the two lakes to the five states:

$$WS = W_B + W_M - E_B - E_M - E_C \tag{2}$$

Let lake Powell's water supply be WS_B, Lake Mead's water supply WS_M, Colorado River's conveying water supply WS_C, Colorado River's evaporation water supply E_C and Arizona's receiving water supply WS_1.

$$WS_B = W_B - E_B \tag{3}$$

$$WS_C = WS_B - E_c - WS_1 \tag{4}$$

$$WS_M = WS_C + W_M - E_M - E_C \tag{5}$$

The area of Lake Powell is S_B, the area of Lake Mead is S_M, the water level of Lake Powell is P, and the water level of Lake Mead is M, so the water storage capacity of the two lakes can be expressed by the following model:

$$W_B = S_B \times L_B \tag{6}$$

$$W_M = S_M \times L_M \tag{7}$$

Pump Output

The pumping volume of Lake Powell is $Pumpage_B$ and that of Lake Mead is $Pumpage_M$, the pumping volume model of the two lakes is:

$$Pumpage_B = \frac{W_B}{W_B+W_M} \times \sum_{i=1}^{5} WD_i \tag{8}$$

$$Pumpage_M = \frac{W_M}{W_B+W_M} \times \sum_{i=1}^{5} WD_i \tag{9}$$

Calculation of Time to Meet Requirements

Assume that the total water consumption for hydraulic power generation in the five states is HP, the time to meet the fixed demand without additional water supply is T, and the evaporation of the lower reaches of Lake Mead is E_D. Then, the calculation model of the time to meet the demand is as follows:

$$E = E_B + E_M + E_C + E_D \tag{10}$$

$$T = \frac{\sum_{i=1}^{5} WD_i}{\frac{W_B+W_M-E-HP}{365}} \tag{11}$$

Additional water supply

Suppose that the required precipitation is R, the year is n, the total evaporation of water resources is E, and the value-added coefficient of evaporation is 3%. The precipitation recharge model is as follows:

$$R = \mathrm{E} \times (1 + 3\%)^{\mathrm{n}-2015} \tag{12}$$

2.2. Solution of Model

Water Demand

The data of residential water demand, industrial water demand, agricultural water demand, and hydroelectric power demand in five states are collected and sorted out, and the following table is obtained [3] as in Table 1 and Table 2:

Tab. 1 Water use by Category by State, 2015 (million kulun/day)

State	Domestic	Agriculture	Industrial	Electric Power
Arizona	24	4568.9	74.42	83.5
Colorado	35.4	9033.3	91.8	37.2
Wyoming	8.93	7806.2	52.54	51.8
New Mexico	24.6	2402	60.2	33.5
California	127	19183	444.8	36.4

Tab. 2 Hydropower generation and water consumption rate by state

State	Hydroelectric power	Power Generation Water
	kWh/year	gal/kWh
Arizona	8763	64.85
Colorado	9130	20.87
Wyoming	1022	136.96
New Mexico	94	6800
California	1176	17.91

Watwe Supply

Water resources information of Lake Powell and Lake Mead is collected and sorted out, and the basic hydrological information of the two lakes is shown in the table below [4] as in Table 3:

Tab. 3 Surface area and evaporation of reservoir and watershed

Reservoir	Evaporation	Surface Area	Evaporation
	in/year	Acres	Gal/year
Hoover dam	80	164000	3.60E+11
Glen Canyon dam	76	169700	3.50E+11
Totals Average			7.10E+11
River	Surface Area	Evaporation	Percentage
	Acres	Gal/year	Difference
Hoover	4000	8.70E+09	2.40%
Glen Canyon	6764	1.40E+10	4.00%
Totals Average		2.30E+10	3.20%

Put the hydrological data of the two lakes into the water supply model for calculation and solution [5], and the water supply of the two lakes to the five states was obtained as follows:

The moisture storage capacity of Lake Powell: $W_B = 169700P$.

The moisture storage capacity of Lake Mead: $W_M = 164000P$.

Five states receive the total water supply from Lake Powell and Lake Mead: $WS = 169700P + 164000 - 2.74 \times 10^9$

The water supply in Lake Powell: $WS_B = 169700P - 1.36 \times 10^9$.

The Colorado River supplies water: $WS_C = 169700P - 8.4 \times 10^{10}$.

Lake Mead supplies water: $WS_M = 169700P + 164000M - 8.54 \times 10^{10}$.

