Stability Study of Event Dynamic Systems Based on Discrete Mathematical Random Variable Modeling

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Abstract. This project intends to consider the stability of a nonlinear control system with unknown dynamics. The system is controlled systematically by using the reverse step method based on Lyapunov's stability principle. This paper also gives the conclusion that when the control law is in a steady state, the output signal in this state will reach uniform boundedness or asymptotic convergence. Finally, a numerical simulation is carried out through an example, and the results show that the algorithm is effective.

Keywords: Non-model dynamics; Reverse stepping; Adaptive stepping; Nonlinear system.

1. Introduction

Due to the lack of modeled dynamic characteristics, it often leads to the distortion and instability of the actual system, so it needs to be dealt with effectively. Scholars have used their dynamic information to control their uncertain dynamics [1] and proposed an efficient adaptive control strategy under bounded conditions. Some researchers use measurable dynamic information to compensate the system and give a new robust adaptive controller [2]. Some researchers proposed a robust backstepping controller based on inversion algorithm. Some scholars have carried out in-depth research on random disturbances by means of control positioning, continuous identification of system models, and stochastic analysis. Aiming at a nonlinear system with unknown dynamics, an output feedback controller which can keep all parameters in the closed-loop system equal bounded or asymptotically convergent is proposed.

2. System description and problem hypothesis

Examining nonlinear systems:

$$\begin{cases} \dot{t}_1 = t_2 + \lambda_{11} t_1 + g_1(\bar{t}_1) + \mu_1(t) \\ \dot{t}_2 = t_3 + \sum_{i=1}^{2} \lambda_{2i} t_i + g_2(\bar{t}_2) + \mu_2(t) \\ \vdots \\ \dot{t}_{n-1} = t_n + \sum_{i=1}^{n-1} \lambda_{n-1,i} t_i + g_{n-1}(\bar{t}_{n-1}) + \mu_{n-1}(t) \\ \dot{t}_n = \theta + \sum_{i=1}^{n} \lambda_{ni} t_i + g_n(\bar{t}_n) + \mu_n(t) \\ f(t) = t_1 \end{cases} \tag{1}$$

Where $t = [t_1, \cdots, t_n]^T \in R^n$ is the system state, $\bar{t}_i = [t_1, \cdots, t_i]^T$, the initial value $t(0) = t_0, \theta \in R$ is the system input, $f(t) \in R$ is the system output, $\lambda_{ij}(i, j = 1, \cdots, n, j \le i)$ is a known constant, the modeled dynamic property $g_i(\cdot)(i = 1, \cdots, n)$ is a nonlinear function dependent on $t_1, \cdots, t_i$, and the unmodeled dynamic property $\mu_i(\cdot)(i = 1, \cdots, n)$ is an unknown nonlinear function [3].

Assuming that $f(t)=t_1$ is measurable and the rest of the state components are unmeasurable, system (1) can be rewritten as a matrix system of equations:

$$\begin{cases} \dot{t} = Kt + G(t) + \phi(t) + D\theta \\ f(t) = Lt \end{cases} \tag{2}$$

Among them:

$$K=\begin{bmatrix} \lambda_{11} & 1 & 0 & \cdots & 0 \\ \lambda_{21} & \lambda_{22} & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \lambda_{n-1,1} & \lambda_{n-1,2} & \lambda_{n-1,3} & \cdots & 1 \\ \lambda_{n1} & \lambda_{n2} & \lambda_{n3} & \cdots & \lambda_{nn} \end{bmatrix}, G(t)=\begin{bmatrix} g_1(\bar{t}_1) \\ g_2(\bar{t}_2) \\ \vdots \\ g_n(\bar{t}_n) \end{bmatrix}, \phi(t)=\begin{bmatrix} \mu_1(t) \\ \mu_2(t) \\ \vdots \\ \mu_n(t) \end{bmatrix}, D=\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}, L=(1\,0\cdots 0)_{1\times n}$$

Accordingly, $G(t)$ and $\phi(t)$ represent the modeled and unmodeled dynamic properties of the system (2), respectively. Aiming at the unmodeled dynamic characteristics of the system, this paper makes the system (1) asymptotically stable by designing a stable controller [4]. First, assume the following for $G(t)$ and $\phi(t)$ [5]:

$(O_1) G(\cdot)$ is known and sufficiently smooth, and $G(0)=0$ is present.

