Experimental study on natural droop of materials with transverse stress

Yuexiang Wu[1, a], Muyang Chen[1, b], Qiyu Zhang[1, c], Hong Gao[2, d]

[1]No.2 High School of East China Normal University

[2]Dalian Maritime University

[a]3354884839@qq.com, [b]1393745700@qq.com, [c]jack_zqy@icloud.com, [d]hgao@dlmu.edu.cn

Abstract. In this study, it is found that the catenary structure of materials with transverse stress is deviated from the hyperbolic cosine function derived by Euler equation. In our experiments, we verify the principle of minimum potential energy and find that the natural suspension potential energy is not necessarily the minimum for non-ideal suspension iron ropes with transverse stress (such as high-voltage ropes or suspension bridges).In other words, the variable of the principle of least action in the suspension line is not necessarily the gravitational potential energy only, but may also be the sum of gravitational potential energy and elastic potential energy in materials with high horizontal stress. Finally, the phenomenological proof of the minimum potential energy of the catenary is given through calculation and drawing.

Keywords: Euler equation, suspension line, principle of minimum potential energy.

1. Introduction

In nature, flexible materials that present bundles will naturally sag, often in the form of catenary, for example, the chain hanging from the railings, the spider silk hanging with dew, the rope hanging from the high voltage line, the ship anchor iron rope used when mooring the ship anchor, and so on.

In architecture, human beings refer to the laws of nature; Different kinds of ropes and even rigid structures also exist in the form of catenary, or simulate the form. Hooke proposed the application of catenings to buildings in 1675. The English translation of his Latin text was, 'As sorry a flexible iron rope so, inverted, stand the touching pieces of an arch.'[1]

Many modern arches have inverted catenary arches[2] such as the Keleti railway station in Budapest, St. Louis Arch in Missouri, USA[3];There are buildings whose roofs mimic the catenaries, such as the Washington Dulles Airport terminal[4]The horizontal tension created by the catenary design eliminates the need for more columns inside the terminal concourse, making the interior space very open, such as the Portugal Pavilion at the 1998 Lisbon World Expo[5], its roof is built in the form of catenary ropes.

2. Theoretical analytical solution of uniform catenary problem

A , B , and λ are all integral constants [6]

$$y = \lambda + A\cosh\frac{x+B}{A} \tag{1}$$

The above equation uses the following conditions: 1. The linear density is constant, 2. The hanging material is not expandable, 3. In the above derivation, suspension points A and B don't need to be of equal height, that is, ideal uniform catenary lines are all hyperbolic cosine lines.

3. Experimental and theoretical comparative study of catenary problem

About the formula of the catenary line, most of the ideal catenary as the premise, whether it form in the shape of catenary line for soft materials of different textures? This is a question worth studying. In addition, the derivation of catenary equation in mathematics is mostly based on the premise that

the total potential energy of the iron rope is the minimum when it is in catenary state. This condition is also demonstrated in textbooks, but there has never been a strict mathematical proof and experimental study on this condition[7-12].In this paper, we will conduct an experimental comparative study on the potential energy value of natural suspension and artificial shape change suspension of chains and iron ropes under different materials and stresses, to explore whether the potential energy value of natural suspension reaches the minimum total potential energy.

3.1. Experimental Scheme

3.1.1 Layout of the experimental site and experimental content

1.Select a $80cm * 80cm$ blank white wall, rope (iron rope, cotton rope, each of three kinds of diameter), iron chain (uneven thickness, different materials), adhesive scale, Weighing scale with 0.01g scale, laser level.

2.Display the horizontal and vertical laser on the wall with the laser level and stick the adhesive scale to the wall along the laser.

3.Record the coordinates of each marking point by the laser level after it is naturally lowered and recorded in a table form.(x_0, y_0)

4.The subjects of natural drooping should conform to the hyperbolic cosine equation (the three undetermined parameters are taken as any three points out of about 50 points in Step 3 to be solved by the following equation) :$y = a\,cosh\frac{x-b}{a} + ca, b, c(x_1, y_1), (x_2, y_2), (x_3, y_3), a, b, c$

$$\begin{cases} a\,cosh\frac{x_1-b}{a} + c - y_1 = 0 \\ a\,cosh\frac{x_2-b}{a} + c - y_2 = 0 \\ a\,cosh\frac{x_3-b}{a} + c - y_3 = 0 \end{cases} \tag{2}$$

5.Calculate total potential energy by formula (this step only appears in the section dealing with verification of minimum potential energy)

$$V = \sum_{i=1}^{N} m_i g y_i = \sum_{i=1}^{N} \frac{m}{N} g y_i = \frac{m}{N} g \sum_{i=1}^{N} y_i \tag{3}$$

(m is the total weight, G is the acceleration due to gravity, and N is the number of segments divided).

3.1.2 Applying Quasi-Newton's Method to solve nonlinear equations (used to solve Equation 2)

In this work, we use quasi-Newton's method to solve nonlinear equations. The $Fortran$ code of Xu Shi liang program is referred. [13]

We verify the convergence of the calculation. In the verification calculation, we find that our program can find the solution of the equation in6 iterations, and the accuracy requirement is 10^{-7}.