Pump Output

The amount of water taken from Lake Powell: $Pumpage_B = \frac{169700P}{169700P+164000M} \times 7.28 \times 10^{11} m^3$

The amount of water taken from Lake Mead: $Pumpage_M = \frac{164000M}{169700P+164000M} \times 7.28 \times 10^{11} m^3$.

Time required for water supply.

In the case of no additional water supply, the time to meet the fixed demand is calculated as the following: $T = \frac{2.66 \times 10^{14}}{169700P+164000M-1.64 \times 10^{10}}$ day.

Additional water supply

Put the data into the model, and the additional water supply can be obtained as follows: $R = 2.78 \times 10^9 \times (1 + 3\%)^{n-2015} m^3$

3. Water Resources Allocation Model

3.1. Analytic Hierarchy Model

This paper first determines the importance of water resources for agriculture, industry, housing, and electricity production in five states by weight classification as in Figure 1.

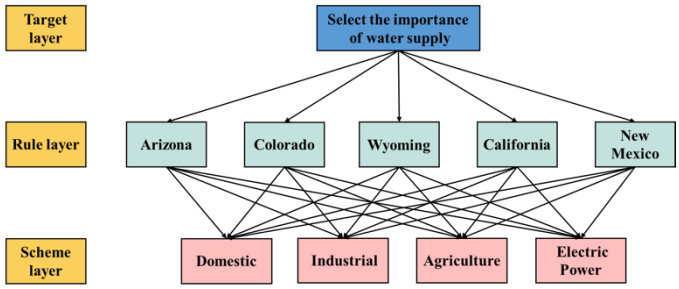


Figure 1. Analytic hierarchy diagram.

Construct a judgment matrix to compare the importance of elements on the same level in pairs. Next, a consistency test is required, and the indicators are shown as follows (the eigenvalue of the judgment matrix is λ) [2]:

$$CI = \frac{\lambda_{max} - n}{n-1} \tag{13}$$

Find the corresponding average random consistency index RI:

Tab. 4 Consistency indicator mapping table

n	1	2	3	4	5	6	7	8	9	10
RI	0	0	0.52	0.89	1.12	1.26	1.36	1.41	1.46	1.49

Consistency test ratio:

$$CR = \frac{CI}{RI} \tag{14}$$

If the consistency test passes, calculate the relative weight by the judgment matrix:

$$\omega_i = \frac{1}{n}\sum_{j=1}^{n} \frac{a_{ij}}{\sum_{k=1}^{n} a_{kj}} (i = 1,2,\dots,n) \tag{15}$$

Based on the data of Model 1 and the established analysis system, the judgment matrix of different water use types in each state is obtained, as shown in the following Table 5[6]:

Tab. 5 Judgment matrix of industrial water

Industrial	Arizona	Colorado	Wyoming	Wyoming	California
Arizona	1	0.17	1,42	1.24	0.81
Colorado	5.98	1	8.47	7.39	4.85
Wyoming	0.71	0.12	1	0.87	0.57
Wyoming	0.81	0.14	1.15	1	0.66
California	1.23	0.21	1.75	1.52	1

Next, the average arithmetic method was used to calculate the weight of different types of water use in five states by using the elements in the judgment matrix, and the results were as follows in Table 6:

Tab. 6 Weight results of different types of water use in each state

	Arizona	California	Wyoming	New Mexico	Colorado
Industrial	0.1030	0.6136	0.0727	0.0838	0.1269
Agriculture	0.1066	0.4455	0.1814	0.0566	0.2099
Domestic	0.1095	0.5776	0.0404	0.1114	0.1612
Electric Power	0.5898	0.2005	0.157	0.0183	0.0344

3.2. Multi-objective Optimization Model

We choose to establish a multi-objective optimization model for the rational allocation of different types of water resources. Its expression is as follows:

$$Z = \max\left[f_1(x), f_2(x)\right] \tag{16}$$

The objective function

The optimal allocation of water resources is a multi-objective optimization decision problem, which requires a model [6].

