The Relationship between the Caster Angle and the Steering Stability and the Measurement Method

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Abstract. The size of the front wheel caster angle of the vehicle is closely related to the stability of high-speed steering. By establishing the vehicle two-degree-of-freedom model and the steering system force input dynamic equation, the relationship calculation formula between the front wheel caster angle and the steering stable limit speed is derived by using the Routh-Hurwitz stability criterion. Taking the specific data of a certain model as an example, the range of speed values that can maintain stability is analyzed. The accuracy of the model will be further verified by means of simulation. This paper also summarizes the relevant model analysis and calibration techniques for the measurement of the back inclination of the main sales of automobiles, and can obtain more accurate experimental data by improving the accuracy of the measurement. Thus providing a guiding basis for the design of the rear inclination angle of the main sales of the car.

Keywords: Vehicle handling stability, caster angle, Vehicle steering stability.

1. Introduction

In today's rapid development of the automobile industry, the four-wheel positioning parameter is still the same Challenges in automotive design calculations. Among them, the size of the caster angle is a very important performance indicator for the car. On the one hand, from the perspective of car design, the caster angle is an important indicator affecting the steering stability of the car. On the other hand, the abnormality of these parameters will have a great adverse impact on the driving safety and handling stability of the vehicle, and precise detection methods are required to ensure that the parameter values remain normal. This article introduces the design and detection method of the caster angle in these two aspects.

Vehicle handling stability is an important evaluation index of vehicle active safety, which is affected by many factors in the process of vehicle steering, such as the position of the vehicle center of mass, suspension performance and main pin positioning parameters. As an important Kingpin parameter, the caster angle is of great significance to the steering stability of a vehicle. According to the study of Reference [1], a smaller caster angle makes steering easier, but less stable at high speeds, while a larger caster angle means a slower steering with a more pronounced return to normal but better stability at high speeds, the determination of the magnitude of the caster angle is very important for the ride comfort of the vehicle. So the balance between high-speed stability and steering lightness is a major issue in the design of the caster angle of the vehicle.

High speed and large steering are a working condition that a car must, it is also an important working condition that tests the stability of the car. References [2-3] respectively established mathematical expressions of stable limit speed and total tire tow distance when the vehicle was steered, but did not perform quantitative analysis of steering stable limit speed and front wheel caster angle. Reference [4] shows that the initial front wheel positioning parameters of the car, including the caster angle, and their dynamic changes have a very large impact on the handling stability. In this paper, a preliminary simulation calculation of the sensitivity of the model to the wheel alignment parameters is carried out through the established vehicle model and the double wishbone independent suspension model. The simulation shows that when designing a vehicle, the different effects of the high-speed vehicle and low-speed vehicle and the initial wheel positioning angle on the dynamic performance should be fully considered. Reference [5,6] further derives the expression of matching the front wheel caster angle with the stable limit speed, establishes a mathematical model of the

positive moment and the positive resistance moment around the main pin after the front wheel is steered, and quantitatively analysis the relationship between the value of the front wheel caster angle and the stable limit speed. The matching expression between the caster angle of the front wheel kingpin and the stable limit vehicle speed is derived. By using this formula, this paper carry out quantitative calculation and experimental research on the prototype vehicle, and obtain the corresponding relationship between the caster angle of the front wheel kingpin and the stable vehicle speed. On the basis of these studies, Reference [7] studies the relationship between vehicle speed and stable angle by numerical analysis from a nonlinear model. Considering the influence of the kingpin caster angle on the handling stability of the vehicle under extreme working conditions, it has theoretical guiding significance for the rational selection of the caster angle in the design.

The above basically establish mathematical equations from the perspective of theoretical derivation to solve the relationship between the caster angle and the driving stability of the vehicle. There have also been many research advances in numerical simulations. Reference [8] uses the ADAMS software to establish a multibody simulation model of the multibody car dynamics, and the different performances of the steering characteristics of the car under these conditions are obtained through simulation calculations at different speeds and different loads. It is found that when the vehicle has understeer characteristics, the steady-state yaw rate does not change much, and the vehicle speed is the main factor for the vehicle's steady-state steering characteristics. The internal relationship between vehicle steering characteristics and vehicle speed, load and tires is revealed, which provides a reference for the analysis of vehicle handling stability. Reference [9] uses ADAMS software to establish a multi-body dynamic model of a SUV model, and conducted front wheel positioning parameters and dynamic simulation studies and vehicle handling stability simulation analysis of the front suspension and vehicle of the model. Specifically, the law of the change of the front wheel positioning parameters of the front suspension with the wheel runout is studied. These studies provide conditions for further study of the caster angle and vehicle steering stability, as well as the verification of theoretical derivation results.