$(O_2) G(t)=G\zeta(t)$. G is a non-singular matrix and $\zeta(t)$ is known and satisfies $\frac{\partial\zeta(t)}{\partial t}+(\frac{\partial\zeta(t)}{\partial t})^T \geq 0, \forall t\in R^n$. $(O_3) \| \phi(t) \| < \alpha\eta f(t)$, where α is unknown and non-negative, and $\eta f(t)\geq 0$ is sufficiently smooth and satisfies $\eta(0)=0$.

3. Controller design

First, the observer is introduced

$$\dot{\hat{t}} = K\hat{t} + S(L\hat{t} - f(t) + G(\hat{t} + W(L\hat{t} - f(t)) + D\theta \tag{3}$$

Here $W=[w_1,\cdots,w_n]^T$ and $S=[s_1,\cdots,s_n]^T$ satisfy:

$$\begin{bmatrix} (K+SL)^T P + P(K+SL) + Q & PG + (I+WL)^T \\ G^T P + (I+WL) & 0 \end{bmatrix} \tag{4}$$

According to formula (2) and formula (3), the observation error $\tilde{t}$ is satisfied

$$\dot{\tilde{t}} = (K+SL)\tilde{t} + G(t) - G(u) + \phi(t) \tag{5}$$

Here's $u=\hat{t}+W(L\hat{t}-f(t))$. It can be seen that $\tilde{t}$ does not depend on the input of the system, so the controller can be designed using formula (3).

Lemma 1 function $U_0=\tilde{t}^T P\tilde{t}$ satisfies:

$$\dot{U}_0 \leq -\pi \| \tilde{t} \|^2 + \Theta\chi(f(t)) f(t)^2 \tag{6}$$

Where $\chi(f(t))\geq 0$ is a smooth function and $\pi>0, \Theta\geq 0$ is a constant [6].

Proof: Remember $\mu=t-u$, from (O_2) to $G(t)-G(u)=G(\zeta(t)-\zeta(t-\mu))\square G\chi(t,\mu)$, here

$$\chi(t,\mu)=\zeta(t)-\zeta(u)=\int_0^1 \left\{\frac{\partial\zeta}{\partial s}\right\}_{s=t+\tau(u-t)} (t-u)d\tau = \int_0^1 \left\{\frac{\partial\zeta}{\partial s}\right\}_{s=t-\tau\mu} \mu d\tau \tag{7}$$

Known by (O_2) and formula (7)

$\mu^T \chi(t,\mu) = \mu^T \int_0^1 \left\{ \frac{\partial \zeta}{\partial s} \right\}_{s=t-\tau\mu}$

$d\tau = \frac{1}{2}\mu^T (\int_0^1 \left\{ \frac{\partial \zeta}{\partial s} + \left(\frac{\partial \zeta}{\partial s} \right)^T \right\}_{s=t-\tau\mu} d\tau)\mu \geq 0$. $(K+SL)^T P + P(K+SL) \leq -Q, PG \leq -(I+WL)^T$ is known by equation (4). $\theta = t-u = t-\hat{t} - W(L\hat{t} - Lt) = (I+WL)\tilde{t}$, hence $\tilde{t}^T PG \leq -\tilde{t}^T (I+WL)^T = -\theta^T$. Taking the derivative of U_0, we can see from (O_3) the above calculation:

$$\dot{U}_0 \leq -(\lambda_{\min}(Q) - \varepsilon_0) \| t \|^2 + \frac{\| P \|^2}{\varepsilon_0} \alpha^2 \bar{\eta}^2 (f(t)) f(t)^2 \tag{8}$$

$\pi \square \lambda_{\min}(Q) - \varepsilon_0, \Theta \square \alpha^2$ and $\chi(f(t)) = \frac{\| P \|^2}{\varepsilon_0} \bar{\eta}^2 (f(t))$, obviously $\pi, \Theta, \chi(f(t))$ both satisfy the conditions of the lemma. Then formula (6) can be obtained from formula (8), and the proof is completed [7].