Here, we consider the principle of "maximum value for the least amount of water," meaning that each state generates an enormous hydro bill for agricultural, industrial, residential, and power production water and the lowest total for each category.

$$max f_1(X) = max \sum_{k=1}^{5} \sum_{j=1}^{3} c_j x_{kj} + \frac{\sum_{k=1}^{5} c_e x_{k4}}{n_k} \tag{17}$$

$$min f_2(X) = min \sum_{k=1}^{5} \sum_{j=1}^{4} \omega_{kj} x_{kj} \tag{18}$$

Convert $f_1(X)$ to extremely small, as shown below:

$$min f_1(X) = min\left(-\sum_{k=1}^{5} \sum_{j=1}^{3} c_j x_{kj} - \frac{\sum_{k=1}^{5} c_e x_{k4}}{n_k}\right) \tag{19}$$

$f_1(x)$—Economic benefits of regional water supply.

$f_2(x)$—Regional water requirement.

j—Water type; (j=1,2,3,4)

k—Five autonomous prefectures; (k=1,2,3,4,5)

c_j—The unit cost of water used for agricultural, industrial, residential, and electric power production. (Dollar per kWh)

The constraint

A state's r domestic water consumption should be capped by the product of its total population and per capita domestic water consumption. The lower limit of water consumption for electric power production should be the water consumption for hydroelectric power generation, and the upper limit should be the sum of water consumption for hydroelectric power generation and thermal power generation. The sum of water used by industry and agriculture in each state shall be capped by the proportion of the total demand of each state to total demand in five states multiplied by the total supply of the five states [7]. The constraints are as follows:

$$\text{s.t.} \begin{cases} 0 \le x_{k1} \le a_k t_k \\ 0 \le x_{k2} + x_{k3} \le \frac{d_k}{T} \text{ WS} \\ w_k \le x_{k4} \le w_k + f_k \end{cases} \tag{20}$$

a—Water consumption per capita; (m3)

t—The total population of a state.

d—The water demand of a state; (m3)

T—Total water demand in five states; (m3)

w—Water demand for hydropower generation; (m3)

f—Water requirement for thermal power generation;(m3)

The final results can be obtained by putting previously collected and obtained relevant data into the model. And since we got $\mathrm{WS} = 169700\mathrm{P} + 164000\mathrm{M} - 2.74 \times 10^9$That means that the total water supply for the five states is related to the water levels of Lake Mead and Lake Powell. However, the lake's water level is dynamic and changing all the time, so the solution of the model can only get the formula, not the final exact value.

Optimal allocation of water in Arizona

$$\begin{aligned} &minf_1 = -2.8x_{11} - 4.1x_{12} - 0.6x_{13} - 0.016x_{14} \\ &minf_2 = 0.103x_{11} + 0.1066x_{12} + 0.1095x_{13} + 0.5898x_{14} \\ &\text{s.t.} \begin{cases} 0 \le x_{11} \le 4.03 \times 10^5 \\ x_{12} + x_{13} \le \frac{0.9089}{4.0001}\ \mathrm{WS} \\ 1.55 \times 10^{10} \le x_{14} \le 1.64 \times 10^{10} \end{cases} \end{aligned} \tag{21}$$

Optimal allocation of water in California

$$\begin{aligned} &minf_1 = -2.8x_{21} - 4.1x_{22} - 0.6x_{23} - 0.049x_{24} \\ &minf_2 = 0.6136x_{21} + 0.4455x_{22} + 0.5776x_{23} + 0.2005x_{24} \\ &\text{s.t.} \begin{cases} 0 \le x_{21} \le 2.24 \times 10^7 \\ x_{22} + x_{23} \le \frac{1.8327}{4.0001}\mathrm{WS} \\ 5.22 \times 10^9 \le x_{24} \le 5.58 \times 10^9 \end{cases} \end{aligned} \tag{22}$$

Optimal allocation of water in Wyoming

$$\begin{aligned} &minf_1 = -2.8x_{31} - 4.1x_{32} - 0.6x_{33} - 0.007x_{34} \\ &minf_2 = 0.0727x_{31} + 0.1814x_{32} + 0.0404x_{33} + 0.157x_{34} \\ &\text{s.t.} \begin{cases} 0 \le x_{31} \le 3.37 \times 10^5 \\ x_{32} + x_{33} \le \frac{0.4515}{4.0001}\mathrm{WS} \\ 3.83 \times 10^9 \le x_{34} \le 4.35 \times 10^9 \end{cases} \end{aligned} \tag{23}$$

Optimal allocation of water in New Mexico

$$\begin{aligned} &minf_1 = -2.8x_{41} - 4.1x_{42} - 0.6x_{43} - 0.015x_{44} \\ &minf_2 = 0.0838x_{41} + 0.0566x_{42} + 0.1114x_{43} + 0.0183x_{44} \\ &\text{s.t.} \begin{cases} 0 \le x_{41} \le 1.4 \times 10^6 \\ x_{42} + x_{43} \le \frac{0.2701}{4.0001}\mathrm{WS} \\ 1.75 \times 10^8 \le x_{44} \le 5.1 \times 10^8 \end{cases} \end{aligned} \tag{24}$$