On the other hand, being able to accurately measure the size of the caster angle and its changes is important for the subsequent maintenance and feedback of the vehicle. Therefore, it is very necessary to build an accurate measurement model. In Reference [10], based on spatial geometric analysis, a mathematical model of inclination verification of the main sales force of the four-wheel aligner was established. Through the analysis and derivation of the geometric relationship of the model, the main factors affecting the accuracy of the inclination of the main sales force are obtained. In Reference [11], the general design of the 3D four-wheel aligner and the principle of measuring the front wheel caster angle are given. On this basis, a commercial-grade calibration device is developed for the 3D four-wheel aligner to verify the accuracy of the 3D four-wheel aligner. The structure is simplified and the accuracy of device detection is improved. According to the analysis of the error detection results of the horizontal turntable and the calibration frame, the feasibility of the four-wheel aligner calibration device is proved. In Reference [12], the design scheme of the measuring instrument with higher accuracy is given through mathematical derivation, which can ensure that the front wheel caster angle has a relatively complete detection mechanism. This can also be combined with the above-mentioned research to better calculate the maximum steering stabilization speed and the design of auxiliary cars.

This paper first establishes the relationship between the caster angle and the maximum steering speed of the car through mathematical derivation, and then summarizes the reference content and introduces the current measurement method of the caster angle. From these two aspects, the caster angle of the car is introduced.

2. METHOD

2.1. The Steering stability model

To determine the relationship between the kingpin caster angle and the maximum stable steering speed, a two-degree-of-freedom linear vehicle dynamic equation with lateral and yaw motion around the center of mass is established:

$$\begin{aligned}(k_1+k_2)\beta+\frac{1}{v}(ak_1-bk_2)\omega_r-k_1\delta=mv(\frac{d\beta}{dt}+\omega)\\(ak_1-bk_2)\beta+\frac{1}{v}(a^2k_1+b^2k_2)\omega_r-ak_1\delta=mab\frac{d\omega}{dt}\end{aligned} \tag{1}$$

The steering system is simplified as the rotation around the front wheel kingpin, and the friction and viscous resistance moments of the system are ignored:

$$2\xi k_1(\beta+\frac{a\omega}{v}-\delta)+I_\delta\frac{d^2\delta}{dt^2}=T_\omega \tag{2}$$

Regarding the determination of the total towing distance of the front wheel, Wei Daogao et al. in the Reference [13] believes that the total drag spacing of the front wheel is composed of the front wheel tire drag distance and the main pin backward drag distance, and the front wheel main pin is approximately through the center of wheel, that is, the main pin tow distance can be described as:

$$\xi=\xi_1+R\sin\alpha \tag{3}$$

while Zhao Yilei et al. in the Reference [1] give a more accurate formula for the air tire drag distance, and the article believes that the tire drag distance is 1/6 of the grounding imprint length.

$$\xi=\xi_{\mathrm{t}}+\xi_{\mathrm{c}}=\frac{l}{6}+\sin\alpha r \tag{4}$$

For the length of the ground imprint in it, the document uses the semi-empirical formula recommended by G.Komd, that is:

$$l=\sqrt{(D-\Delta)\Delta} \tag{5}$$

Where

$$\Delta=\frac{CQ(0.5G_1)^{0.85}}{b^{0.7}D^{0.45}p^{0.6}} \tag{6}$$

Where: C is the coefficient related to the tire structure; Q is the coefficient related to the tire width; G1 is the vertical load of the front axle; b is the tire width, mm; p is the tire air pressure, kPa; D is the width of the tire ground contact stress, mm.

This paper takes the formula (3) for the further analysis.