Because of $\frac{\| P \|^2}{\varepsilon_0} \alpha^2 \eta^2 (f(t)) \geq 0$, $\dot{U}_0$ is not necessarily negative definite, so to make the observation error convergence, it is necessary to design a suitable control input to make $\dot{U}_0$ negative definite. Consider the system:

$$\begin{cases} \dot{\tilde{t}} = (K+SL)\tilde{t} + G\chi(t,z) + \phi(t) \\ \dot{f}(t) = \hat{t}_2 + \tilde{t}_2 + \lambda_{11} t_1 + g_1(f(t)) + \mu_1(t) \\ \dot{\hat{t}}_2 = \hat{t}_3 - s_2 \tilde{t}_1 + \sum_{i=1}^{2} \lambda_{2i} \hat{t}_i + \hat{g}_2(\hat{\bar{t}}_2, f(t)) \\ \vdots \\ \dot{\hat{t}}_{n-1} = \hat{t}_n - s_{n-1} \tilde{t}_1 + \sum_{i=1}^{n-1} \lambda_{n-1,i} \hat{t}_i + \hat{g}_{n-1}(\hat{\bar{t}}_{n-1}, f(t)) \\ \dot{\hat{t}}_n = \theta - s_n \tilde{t}_1 + \sum_{i=1}^{n} \lambda_{ni} \hat{t}_i + \hat{g}_n(\hat{t}, f(t)) \end{cases} \tag{9}$$

Here's $\hat{g}_i \triangleq g_i(\hat{\bar{t}}_i - \bar{w}_i \tilde{t}_1), i = 2,3,\cdots,n,$ $\hat{\bar{t}}_i = (\hat{t}_1, \hat{t}_2, \cdots, \hat{t}_i)^T, \bar{w}_i = (w_1, \cdots, w_i)^T$. It is obviously possible to design a controller such that all signal convergence of system (9) and system (1) is consistent. Define a new state transition:

$$\begin{cases} z_1 = f(t) \\ z_2 = \hat{t}_2 - \varepsilon_1(f(t), \hat{\Theta}) \\ z_i = \hat{t}_i - \varepsilon_{i-1}(\hat{\bar{t}}_{i-1}, f(t), \hat{\Theta}), i = 3, \cdots, n \end{cases} \tag{10}$$

Where: $\varepsilon_0 = 0, z_{n+1} = 0$; Virtual control $\varepsilon_i (i = 1, \cdots, n-1)$ is the fully smooth function to be determined, and $\varepsilon_1(0) = \varepsilon_2(0,0) = \cdots = \varepsilon_n(0,0) = 0; \hat{\Theta}$ is the estimate of Θ. Actual control $\varepsilon_n \square \theta(\hat{t}, f(t))$ to be determined. Thus system (10) becomes [8]:

$$\begin{cases} \dot{\tilde{t}} = (K+SL)\tilde{t} + G\chi(t,z) + \phi(t) \\ \dot{z}_1 = z_2 + \varepsilon_1 + g_1 + \lambda_{11}t_1 + \tilde{t}_2 + \mu_1(t) \\ \dot{z}_2 = z_3 + \varepsilon_2 + \hat{g}_2 - s_2\tilde{t}_1 + \sum_{i=1}^{2}\lambda_{2i}\hat{t}_i - \dfrac{\partial\varepsilon_1}{\partial f(t)}(\hat{t}_2 + \tilde{t}_2 + g_1 + \lambda_{11}t_1 + \mu_1(x)) - \dfrac{\partial\varepsilon_1}{\partial\hat{\Theta}}\dot{\hat{\Theta}} \\ \vdots \\ \dot{z}_n = \theta + \hat{g}_n - s_n\tilde{t}_1 + \sum_{i=1}^{n}\lambda_{ni}\hat{t}_i - \sum_{i=1}^{n-1}\dfrac{\partial\varepsilon_{n-1}}{\partial\hat{t}_i}\dot{\hat{t}} - \dfrac{\partial\varepsilon_{n-1}}{\partial f(t)}(\hat{t}_2 + \tilde{t}_2 + g_1 + \lambda_{11}t_1 + \mu_1(t)) - \dfrac{\partial\varepsilon_{n-1}}{\partial\hat{\Theta}}\dot{\hat{\Theta}} \end{cases} \tag{11}$$

Next, the Backstepping method is used to design the controller in n steps to make the system (11) asymptotically stable [9]. The first choice is Lyapunov function $U_0 = \tilde{t}^T P\tilde{t}$, where $P = P^T$ satisfies equation (4), then U_0 satisfies equation (6) in Lemma 1, where $\varepsilon_0 > 0$ is undetermined.