Optimal allocation of water in Colorado

$$\begin{aligned} &minf_1 = -2.8x_{51} - 4.1x_{52} - 0.6x_{53} - 0.015x_{54} \\ &minf_2 = 0.1269x_{51} + 0.2099x_{52} + 0.1612x_{53} + 0.0034x_{54} \\ &\text{s.t.} \begin{cases} 0 \le x_{51} \le 3.77 \times 10^6 \\ x_{52} + x_{53} \le \frac{0.5324}{4.0001}\mathrm{WS} \\ 3.72 \times 10^8 \le x_{54} \le 9.49 \times 10^8 \end{cases} \end{aligned} \tag{25}$$

4. Water supply priority analysis model

4.1. Model Establishment

Five state water priorities

Normalize the weight of water demand of each state calculated in model 2. Divide the weight of total water demand of the kth state by the weight of total water demand of the five states to obtain the water supply priority grade coefficients. Arrange the priority grade coefficients from large too small to obtain the water supply priority grade of the five states.

$$\text{Class}_{\ k} = \frac{\sum_{j=1}^{5} \omega_{kj}}{\sum_{k=1}^{5} \sum_{j=1}^{5} \omega_{kj}} \tag{26}$$

Water supply priority of each industry in each state

For industrial water demand weights obtained from model 2 states normalized processing, it will be the first one state of a particular industry water demand weight divided by the total water requirement of the state weight, get a particular state of the industrial water supply priority level coefficient, arrange priority coefficient from big to small, get a particular state of the industrial water supply priority level.

$$\text{Class}_{\ k} = \frac{\omega_{kj}}{\sum_{j=1}^{5} \omega_{kj}} \tag{27}$$

4.2. Solution of Model

Five state water priorities

According to the weight data calculated in Model 2 and substituted into the solution model of water supply priority of five states, the water supply priority grade coefficients and priority grades of five states are calculated as follows as in Table 7:

Tab. 7 Water supply priority grade coefficient and priority grade in five states

Priority coefficient	Priority sorting
0.4593	California
0.2272	Arizona
0.1331	Colorado
0.1129	Wyoming
0.0675	New Mexico

Water supply priority of each industry in each state

The weight data calculated in Model 2 is substituted into the solution model of water supply priority of each water use type in five states. The water supply priority grade coefficient and priority grade of each industry in each state is calculated as shown in the following Table 8:

Tab. 8 Water supply priority grade coefficient and priority grade of each water use type in five states

State	Domestic	Agriculture	Industrial	Electric Power
Arizona	0.12	0.12	0.11	0.65
California	0.31	0.24	0.33	0.11
Wyoming	0.09	0.40	0.16	0.35
New Mexico	0.41	0.21	0.31	0.07
Colorado	0.30	0.39	0.24	0.06

It can be seen from the table that the water supply priority coefficient of agriculture and industry in Arizona is low, both less than 0.15. In the case of a lack of water resources, the scale of local agriculture and industry can be appropriately reduced to save water resources.

Similarly, New Mexico, Colorado, and California should scale back their power industries if water is scarce, and Wyoming should scale back its industry to reduce demand.

5. Summary

First of all, this paper is collected five states involved in the demand of water resources and hydropower resources, primary hydrological conditions, and lake Powell and lake mead, secondly according to lake location, involves the basin and factors such as evaporation of lake Powell and mead lake water resources supply situation is analyzed, five states can get the final supply of water. Next, determine how much water should be extracted from each lake based on the amount of water stored in each lake. Then, the water demand and daily water supply data of five states are used to solve the problem of the time to meet the fixed demand without additional water supply. Finally, the extra water supply is solved by considering the precipitation factor. This paper solved the weight of different supply types in each state through the Analytic Hierarchy Process to determine the water consumption priorities of five states. Finally, according to the principle of "highest economic efficiency and least total water consumption," we get the optimal allocation of different water types in each state. This paper normalized the weight results solved according to the water resources demand of five states and calculated the priority coefficient. Then we rank the water supply priority levels of five states and each industry in a state. Finally, the water supply scheme is given according to the priority levels.

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