The expression (3) is substituted for the expression (2)

$$\begin{aligned}&2(\xi_1+R\sin\alpha)k_1(\beta-\delta)+\\&\frac{2}{v}ak_1(\xi_1+R\sin\alpha)\omega+I_\delta\frac{d^2\delta}{dt^2}=T_\omega\end{aligned} \tag{7}$$

Take the steering System dynamics equation as a complement to the vehicle dynamics equation, take the (1),(7) equations as a set of equations, apply the Laplace transform to them to get the characteristic equation:

$$\begin{vmatrix} k_1+k_2-mvs & ak_1-bk_2-mv^2 & -k_1 \\ ak_1-bk_2 & a^2k_1+b^2k_2-mvab & mabs \\ 1 & a & \frac{I_\delta s^2}{2k_1(\xi_1+R\sin\alpha)}-1 \end{vmatrix} = 0 \quad (8)$$

Using the Routh-hurwitz stability criterion, the matching expression between the caster angle and the maximum steady speed of the vehicle steering is obtained [7]:

$$\alpha = \arcsin\left(\frac{I_\delta(k_1a+k_2b)}{k_1mabR}\left(2-\frac{k_2l^2}{mav^2}\right)-\frac{\xi_1}{R}\right) \quad (9)$$

It can be seen from this equation that when the caster angle is reduced, the maximum stable steering speed will increase accordingly. Therefore, the minimum caster angle at different speeds can be determined by this equation. When the caster angle of the vehicle is less than this value, the vehicle will be unstable. At the same time, through this formula, it is understood that the design calculation of the front wheel caster angle should consider its reasonable matching relationship with the stable steering speed.

Table 1. Parameters in the equation

Parameter	Meaning
ξ	Total towing distance for the front wheels
I δ	Steering system moment of inertia
T w	Steering wheel input torque
ξ1	Pneumatic tire dragging distance
R	Wheel roll radius
α	Front wheel caster angle
k1, k2	Front and rear wheel side stiffness
β	Centroid lateral declination
v	stable limiting Steering speed
a, b	The distance from the centroid to the front and rear axes
ω	The angular velocity of the pendulum around the centroid
δ	Front wheel steering angle
m	Mass of the vehicle

Table 1 lists the physical quantities used in the derivation of formulas and their physical significance.

2.2. The caster angle measurement method

For this highly accurate equation, there are requirements for the measurement accuracy of the front wheel caster angle. When the measurement accuracy is improved, a more accurate maximum steering speed can be obtained, which further assists the design of the car. Four-wheel aligner is now commonly used to measure the value of the front wheel caster angle. In this regard, the error analysis

and calibration method of the measurement model are given in the Reference [12]. The accurate measurement model of the front wheel caster angle was established by mathematical deduction：

$$(P^2+1)\sin^2\lambda_1 - 2PQ\sin\lambda_1 + Q^2 - 1 = 0 \tag{10}$$

Where

$$P = (\tan\beta\cos\delta + \sin\delta\tan\gamma)^{-1} \tag{11}$$

$$Q = P(\sin\alpha + \cos\alpha\tan\gamma) \tag{12}$$

Here the α is the wheel camber, the β is the value of the kingpin inclination angle, and λ_1 is the sensor measurement angle when the wheel turns right. From the formula, it can be found that the front wheel caster angle and the kingpin inclination angle are coupled. For the four-wheel aligner currently used, a linear formula is used to measure the caster angle of the kingpin, without considering the influence of the kingpin inclination angle. Only the relationship between the kingpin caster angle and the sensor measurement angle λ is shown. Through further experimental research, it is found that when the kingpin inclination angle is large, the existing linear model will produce a large error when measuring the front wheel caster angle, which cannot meet the measurement accuracy and needs to be improved.

This reference also gives the source of the measurement error of the kingpin caster angle, pointing out that it is mainly composed of the linearity system error and the zero point error. The zero point error is mainly caused by the misalignment of the initial calibration of the sensor. On the one hand, the systematic error is due to that when the kingpin inclination is large, it has a significant influence on the measurement of the kingpin's backward inclination, resulting in a large error in the measurement of the kingpin's backward inclination. On the other hand, there is a nonlinear relationship between the kingpin caster angle and the angle measured by the caster sensor due to the increase of the kingpin caster angle. The analysis of these errors also provides ideas for the subsequent improved design of the measurement of the kingpin caster angle.