Step 1 selects Lyapunov function $U_1 = U_0 + \frac{1}{2}\tilde{\Theta}^2 + \frac{1}{2}z_1^2$, where $\tilde{\Theta} = \hat{\Theta} - \Theta$ is the parameter estimation error. According to equation (10)

$$\dot{U}_1 = \dot{U}_0 + \tilde{\Theta}\dot{\tilde{\Theta}} + z_1(z_2 + \varepsilon_1(f(t),\hat{\Theta}) + g_1 + \lambda_{11}t_1 + \tilde{t}_2 + \mu_1(t)) \tag{12}$$

Bring formula (6) into formula (12) and notice $\dot{\tilde{\Theta}} = \dot{\hat{\Theta}}$, get

$$\dot{U}_1 \le -\pi \| \tilde{t} \|^2 + \frac{\| P \|^2}{\varepsilon_0}\bar{\eta}^2(f(t))\Theta f(t)^2 + \tilde{\Theta}\dot{\hat{\Theta}} + z_1(z_2 + \varepsilon_1 + g_1 + \lambda_{11}t_1) + z_1\tilde{t}_2 + z_1\mu_1(t) \tag{13}$$

The use of formula principle has

$$z_1\tilde{t}_2 = \varepsilon\tilde{t}_2^2 + \frac{1}{4\varepsilon}z_1^2 - (\sqrt{\varepsilon}\tilde{t}_2 - \frac{1}{2\sqrt{\varepsilon}}z_1)^2 \tag{14}$$

Here $\varepsilon > 0$ is to be determined [10].

$$z_1\mu_1(t) \le \frac{1}{2}z_1^2 + \frac{1}{2}\Theta\eta^2(f(t)) = \frac{1}{2}z_1^2 + \frac{1}{2}\Theta\bar{\eta}^2(f(t)) \tag{15}$$

Put equation (14) and (15) into equation (13), and add and subtract $z_1^2 + \frac{n-1}{2}z_1^2\bar{\eta}^2(f(t))\hat{\Theta}$ to the right of equation (13)

$$\dot{U}_1 \le -z_1^2 + z_1z_2 - (\sqrt{\varepsilon}\tilde{t}_2 - \frac{1}{2\sqrt{\varepsilon}}z_1)^2 - \pi\| \tilde{t} \|^2 - \frac{n-1}{2}z_1^2\bar{\eta}^2(f(t))\hat{\Theta} + \varepsilon\tilde{t}_2^2 + z_1(\varepsilon_1 + z_1 + g_1 +$$

$$\lambda_{11}t_1 + \frac{1}{4\varepsilon}z_1 + \frac{1}{2}z_1 + z_1\bar{\eta}^2(f(t))(\frac{\| P \|^2}{\varepsilon_0} + \frac{n}{2})\hat{\Theta}) + (\dot{\hat{\Theta}} - (\frac{\| P \|^2}{\varepsilon_0} + \frac{1}{2})\eta^2(f(t))\tilde{\Theta}$$

Select virtual controller ε_1 as

$$\varepsilon_1(f(t),\hat{\Theta}) = \{-z_1 - g_1 - \lambda_{11}t_1 - \frac{1}{4\varepsilon}z_1 - \frac{1}{2}z_1 - z_1\bar{\eta}^2(f(t))(\frac{\| P \|^2}{\varepsilon_0} + \frac{n}{2})\hat{\Theta}\}_{z_1 = f(t)} \tag{16}$$

(O_1) gets $\varepsilon_1(0,\hat{\Theta}) = 0$. Substituting $\varepsilon_1(f(t),\hat{\Theta})$ into equation (16) yields

$$\dot{U}_1 \le -z_1^2 + z_1 z_2 - (\sqrt{\varepsilon}\tilde{t}_2 - \frac{1}{2\sqrt{\varepsilon}} z_1)^2 - \pi \| \tilde{t} \|^2 + \varepsilon \tilde{t}_2^2 - \frac{n-1}{2} z_1^2 \bar{\eta}(f(t))\hat{\Theta} +$$

$$(\dot{\hat{\Theta}} - (\frac{\| P \|^2}{\varepsilon_0} + \frac{1}{2})\eta^2(f(t))\tilde{\Theta}$$