3.2. Uniform rope catenary experiment

3.2.1 Natural catenary experiment and theoretical comparison of iron ropes with constant height hanging points

The horizontal stress of the rope is larger than that of the chain. This section explores the influence of horizontal stress and the horizontal deformation generated by the stress on the form of the overhang line at different scales. In order to make a clear contrast, two kinds of rope materials are selected in this section: three kinds of cotton rope with different diameters and three kinds of iron rope with different diameters. In the experiment, the number of points of the cotton rope is as similar as possible to better compare the corresponding deviation degree.

In this section, in order to quantitatively compare the deviation degree between different diameters of cotton rope and iron rope and the theoretical curve, the goodness of fit between the theoretical curve and the experimental curve in each experiment in this section is obtained with the program to

qualitatively compare the deviation degree between the rope and the actual catenary in this section. The formula is the goodness-of-fit index selected in this section(3) [14] The formula.

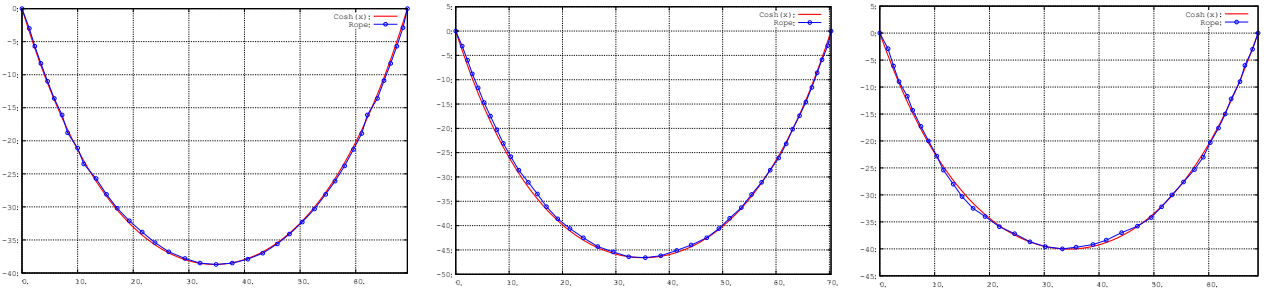
$$R_{NL} = 1 - \sqrt{\frac{\sum(y_i - \hat{y}_i)^2}{\sum {y_i}^2}} \tag{4}$$

Where $y_i$is the original data and $\hat{y}_i$ is the data provided by the fitted curve. And with each$N$ element are regarded as two points in the N dimensional space, namely, $\sqrt{\frac{\sum(y_i - \hat{y}_i)^2}{\sum {y_i}^2}}$ is the distance between two points, $\{y_i\}$ and $\{y_i\}$, divided by the distance from the origin, and the relative error is taken into account. The goodness-of-fit index $R_{NL} \leq 1$, the closer it is to 1, the higher the fit degree R_{NL} is. It has the advantages of high resolution and high accuracy.

When it comes to the cotton rope with natural hanging points of equal height, its horizontal stress is small, and the characteristics of the horizontal stress value does not change much with the diameter determine the need for a quantitative comparison of the degree of deviation of each hanging line from the theoretical curve. At the same time, the quantitative value provided by the goodness-of-fit can also better reflect the degree of deviation from the theoretical curve among different dangling lines when testing the material of iron rope with large horizontal stress and obvious variation of horizontal stress value with diameter.

In this section, the fitting degree of cotton rope and iron rope to the theoretical curve will be compared with the fitting degree of the corresponding thin/medium/thick diameter chains. This helps to visualize the difference between a hypothetical chain that fits the theory perfectly and a hypothetical rope that does not fit the theory well.

The following is an experiment with a natural catenary of equal height with cotton rope. The blue curve is the actual measured data obtained in the experiment, and the red curve is the theoretical curve of the hyperbolic cosine equation.



A. Practical fitting of experiment and ideal state of thin cotton rope

B. Practical fitting of experiment and ideal state of medium cotton rope

C. Practical fitting of experiment and ideal state of thick cotton rope

Figure 1. Practical fitting of experiment and ideal state of thick/medium/thin cotton rope

A: Thin cotton rope hanging naturally

39 points of the thin cotton rope are taken, and $(0,0), (34.9, -38.7), (69.3,0)$ will be used as the three characteristic points to find the parameters, and the solution obtained is: $a = 19.8652, b = 34.65, c = -58.5668$ As shown in FIG.1, for the thin cotton rope with less horizontal stress, the rope produced by it fid the theoretical curve. The goodness-of-fit index of the thin cotton rope is $R_{NL} = 0.988674700$compared with $R_{NL} = 0.985538304$ of thin cotton rope, and the difference is 0.003136396. This is the reflection of the weak stress and elastic deformation of the thin cotton rope itself.