Reference [11] presents a detailed study of the traditional four-wheel aligner calibration instrument, found that the traditional calibration instrument could not accurately measure the value of the main pin inclination. Therefore, a new 3D four-wheel aligner calibration device scheme is proposed, which is realized by independent adjustment of the kingpin caster angle and the inclination angle of the kingpin based on the projection of the kingpin of the vehicle in the horizontal and vertical planes, and the mechanical design is given. Especially, this paper proposes a cross shaft structure, so that the adjustment of the front wheel caster angle and the kingpin inclination angle can be completed independently, avoiding interference when adjusting each of the two angles. This reference also proposes a photoelectric encoder to read the detection results, which simplifies the structure and improves the detection accuracy of the device.

The comprehensive application of these studies can solve the problems existing in the existing four-wheel aligner and improve the measurement accuracy of the front wheel caster angle.

3. Results

Based on the above equation in 2.1, we also plotted the equation and got the following figure. The formula is programmed in Python and plotted with the test data of the vehicle. Test data refer to the values used in Reference [5]. Table 2 lists the specific values used in this study. The changing image of the caster angle of the Kingpin and the maximum speed of the vehicle can be:

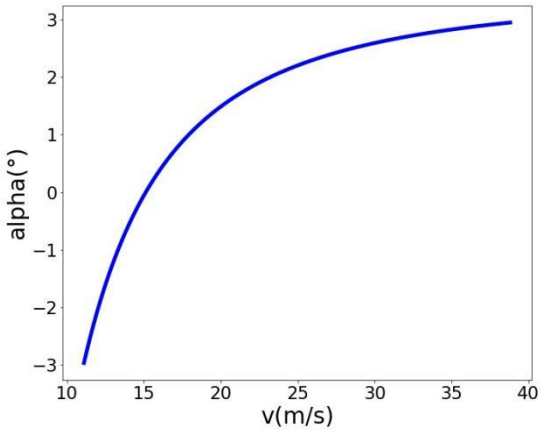


Figure 1. caster angle with max steering speed

Table 2. Parameters of a sample vehicle for calculating caster angle

m /kg	a/ mm	b/ mm	k1/ N·rad-1	k2/N ·rad-1	l/ mm	I δ/kg	ξ1 /mm	R/ mm	v/k m·h-1
1600	1350	1300	44950	55060	2650	3 0	- 5~20	37 0	40 ~140

Figure 1 indicates that when the speed is low, the car kingpin caster angle size and speed is positively correlated, but when the speed is high, the change of caster angle of kingpin tends to smooth. It can be seen that if you want to increase the maximum steering stabilization speed, the value of increasing the front caster angle is obvious at the beginning, and when it continues to increase, there is a boundary effect. Therefore, the proper selection of the value of the main sales back inclination angle will play a very important role in the design of the car.

Taking the test data of this model as an example, the design value of the caster angle of the front wheel kingpin is 2.0 ° ± 20'. When the speed is 27 m/s, that is, 97.2 km/h, the value of the caster angle reaches the upper limit of the design value. Therefore, it can be judged that the safety of steering can be ensured when the driving speed is below 97.2 km/h.

This figure depicts the relationship between the car's caster angle and the extreme turning speed, and through this figure, we can derive the relationship of the speed when the caster angle is different. This can also assist the car in the design of the car's caster angle by the value of the maximum turning speed.

4. Conclusion

From the perspective of mathematical modeling, the research of previous references is synthesized. The two-degree-of-freedom vehicle model and its formula are widely used in the design of vehicle driving system, so the derived model is also universal, using this more accurate model design, more accurate relationship between caster angle and steering speed can be obtained, which is convenient for high precision design of automobile. At the same time, the reasonable speed range is more beneficial to the driver to improve the handling stability and ride comfort, improve the driving experience.

On the other hand, this paper describes the relationship between the caster angle and the steering stability of the car and the progress of the measurement technology of the caster angle, and the combination of these two aspects helps to more fully understand the important role of the selection of the backlash angle of the main salesman for the design of the car, and considers the allowable error range of the design through the accuracy of the measurement.

In this problem, the size of the drag distance is a fixed value, but in the actual working condition, the car's no-load or full load, steering angle and so on will affect the drag distance length. In the future,

the values of drag distance under different working conditions can be further determined by experiment or simulation. In this part, using relevant software for the establishment and testing of virtual prototypes can be considered, such as the use of ADAMS for analysis, and ensure that the simulation content is more consistent with the actual situation.

In addition, for the measurement of the caster angle, it can also be seen from the references that in terms of measurement accuracy, existing measuring instruments are still not satisfied in some cases. This requires more design improvements and experimental validation.

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