Step i. Repeat the procedure to select the virtual controller

$$\varepsilon_i(\hat{\bar{t}}_i, f(t), \hat{\Theta}) = \{-z_{i-1} - z_i - \hat{g}_i + s_i\tilde{t}_1 + \sum_{j=1}^{i-1}\frac{\partial \varepsilon_{i-1}}{\partial \hat{t}_j}\dot{\hat{t}}_j - \sum_{j=1}^{i}\lambda_{ij}\hat{t}_j + \frac{\partial \varepsilon_{i-1}}{\partial f(t)}(\hat{t}_2 + g_1 + \lambda_{11}t_1) +$$

$$\frac{\partial \varepsilon_{i-1}}{\partial \hat{\Theta}}\dot{\hat{\Theta}} - \frac{1}{4\varepsilon}(\frac{\partial \varepsilon_{i-1}}{\partial f(t)})^2 z_i + \frac{1}{2\varepsilon}\frac{\partial \varepsilon_{i-1}}{\partial f(t)}(z_1 - \sum_{j=1}^{i-2}\frac{\partial \varepsilon_j}{\partial f(t)} z_{j+1}) - \frac{1}{2}(\frac{\partial \varepsilon_{i-1}}{\partial f(t)})^2 z_i\}_{z_1 = f(t), \cdots, z_i = \hat{t}_i - \varepsilon_{i-1}}$$

Here $\dot{\hat{\Theta}} = (\frac{\| P \|^2}{\varepsilon_0} + \frac{n}{2})\eta^2(f(t))$ is an indirect estimate of parameter α, and the controller design is thus completed. Now consider that parameters ε_0 and $\varepsilon \cdot \varepsilon_0$ and ε satisfy: $\begin{cases} \varepsilon_0 > 0 \\ \varepsilon > 0 \\ \pi \square \lambda_{\min}(Q) - \varepsilon_0 > 0 \\ \pi > \varepsilon \end{cases}$.

Since Q is positive definite, $\lambda_{\min}(Q) > 0$, ε_0 and ε must exist, and $\varepsilon + \varepsilon_0 < \lambda_{\min}(Q)$.

4. Simulation Examples

Consider a three-order system:

$$\begin{cases} \dot{t}_1 = t_2 + \alpha t_1^2 \cos t_3 \\ \dot{t}_2 = -t_2 + t_3 - t_2^3 \\ \dot{t}_3 = -\frac{1}{2}t_3 - t_3^3 - t_2^2 t_3 + \theta \\ f(t) = t_1 \end{cases}$$

Let's call this $\alpha = 1$. This system can be written in the form of system (2) and satisfies the assumptions. Solving the matrix inequality (4) yields:

$$P = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, S = \begin{bmatrix} -\frac{9}{2} \\ -\frac{15}{4} \\ -\frac{15}{8} \end{bmatrix}, W = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, Q = \begin{bmatrix} 36 & -\frac{1}{4} & \frac{15}{8} \\ -\frac{1}{4} & 2 & -1 \\ \frac{15}{8} & -1 & 1 \end{bmatrix}$$

Design observer:

$$\begin{cases} \dot{\hat{t}}_1 = \hat{t}_2 - \frac{9}{2}(\hat{t}_1 - t_1) \\ \dot{\hat{t}}_2 = \hat{t}_3 - \hat{t}_2 - \frac{15}{4}(\hat{t}_1 - t_1) - \hat{t}_2^3 \\ \dot{\hat{t}}_3 = \theta - \frac{1}{2}\hat{t}_3 - \frac{15}{8}(\hat{t}_1 - t_1) - \hat{t}_3^3 - \hat{t}_2^2 \hat{t}_3 \end{cases}$$

According to the design steps given above, control θ is:

$$\theta = -z_2 - z_3 + \hat{t}_3^3 + \hat{t}_2^2\hat{t}_3 + \frac{15}{8}(\hat{t}_1 - t_1) + \frac{1}{2}\hat{t}_3 + \frac{\partial \varepsilon_2}{\partial \hat{t}_2}\dot{\hat{t}}_2$$

$$\dot{\hat{\Theta}} = (\frac{\| P \|^2}{\varepsilon_0} + \frac{3}{2})\eta^2(f(t))$$