B: Medium cotton rope hanging naturally

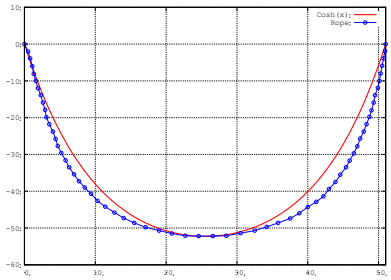
41points of the medium cotton rope are taken, and $(0,0), (33.5, -46.6), (70.3,0)$ will be used as the three characteristic points to find the parameters, and the solution obtained is: $a = 18.026, b = 35.15, c = -64.6294$As shown in FIG.1, for the medium cotton rope with less horizontal stress, the rope produced by it fid the theoretical curve. The goodness-of-fit index of the medium cotton rope is $R_{NL} = 0.980098009$ compared with $R_{NL} = 0.989943981$ of medium cotton rope, and the

difference is0.009845972. This is the reflection of the weak stress and elastic deformation of the medium cotton rope itself.

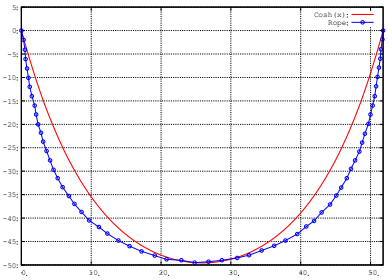
C: Thick cotton rope hanging naturally

38points of the thick cotton rope are taken, and $(0,0),(33.3,-40),(69,0)$ will be used as the three characteristic points to find the parameters, and the solution obtained is: $a=19.2793, b=34.5, c=-59.3167$As shown in FIG.1, for the thick iron cotton with less horizontal stress, the rope produced by it fid the theoretical curve. The goodness-of-fit index of the thick cotton rope is $R_{NL}=0.981392086$ compared with $R_{NL}=0.989615142$ of thick cotton rope, and the difference is0.008223056. This is the reflection of the weak stress and elastic deformation of the thick cotton rope itself.

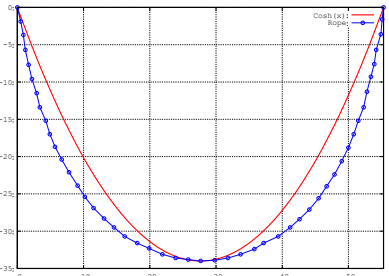
For cotton rope, a material with greater horizontal stress than iron chain, a certain elastic deformation will be generated, and the deviation of the catenary to the theoretical curve increases with the increase of the diameter. The goodness-of-fit difference between the thick cotton rope and the thick chain is smaller than that of the chain in the medium cotton rope comparison. In the following, the iron rope was taken as the control group to explore the deviation of the measured catenary from the theoretical curve caused by different diameters under the significant increase of horizontal stress. The catenary of the iron rope material simulates all kinds of ropes in practice. Compared with cotton rope, more points are taken to obtain more accurate comparison of deformation degree of iron rope.



D. Practical fitting of experiment and ideal state of thin iron rope



E. Practical fitting of experiment and ideal state of medium iron rope



F. Practical fitting of experiment and ideal state of thick iron rope

Figure 2. Practical fitting of experiment and ideal state of thick/medium/thin iron rope

D: Thin iron rope hanging naturally

64points of the thin iron rope are taken, and $(0,0),(26.6,-52.2),(51,0)$ will be used as the three characteristic points to find the parameters, and the solution obtained is: $a=10.209, b=25.5, c=-62.4683$ As shown in FIG.2, for the thin iron rope with less horizontal stress, the rope produced by it fid the theoretical curve. The goodness-of-fit index of the thin iron rope is $R_{NL}=0.870019495$ compared with $R_{NL}=0.988674700$ of thin iron rope, and the difference is0.118655205. It reflects the obvious deviation between the horizontal stress of the thin iron iron rope and the theoretical curve.

E: Medium iron rope hanging naturally

62points of the medium iron rope are taken, and $(0,0),(24.8,-49.5),(51.8,0)$ will be used as the three characteristic points to find the parameters, and the solution obtained is: $a=10.7477, b=25.9, c=-60.304$ As shown in FIG.2, for the medium iron rope with less horizontal stress, the rope produced by it fid the theoretical curve. The goodness-of-fit index of the medium iron rope is $R_{NL}=0.812448740$ compared with $R_{NL}=0.989943981$ of medium iron rope, and the difference is0.177495241. It reflects the obvious deviation between the horizontal stress of the medium iron rope and the theoretical curve.

F: Thick iron rope hanging naturally

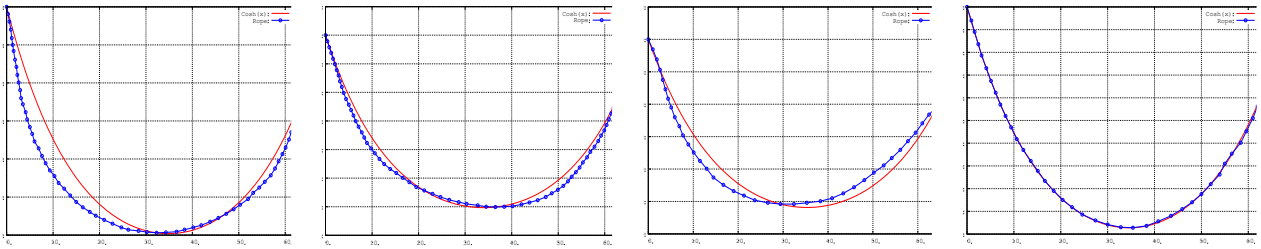
62points of the thick iron rope are taken, and $(0,0),(28.1,-33.1),(55.1,0)$ will be used as the three characteristic points to find the parameters, and the solution obtained is: $a=15.0449, b=27.55, c=-48.155$As shown in FIG.2, for the thick iron rope with less horizontal stress, the rope

produced by it fid the theoretical curve. The goodness-of-fit index of the thick iron rope is $R_{NL} = 0.851326764$ compared with $R_{NL} = 0.989615142$ of thick iron rope, and the difference is0.138288378. It reflects the obvious deviation between the horizontal stress of the thick iron rope and the theoretical curve.