Here $z_1 = f(t), z_2 = \hat{t}_2 - \varepsilon_1(f(t), \hat{\Theta}), z_3 = \hat{t}_3 - \varepsilon_2(\hat{t}_1, \hat{t}_2, f(t), \hat{\Theta})$ and $\varepsilon_1, \varepsilon_2$ are:

$$\varepsilon_1 = -f(t) - \frac{1}{4\varepsilon} f(t)(\frac{\| P \|^2}{\varepsilon_0} + \frac{3}{2})\overline{\eta}^2(f(t))\hat{\Theta}$$

$$\varepsilon_2 = -z_2 + \hat{t}_2^3 + \frac{15}{4}(\hat{t}_1 - t_1) + \hat{t}_2 + \frac{\partial \varepsilon_1}{\partial f(t)}\hat{t}_2 -$$

$$\frac{1}{4\varepsilon}(\frac{\partial \varepsilon_1}{\partial f(t)})^2 z_2 + \frac{1}{2\varepsilon}\frac{\partial \varepsilon_1}{\partial f(t)} z_1 - z_1 - \frac{1}{2} z_2(\frac{\partial \varepsilon_1}{\partial f(t)}) + \frac{\partial \varepsilon_1}{\partial \hat{\Theta}}\dot{\hat{\Theta}}$$

Set the initial state $t_1(0) = 0, t_2(0) = 0.2, t_3(0) = 0.5, \hat{t}_1(0) = \hat{t}_2(0) = \hat{t}_3(0) = 0, \hat{\Theta}(0) = 0$; Parameter $\varepsilon_0 = 0.14, \varepsilon = 0.18$. The simulation results are shown in Figure 1-2. Figure 1-2 verifies the correctness of the results in this paper, indicating that the controller is effective.

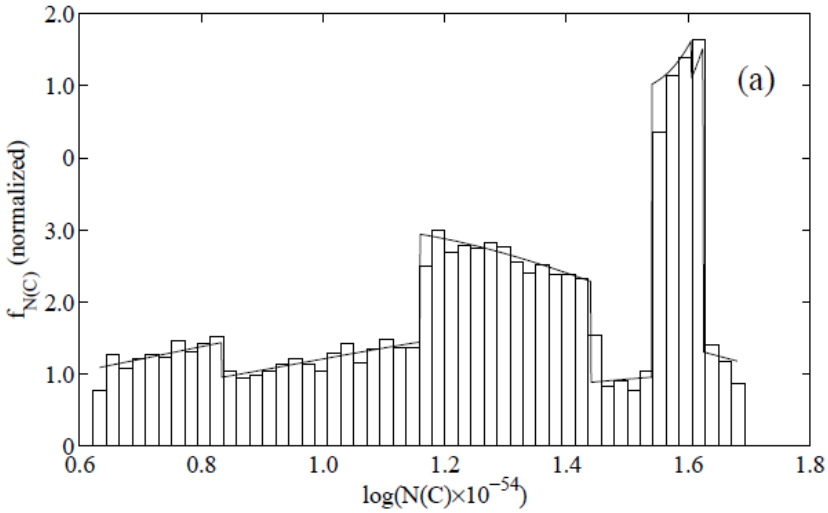

Fig. 1 System state t_1 (solid line) and observer state $\hat{t}_1$ (dashed line)

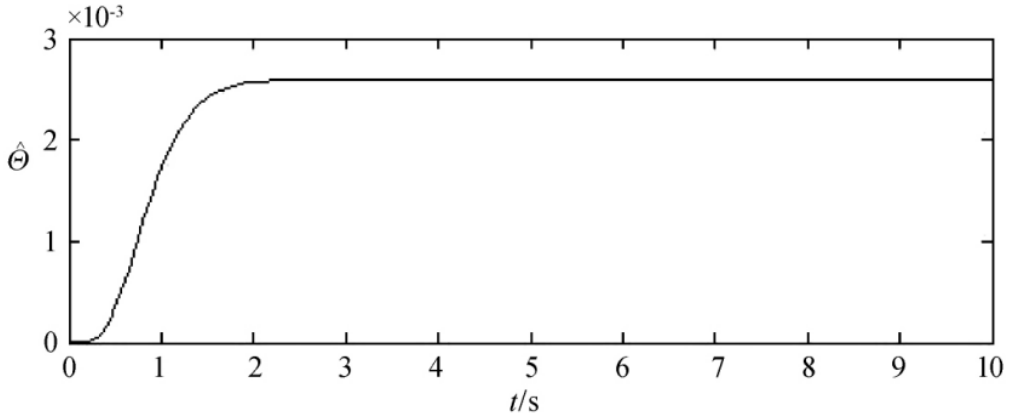

Fig. 2 Estimate $\hat{\Theta}$ of parameter Θ

5. Conclusion

A new multidimensional Popov criterion is proposed. When multiple unmodeled dynamics occur at the same time, the paper gives a robust absolute stability criterion as simple and clear as the multivariate Popov criterion. The solution of the multidimensional Popov criterion is a method to solve the optimal problem.

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