3.2.2 Natural suspension experiment and theoretical comparison of ropes' ends hanging at different height

In this period, we use thin rope, medium rope, thin cotton rope and thick cotton rope with different diameters to carry out the experiment of hanging each end of the rope at different height. For anchor ropes, thin rope, thick rope, thin rope, cotton rope, and thick cotton rope are used.

The experiment of hanging each end of the rope at different height is given below. The blue curve is the data measured in the experiment, and the red curve is the theoretical curve of the hyperbolic cosine equation.



A. Practical fitting of experimental and ideal state of medium iron rope

B. Practical fitting of experimental and ideal state of thin iron rope

C. Practical fitting of experimental and ideal state of thick cotton rope

D. Practical fitting of experimental and ideal state of thin cotton rope

Figure 3. Practical fitting of experiment and ideal state of thin/medium rope and thin/thick cotton rope

A: Ends of the medium iron rope hanging at different height naturally

62points of the medium iron rope are taken, and $(0,0), (34.1, -58.2), (61.3, -28.2)$ will be used as characteristic points. The solution obtained is:$a = 15.3369, b = 34.5032, c = -73.5422$. As shown in FIG. 3, for the medium iron rope with the strongest horizontal stress, the curve generated by it basically do not fit the theoretical curve, and only a few points in the right of the lowest point are on the theoretical curve, which completely do not conform to the theoretical prediction, showing a conclusion similar to the experiment of the isometric iron rope. The goodness-of-fit index of the medium iron rope is $R_{NL} = 0.862172008$compared with $R_{NL} = 0.993817985$of the medium iron rope, and the difference is0.131645977. It reflects the obvious deviation between the horizontal stress of the medium iron rope and the theoretical curve.

B: Ends of the thin iron rope hanging at different height naturally:

64points are taken for the thin iron rope, and $(0,0), (36.5, -60.1), (62.9, -21.5)$ will be used as the three characteristic points. The solution obtained is: $a = 14.6995, b = 34.0035, c = -75.012$. As shown in FIG. 3, for the thin iron rope with the second strongest horizontal stress, the curve produced by it also do not fit the theoretical curve, showing a similar conclusion to the experiment of the isometric iron rope. The goodness-of-fit index of the thin rope is $R_{NL} = 0.943470597$compared with $R_{NL} = 0.991088450$ of the thin rope, and the difference is0.047617853. It reflects the obvious deviation between the horizontal stress of the thin rope and the theoretical curve.

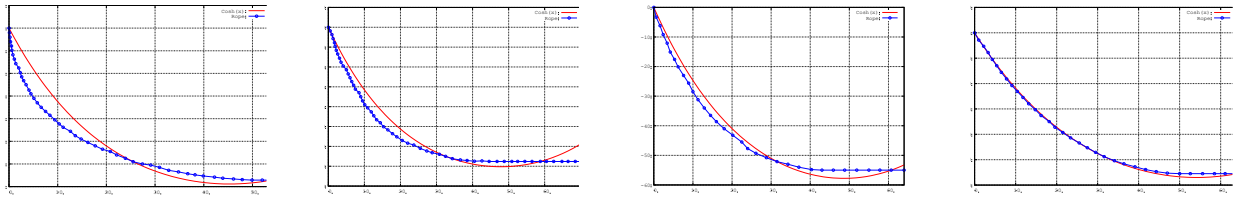
C: Ends of the thick cotton rope hanging at different height naturally:

38points are taken for the thick cotton rope, $(0,0), (29.4, -50.8), (64.3, -20.7)$ will be used as the characteristic points for the calculation. The solution obtained is:$a = 17.0346, b = 35.3405, c = -68.881$. As shown in FIG. 3, for the thick cotton rope with a certain horizontal stress, the curve produced by it do not conform to the theoretical curve at all, and only the feature points are at the position of coincidence. On the one hand, it may be due to the irregular elastic deformation of the thick cotton rope itself. The goodness-of-fit index of the thick cotton rope is $R_{NL} = 0.910009384$compared with $R_{NL} = 0.992560863$ of the high and low hanging of the thick iron rope, and the difference is0.082551479. This reflects the obvious deviation between the horizontal

stress of the rough cotton rope and the theoretical curve, but the deviation is smaller because the horizontal stress is less than that of the medium iron rope.

D: Ends of the thin cotton rope hanging at different height naturally:

39points are taken from the thin cotton rope$(0,0),(35.4,-48.6),(63,-19)$ will be used as the characteristic points, and the solution obtained is:$a=17.0937,b=34.57,c=-65.7138$.As shown in FIG. 3, the curve produced by the thin cotton rope with small horizontal stress conform to the theoretical curve and tend to the analytical solution. The goodness-of-fit index of the thin cotton rope is $R_{NL}=0.992502093$compared with $R_{NL}=0.991088450$ of the thin iron rope, and the difference is. -0.001413643. It can be seen that there is basically no difference.



E. Practical fitting of experimental and ideal state of thick iron anchor rope

F. Practical fitting of experimental and ideal state of thin iron anchor rope

G. Practical fitting of experimental and ideal state of thick cotton anchor rope

H. Practical fitting of experimental and ideal state of thin cotton anchor rope

Figure 4. Practical fitting of experiment and ideal state of thin/thick iron rope and thin/thick cotton anchor rope

E: Thick iron anchor rope hanging naturally:

49points are taken for the thick iron rope, $(0,0),(25.5,-59),(53.9,-67.1)$ will be used as the three characteristic points. The solution obtained is: $a=21.3891,b=45.3098,c=-90.2374$. As shown in FIG. 4, for the thick iron rope with the largest horizontal stress, the curve produced by it does not fit the theoretical curve at all. The goodness-of-fit index of the thick iron rope is $R_{NL}=0.847534239$compared with $R_{NL}=0.993073523$ of the thick rope ship anchor rope, and the difference is0.145539284. It shows that the horizontal stress of the thick iron rope leads to obvious deviation from the theoretical curve.

F: Thin iron anchor rope hanging naturally:

57points of the thin iron rope are taken, and$(0,0),(34.4,-66.2),(58.8,-67.6)$ will be used as characteristic points. The solution obtained is:$a=23.0659,b=47.8631,c=-93.3078$. As shown in FIG.4, for the thin iron rope with less horizontal stress, the curve produced by it still does not fit the theoretical curve at all. The goodness-of-fit index of the thin iron rope is $R_{NL}=0.913465083$ compared with $R_{NL}=0.988196135$ of thin iron anchor rope, and the difference is0.074731052. It shows that the horizontal stress of the thin rope leads to obvious deviation from the theoretical curve, but because the horizontal stress of the thin rope is smaller than that of the thick rope, the degree of deviation from the theoretical curve is also reduced.

G: thick cotton anchor rope hanging naturally:

33points are taken for the thick iron rope, $(0,0),(31.5,-52.1),(60.7,-55)$ will be used as the three characteristic points for the calculation, and the solution obtained is: $a=26.7554,b=48.626,c=-84.5263$. As shown in FIG. 4, for the thick cotton rope with smaller horizontal stress, the curve produced by it still does not fit the theoretical curve very well. The goodness-of-fit index of the thick cotton rope is $R_{NL}=0.940017939$compared with $R_{NL}=0.993073523$ of the thick anchor rope, and the difference is0.053055584. It shows that the horizontal stress of the rough cotton rope leads to obvious deviation from the theoretical curve, but because the horizontal stress of the rough cotton rope is smaller than that of the thin iron rope, the degree of deviation from the theoretical curve continues to decrease.

H: Natural suspension of thin cotton rope ship anchor rope:

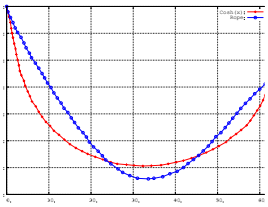
34 are taken for the thin cotton rope, $(0,0),(31.6,-48.8),(64,-55.5)$ will be used as the three characteristic points. The solution obtained is: $a=32.3537,b=54.1775,c=-89.3563$. As shown in FIG. 4, for the thin cotton rope with small horizontal stress, the curve produced by it is

relatively close to the theoretical curve, with only the rightmost part slightly warped. The goodness-of-fit index of the thin cotton rope is$R_{NL} = 0.984460533$compared with $R_{NL} = 0.988196135$ of the thin anchor rope, and the difference is0.003735602. The deviation between the horizontal stress of thin cotton rope and the theoretical curve is basically small.

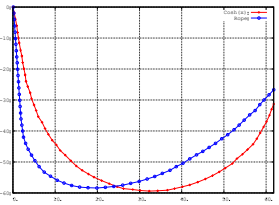
The results of four groups of experiments show that for the iron rope with small variation of horizontal stress with diameter, if the anchor rope experiment is set as the thin rope in which the points are taken relatively dense, in large horizontal span, and low vertical span, then it can be more consistent with the theoretical curve. But for medium and thick ropes of small horizontal span, large vertical span, and relatively sparse points taken, the deviation from the theoretical curve was both on the right end of both ropes, which proved that the bottom end of some anchor ropes in real life would also have the corresponding deviation from the theoretical curve.

3.2.3 Experiment and verification of minimum potential energy of non-iso-height non-natural suspension rope

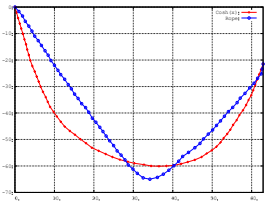
In this section, the thinnest and thickest ropes are selected to reflect the difference of potential energy changes under different diameters to the greatest extent. The shapes of ropes in each experiment are manually changed twice. For the first time, the lowest point of the original natural hanging rope is given a downward tension to mimic the common hanging rope and suspension bridge in life. For the second time, a downward tension is given to a certain point on one side of the original natural suspension rope, mimicking catenary under irregular force in life, such as the cable hanging the cable car. For the ship anchor rope because it is not possible to give a downward pull to the lowest point, we choose to give a lateral pull force to a point in the medium section before the anchor rope reaches the bottom level, so that the force point is lower than its original position, and mimic the situation that the middle section of the anchor rope is fixed. After the force added, the potential energy of the force point itself is lower than the original potential energy, and the change of the total potential energy of the rope is unknown. The data graphs are given below, and then the potential energy change is discussed separately. In this section, the two hanging points of the thick iron rope and the thin iron rope are consistent with the corresponding ones in to ensure the accuracy of the potential energy comparison. In this section, the potential energy of the ropes hanging at different height is discussed first, then the potential energy of the ship anchor rope. The blue curve is the data measured in the stretching experiment, and the red curve is the data of the natural suspension catenary.



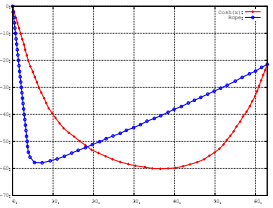
A. Practical fitting of experimental and ideal state of medium iron rope



B. Practical fitting of experimental and ideal state of medium iron rope



C. Practical fitting of experimental and ideal state of thin iron rope



D. Practical fitting of experimental and ideal state of thin iron rope

Figure 5. Comparison of two unnatural droop patterns of thick iron rope and thin iron rope with their natural shape

62 points are taken for the medium iron rope, the weight is $49.14g$, when it is hung naturally$\sum_{i=1}^{62} y_i = -24.23m$ and the gravitational potential energy is$-188.16J$. When the iron rope is pulling down$\sum_{i=1}^{62} y_i = -24.39m$, and the gravitational potential energy is$-189.48J$, which is smaller than the potential energy of the natural suspension; when the iron rope is laterally pulled down$\sum_{i=1}^{62} y_i = -23.76m$, the gravitational potential energy is$-184.56J$, which is greater than the potential energy under the natural suspension.

64points are taken for the thin iron rope. The weight of the iron rope is$6.12g$, and the gravitational potential energy is -23.07$J$when it is suspended naturally$\sum_{i=1}^{64} y_i =$ -24.61m. When the thin iron rope is pulled down$\sum_{i=1}^{64} y_i = -24.04m$, the gravitational potential energy is$-22.53J$, which is

greater than the potential energy of the natural suspension; When the thin steel rope is laterally pulled down$\sum_{i=1}^{64} y_i = -22.51m$, the gravitational potential energy is$-21.09J$, which is greater than the potential energy when it is suspended naturally.

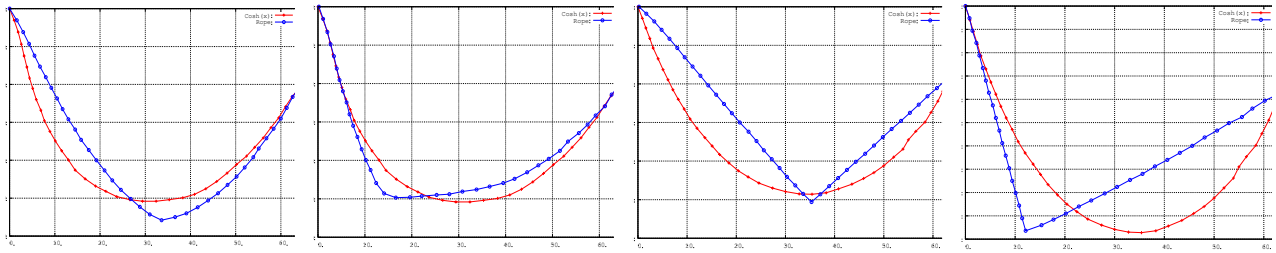
Note here that the potential energy of the medium iron rope with bigger stress down pulled is smaller than that of the natural hanging status, which is consistent with the conclusion in(3.3.2), that the transverse stress leads to the accumulation of elastic potential energy and does not reach the minimum value.

A visual comparison is given in the table below, from which it is observed that, for the medium rope$V_{downward} < V_{natural} < V_{lateral}$, which is consistent with the conclusion obtained for the medium-high hanging points in(3.3.2). It is proved again that the premise that the potential energy of a naturally suspended catenary is minimum under the principle of least action is not true in some cases where the horizontal stress is not negligible.

Table 1. The lowest potential energy of ropes with different diameters

Type / hang	Medium iron rope	Thin iron rope
Natural	$-188.16J$	-23.07J
Down pulled	$-189.48J$	$-22.53J$
Laterally pulled	$-184.56J$	$-21.09J$

The cotton rope case is discussed below.



E. Practical fitting of experimental and ideal state of thick cotton rope

F. Practical fitting of experimental and ideal state of cotton rope

G. Practical fitting of experimental and ideal stateexperimental and ideal state of thin cotton rope

H. Practical fitting of experimental and ideal state of thin cotton rope

Figure 6. Comparison of the two unnatural drooping patterns of thick and thin cotton rope with the original shape

38points of thick cotton rope are taken, the weight is$25.13g$, and the gravitational potential energy is $-81.76J$when hanging naturally$\sum_{i=1}^{38} y_i = -12.62m$. When the thick cotton rope is pulled down$\sum_{i=1}^{38} y_i = -12.91m$, and the gravitational potential energy is$-83.70J$, which is smaller than the potential energy when it is suspended naturally; When the cotton rope is pulled laterally down $\sum_{i=1}^{38} y_i = -12.59m$, the gravitational potential energy is$-81.60J$, and is greater than the potential energy of the natural suspension.

39points of the cotton rope, and the weight is$0.8g$, and the gravitational potential energy is $-2.45J$ when it is suspended naturally $\sum_{i=1}^{39} y_i = -12.17m$. When the cotton rope is pulled down$\sum_{i=1}^{39} y_i = -10.85m$, the gravitational potential energy is$-2.24J$, which is greater than the potential energy when the rope is suspended naturally. When the thin cotton rope is laterally pulled down$\sum_{i=1}^{39} y_i = -11.25m$, the gravitational potential energy is$-2.26J$, which is greater than the potential energy when the rope is suspended naturally.

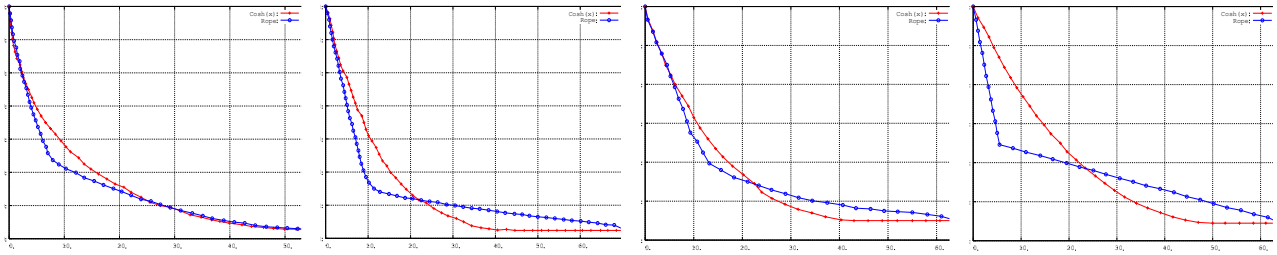
Notice here that the potential energy of the thick cotton rope with high stress down pulled is smaller than that of the natural hanging status, which is consistent with the conclusion in (3.3.2). The transverse stress leads to the accumulation of elastic potential energy and does not reach the minimum value.

The following table provides an intuitive comparison, from which it is observed that, for the thick cotton rope$V_{downward} < V_{natural} < V_{lateral}$, the medium iron ropes are same to the medium iron ropes in this section(3.3.2) It is proved that the premise of the minimum potential energy under the principle of least action for a naturally suspended dangling rope is not true in some cases where the horizontal stress is not negligible.

Table 2. Verification of the lowest potential energy of high and low ropes of cotton ropes with different diameters

Type / hang	Thick cotton rope	Thin cotton rope
Natural	$-81.76J$	$-2.45J$
Down-pulled	$-83.70J$	$-2.24J$
Laterally-pulled	$-81.60J$	$-2.26J$

Below explore the conclusions related to the ship's anchor rope.



I. Practical fitting of experimental and ideal state of thick iron rope

J. Practical fitting of experimental and ideal state of thin iron rope

K. Practical fitting of experimental and ideal state of thick cotton rope

L. Practical fitting of experimental and ideal state of thin cotton rope

Figure 7. Comparison of two unnatural droop patterns of thick and thin cotton rope with the original shape

50 points of the thick iron rope of the ship anchor rope are taken, the weight is$397.4g$, and the gravitational potential energy is $-1613.36J$when it is suspended naturally$\sum_{i=1}^{50} y_i = -20.30m$. When the the ship anchor rope thick iron rope is laterally pulled down$\sum_{i=1}^{50} y_i = -21.17m$, the gravitational potential energy is$-1649.17J$, which is smaller than the potential energy under natural suspension.

57 points of the thin iron rope of the ship anchor rope are taken, the weight is $6.12g$ and the gravitational potential energy is $-27.50J$when it is suspended naturally$\sum_{i=1}^{57} y_i = -26.13m$. When the thin iron rope of the anchor rope is laterally pulled down$\sum_{i=1}^{57} y_i = -25.23m$, the gravitational potential energy is$-26.55J$, which is greater than the potential energy under natural suspension.

33points of the thick cotton rope of the ship anchor rope are taken, the weigh is$25.13g$, and the gravitational potential energy is $-91.97J$when it is suspended naturally$\sum_{i=1}^{33} y_i = -12.32m$. When the ship anchor rope cotton rope is laterally pulled down$\sum_{i=1}^{33} y_i = -12.06m$, the gravitational potential energy is$-89.99J$, which is greater than the potential energy under natural suspension.

34points of the thin cotton rope of the ship anchor rope are taken, the weight is$0.8g$, and the gravitational potential energy is $-2.82J$when it is suspended naturally$\sum_{i=1}^{34} y_i = -12.23m$. When the ship anchor rope thin cotton rope is laterally pulled down$\sum_{i=1}^{34} y_i = -11.93m$, the gravitational potential energy is$-2.75J$, which is greater than the potential energy when the ship anchor rope thin cotton rope is pulled down on the side.

Note here that the lateral pull-down potential energy of the thick iron anchor rope is smaller than that of the non-stressed state, which is consistent with the conclusion in (3.3.2) and this section that the transverse stress leads to the accumulation of elastic potential energy and does not reach the minimum value.

The following table gives an intuitive comparison, from which it is observed that for thick iron rope, the conclusion is consistent with the medium iron rope, and the thick cotton rope in(3.3.2) It is proved that the premise of the minimum potential energy of the natural hanging rope under the principle of least action is not true in some cases where the horizontal stress is not negligible. So far, this conclusion has been experimentally observed in three types of draping ropes: constant hanging points, high and low hanging points, and ship anchor ropes. It can be proved that most of the materials with high horizontal stress in daily life do not conform to the assumption of minimum potential energy when they naturally sag.

Table 3. Verification of the minimum potential energy of ship anchor ropes with iron and cotton ropes of different diameters

Type / hang	Thick cotton rope	Thin cotton rope	Thick rope	Thin iron rope
Natural	$-91.97J$	$-2.82J$	$-1613.36J$	$-27.50J$
Laterally pulled	$-89.99J$	$-2.75J$	$-1649.17J$	$-26.55J$

4. Conclude and look ahead

To sum up, for materials with large horizontal stress, such as medium thick iron rope, medium iron rope, and thick cotton rope, none of them meet the assumption of the minimum potential energy theory when under natural non-stressed state in the potential energy verification experiment. The transverse horizontal stress leads to the accumulation of elastic potential energy, so that the minimum potential energy value is not reached. The elastic potential energy, which cannot be ignored, leads to changes in gravitational potential energy, which may not be the only variable of the least action principle in the suspension. Perhaps for the suspension with large horizontal stress, the potential energy in the least action principle is the sum of gravitational potential energy and elastic potential energy.

The fact that the hyperbolic cosine based on the premise of the minimum potential energy equation could not completely applied to ropes of certain materials proves that the bigger horizontal stress of the material the suspension rope has, the bigger the deviation from the obtained theoretical curve hyperbolic cosine equations is.

In this paper, a large number of experiments have found that the material with high horizontal stress does not conform to the premise of the principle of minimum gravitational potential energy under natural suspension, so the fit degree to the theoretical curve is low, and the deviation degree is increasing with the increase of horizontal stress. Therefore, the results of the verification of the minimum potential energy principle in the experiment and the fitting of the theoretical curve with the natural suspension can support each other.

References

[1] Block P, DeJong M, Ochsendorf J. As hangs the flexible line: Equilibrium of masonry arches[J]. Nexus Network Journal, 2006, 8(2): 13-24.

[2] flickr. [EB/OL]. (Unknown) [2022-07-29] https://www.flickr.com/photos/20792787@N00/3241785698

[3] Smithsonian Magazine. [EB/OL]. (Unknown)[2022-7-29] https://www.smithsonianmag.com/history/story-st-louis-gateway-arch-180956624/

[4] Visit Loudoun. [EB/OL]. (Unknown) [2022-7-29] https://www.visitloudoun.org/listing/washington-dulles-international-airport-%28iad%29/192

[5] e-architect. [EB/OL]. (Unknown) [2022-7-29] https://www.e-architect.com/portugal/portuguese-pavilion-expo-lisbon

[6] Liang Kunmiao. Mechanics: the next volume of theoretical mechanics [M]. 4th edition. Beijing: Higher Education Press 2010:239-241.

[7] Wikipedia. Catenary [EB/OL]. (2022-06-10)[2022-07-25]. https://en.wikipedia.org/wiki/Catenary

[8] Zhou Peiyuan. Theoretical Mechanics [M] 2nd edition. Beijing: Science Press 2012

[9] Ju Guoxing. Study guide and exercise analysis of theoretical mechanics: for science [M] Second edition. Beijing: Science Press 2018:475-476.

[10] Jin Shangnian and Ma Yongli. Theoretical mechanics [M] 3rd ed. Beijing: Higher Education Publishing House 2002

[11] Zhang Jianshu, Sun Xiuquan, Zhang Zhengjun. Theoretical Mechanics [M] 1st ed. Beijing: Science Press 2005

[12] Zhou Lezhu. A concise tutorial on theoretical mechanics [M]. Beijing: Beijing University Press 2005

[13] Xu Shiliang. FORTRAN common algorithm program set [M]. Tsinghua University Press 1995

[14] Zhang S. Q. A discussion of the goodness-of-fit index of curve regression [J]. Chinese Health Statistics, 2002, 19(1): 9-11.